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Cost sharing: efficiency and implementation

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Abstract

We study environments where a production process is jointly shared by a finite group of agents. The social decision involves the determination of input contribution and output distribution. We define a competitive solution when there is decreasing-returns-to-scale which leads to a Pareto optimal outcome. Since there is a finite number of agents, the competitive solution is prone to manipulation. We construct a mechanism for which the set of Nash equilibria coincides with the set of competitive solution outcomes. We define a marginal-cost-pricing equilibrium (MCPE) solution for environments with increasing returns to scale. These solutions are Pareto optimal under certain conditions. We construct another mechanism that realizes the MCPE. © 1999 Elsevier Science S.A. All rights reserved.

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1. Introduction

The sharing of costs is prevalent in many facets of economic activity. Large enterprises allocate overhead costs among various departments. Members of a university share the cost of a software site license. The parties watching a pay-per-view boxing match share the fee.

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Issues of cost sharing surface explicitly as well as implicitly. The problem arises explicitly whenever a group of individuals jointly uses a common resource or undertakes a joint project (for an excellent survey, see Young, 1994). A cost-sharing method arises implicitly in any private-ownership competitive economy. In such an economy, an individual's share of the production costs of a firm is the amount he pays for goods purchased minus the profits he earns from shares owned in that firm.

The properties of the cost-sharing method are of major concern. Does it lead to efficient outcomes? Is the outcome unique? Can it be manipulated by the individuals involved? Moulin and Shenker (1992) provide the serial cost-sharing method and have demonstrated its appeal as far as manipulability and uniqueness are concerned. Moulin and Shenker (1994) and Moulin and Watts (1997) analyze more traditional methods such as average cost sharing. Both serial and average cost sharing do not guarantee Pareto optimality. Our contribution is to present two mechanisms that will generate Pareto optimal outcomes for several classes of environments. We start by observing that in a neoclassical economy, the implicit cost sharing mentioned above implies efficiency. A cost-sharing method that attempts to replicate a neoclassical economy by creating a fictitious firm would have two shortcomings: manipulation may exist with a finite number of individuals and convexity of the technology is required. Both manipulation and non-convexity may lead to an undesirable outcome, while non-convexity may also lead to nonexistence of equilibria. We suggest two cost-allocation mechanisms that obtain efficient outcomes by imitating implicit cost sharing and addressing both problems.

The manipulation issue is resolved in part by eliminating the market power possessed by the individuals. To address the nonexistence issue, we resort to marginal-cost-pricing equilibria that exist for a large class of non-convex environments (see Brown, 1991).

The paper proceeds as follows. In Section 2, we present the competitive solution. In Section 3, the competitive solution is implemented and the resulting mechanism is compared to other cost-sharing methods. In Section 4, we address the problems created by increasing returns to scale. Finally, in Section 5, we conclude the paper and mention further directions of research.

2. Allocation of costs—the competitive solution

The large variety of cost-allocation problems makes it intractable to present a general method to allocate costs efficiently. In this section, we define the class of environments that we analyze in this paper. Then, we introduce the competitive solution concept for this class. We show that for a subclass the competitive solution yields efficient allocations only when ignoring possible manipulation by individuals.

We consider a class of cost allocation problems [1] where there is a finite number N of individuals, greater than 2, that consume two goods, x and y, and have access to technology $c: \Re_+ \to \Re_+$, where $c(y')$ is the cost of producing y' units of good y. The preferences of each individual i can be represented by a utility function $u^i(x^i, y^i)$ where utility is strictly increasing, differentiable, concave and satisfies the Inada conditions ($\lim_{x^i \to 0} u_1^i(x^i, y^i) = \lim_{y^i \to 0} u_2^i(x^i, y^i) = \infty$, $\lim_{x^i \to \infty} u_1^i(x^i, y^i) = \lim_{y^i \to \infty} u_2^i(x^i, y^i) = 0$). The individuals are endowed with strictly positive amounts w^i of good x and none of good y. An allocation is given by a $2N+2$-tuple $((x^i, y^i)_{i=1}^N, x_p, y_p)$ where the first N components denote the individuals' consumption levels and the last two components the production levels. The allocation is feasible if:

$$\sum_{i=1}^{N} x^i + x_p \leq \sum_{i=1}^{N} w^i$$

$$\sum_{i=1}^{N} y^i \leq y_p$$

$$c(y_p) \leq x_p.$$

A feasible allocation is Pareto optimal if there does not exist a feasible allocation which makes no individual worse off and at least one individual strictly better off.

We will say the cost-allocation problem belongs to class D (I) if the cost function is differentiable and convex (concave), with $c(0) = 0$. In the first case, we are in the decreasing-returns-to-scale scenario, whereas in the second case, production is characterized by increasing returns to scale.

In order to define a competitive solution, we need to create a firm that owns the technology and endow individuals with strictly positive ownership shares a^i. A competitive solution is a feasible allocation $((x^{i\prime}, y^{i\prime})_{i=1}^N, x'_p, y'_p)$ and a price p' (for good y in terms of good x) such that given the price p', the firm is maximizing its profits and the individuals are maximizing their utility subject to their budget constraints:

(x'_p, y'_p) solves

$$\max_{x_p, y_p} p' y_p - x_p$$

$$\text{s.t. } c(y_p) \leq x_p$$

[1] These problems encompass both traditional cost sharing (when individuals demand outputs and the mechanism determines inputs) and surplus sharing (when individuals supply inputs and the mechanism determines outputs). Our mechanism can be interpreted as a hybrid construction since it determines both inputs and outputs.

$(x^{i\prime}, y^{i\prime})$ solves

$$\max_{x^i, y^i} u^i(x^i, y^i)$$
$$\text{s.t. } x^i + p' y^i \leq w^i + a^i \pi_p$$

where π_p denotes the profits of the 'firm'.

When the competitive solution exists (this is guaranteed only in class D), the standard arguments leading to the First Welfare Theorem show that the competitive solution yields a Pareto optimal allocation.

Proposition 1. *All competitive solutions for a given cost allocation problem are Pareto optimal*.

Restricting attention to the class D of cost-allocation problems, the competitive solution exists and generates an allocation with an implicit sharing of costs. The cost of production implicitly imposed on individual i is the difference between i's expenditures on y and i's share in the profit. The specific allocation realized depends upon the ownership structure and may not possess an axiomatic characterization like several cost allocation methods put forward in the literature. It is, however, Pareto optimal. This may seem to be a viable method to reach an efficient cost allocation, but strategic behavior by the individuals may undermine the efficiency. An individual, by misrepresenting his preferences, may be able to secure an outcome preferable to the competitive cost allocation achieved with his true preference. Several papers have addressed the incentives problem inherent in the Walrasian paradigm for pure exchange economies (Hurwicz, 1979; Schmeidler, 1980; Postlewaite and Wettstein, 1989) and for production economies (Hong, 1995). In Section 3, we offer a continuous and feasible mechanism that would realize the competitive solution to the cost allocation problem. All the Nash equilibria of this mechanism yield Pareto optimal outcomes.

3. Realization of the competitive solution (mechanism A)

Mechanism A consists of an n-tuple of strategy sets and an outcome function mapping strategies into allocations. The strategy space of individual i is $S^i = \Re_{++} \times \Re^2 \times \Re \times \Re_{++}$ with a generic element denoted by (p^i, c^i, t^i, r^i). The first component is a price for commodity y submitted by individual i, the second is a net consumption bundle, the third is an input level into the production process, and the fourth is a number used in averaging out the possibly conflicting demands of all the individuals.

We will now outline the way mechanism A operates informally, before we formally describe it. The mechanism constructs an average price based on the announced prices and an average production plan based on the announced production plans. The required amount of input, specified in the production plan, is collected from the individuals and used in production. Individual budget sets are constructed based on the average price and the profits generated from the production plan. The consumption bundles requested are projected onto these budget sets. The resulting bundles may not be feasible in the aggregate, but aggregate feasibility is reached by scaling down the bundles.

We assume the individuals are completely informed as regards the technology, preferences and endowments. We also assume the designer knows the individuals' initial endowments, but we do not assume that the designer knows the technology. [2] Notice that the firm is a fictitious entity, which is created in order to define the outcome function. It is thus controlled by the designer and has no strategic role.

We show that mechanism A has Nash equilibria that are all competitive solutions, thereby, yielding an efficient solution to the problem of cost allocation. In order to present more clearly the formal description of the mechanism, we will proceed in several steps even though the mechanism itself is a one-stage game.

Step 1: An average price $\bar{p}$ is constructed as follows.

Define:

$$\alpha^i = \sum_{t,t' \neq i} |p^t - p^{t'}|^2; \qquad \alpha = \sum_{i=1}^{N} \alpha^i$$

$$\beta^i = \frac{\alpha^i}{\alpha} \qquad \alpha > 0$$

$$= \frac{1}{N} \qquad \alpha = 0$$

$$\bar{p} = \sum_{i=1}^{N} \beta^i p^i$$

The construction of $\bar{p}$ implies that if all individuals other than individual i announce the same price q, the average price constructed will be q and furthermore, individual i's announcement will have no effect on the price reached.

Step 2: The production plan (x_p, y_p) used by the mechanism is determined by:

$$(x_p, y_p) = \left(\min\left\{ \sum_{i=1}^{N} w^i, \max\left\{ 0, \sum_{i=1}^{N} t^i \right\} \right\}, c^{-1}\left(\min\left\{ \sum_{i=1}^{N} w^i, \max\left\{ 0, \sum_{i=1}^{N} t^i \right\} \right\} \right) \right)$$

The appearance of the cost function in the production plan does not imply that the designer needs to know the technology. The operation of the mechanism

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[2] Knowledge of initial endowments can be relaxed at the cost of a more complicated mechanism that would handle destruction and withholding of initial endowments as in the work of Hong (1995). Furthermore, note that each individual needs only to know the set of preferences and endowments that exist in the entire society and not the specific preference or endowment for any particular (other) individual.

reveals the value of c at a single point—the production point yielded by the choices of the individuals.

Step 3: N individual budget sets are constructed (elements are net trades), based on the average price and the production plan:

$$B^i(\bar{p}) = \left\{ (z_1, z_2) \in \Re^2 \middle| \begin{array}{l} z_1 + \bar{p} z_2 \leq a^i(\bar{p} y_p - x_p) \\ z_1 + w^i \leq \sum_{i=1}^{N} w^i - x_p;\ z_1 + w^i \geq 0 \\ z_2 \leq y_p;\ z_2 \geq 0 \end{array} \right\}$$

Let v^i be the closest point in B^i to c^i. In order to insure the final allocation is feasible, the following set J is constructed:

$$J = \left\{ r \in \Re_{++} \middle| \begin{array}{l} r \cdot r^i \leq 1 \qquad \text{for } i = 1\ldots, N \\ r \sum_{i=1}^{N} r^i (v_1^i + w^i) \leq \sum_{i=1}^{N} w^i;\ r \sum_{i=1}^{N} r^i v_2^i \leq y_p \end{array} \right\}$$

Let $\hat{r} = \max_{r \in J} r$.

The N bundles allocated to the individuals by the mechanism are:

$$\gamma_1^i = \hat{r} \cdot r^i (v_1^i + w^i); \quad \gamma_2^i = \hat{r} \cdot r^i v_2^i \qquad \text{for } i = 1, \ldots, N.$$

The mechanism in addition to the individuals' utility functions constitutes a well defined game. We analyze the Nash equilibria of games resulting from our mechanism A. Several other solution concepts like subgame perfect equilibria (Moore and Repullo, 1988; Abreu and Sen, 1990 and more recently Varian, 1994), equilibria in undominated strategies (Palfrey and Srivastava, 1991) and virtual 'equilibria' (Matsushima, 1988 and Abreu and Sen, 1991) have been used to analyze mechanisms in the literature. Mechanisms relying on these solution concepts may require more stringent informational assumptions or larger strategy spaces. Next, we will show that all Nash equilibria generated by our mechanism give rise to Pareto optimal allocations.

Proposition 2. *For any cost allocation problem in D, the Nash equilibria of the mechanism constructed above yield a competitive solution that is Pareto optimal.*

Proof. Denote $\bar{p}'$, x_p', y_p', $r^{i\prime}$, $\hat{r}'$ and $(x^{i\prime}, y^{i\prime})_{i=1}^{N}$ as the values and allocations generated at the Nash equilibrium point. We show this is a competitive solution via the following lemmata.

Lemma 1. *Individual i can get arbitrarily close to any point θ in $B^i(\bar{p}')$.*

Proof. Announcing the net trade leading to θ as c^i and a large enough r^i will generate an outcome arbitrarily close to θ. The large r^i nullifies the effect of all the other terms in the construction, and the calculation of the final bundles allocated to the individuals will leave individual i arbitrarily close to θ. ■

Lemma 2. *Individual i can generate a* $B^i(\bar{p}')$ *that contains net trades leading to strictly positive consumption bundles for himself given any choice of strategies by the other individuals.*

Proof. Since $w^i > 0$ and $\bar{p}' > 0$ individual i can, by sending in an appropriate t^i, force a production plan that has x_p and y_p strictly positive and yields a positive income level for consumer i (even if profits are always negative, it is possible to choose a small enough production level to guarantee positive income). This implies that $B^i(\bar{p}')$ contains net trades that lead to strictly positive consumption bundles for individual i. ■

Lemma 3. *The equilibrium allocation must be strictly interior* $((x^{i\prime}, y^{i\prime})_{i=1}^N \in \Re^{2N}_{++})$.

Proof. Assume by way of contradiction, there exists an individual i for whom $(x^{i\prime}, y^{i\prime}) \notin \Re^2_{++}$. By Lemma 2, individual i can, by sending in a possibly different t^i, obtain a $B^i(\bar{p}')$ that contains net trades leading to strictly positive consumption bundles. Any one of those consumption bundles is strictly preferred to $(x^{i\prime}, y^{i\prime})$. By Lemma 1 and continuity of preferences, there exists an obtainable consumption that is preferred to the equilibrium consumption. This contradicts that individual i was playing a Nash equilibrium strategy. Hence, the equilibrium outcome entails a strictly interior allocation. ■

Lemma 4. *The production plan* (x'_p, y'_p) *maximizes profits under price* $\bar{p}'$.

Proof. Assume, by way of contradiction, there is a production plan $(\tilde{x}_p, \tilde{y}_p)$ that yields higher profits. Any point of the form $(\lambda x'_p + (1-\lambda)\tilde{x}_p, \lambda y'_p + (1-\lambda)\tilde{y}_p)$ with $0 < \lambda < 1$ yields higher profits by convexity of the cost function. By Lemma 3, there exists a λ close enough to 1, where such a point is feasible. By Lemma 2, any individual could obtain this point by altering the t^i message. Thus, individual i expands the $B^i(\bar{p}')$ set, and by Lemma 1 and continuity of preferences can obtain a preferred outcome, in contradiction to the original outcome being an equilibrium outcome. ■

Lemma 5. *The consumption plan* $(x^{i\prime}, y^{i\prime})$ *maximizes individual i's utility subject to the budget constraint with price* $\bar{p}'$ *and production plan* (x'_p, y'_p).

Proof. Assume, by way of contradiction, there is a consumption plan $(\tilde{x}^i, \tilde{y}^i)$ that satisfies individual i's budget constraint and is strictly better than $(x^{i\prime}, y^{i\prime})$. By Lemma 3, only the first constraint in $B^i(\bar{p}')$ can be binding at the point $(x^{i\prime}, y^{i\prime})$. By this fact, there exists a λ close enough to 1 such that $(\lambda x^{i\prime} + (1-\lambda)\tilde{x}^i, \lambda y^{i\prime} + (1-\lambda)\tilde{y}^i)$ belongs to $B^i(\bar{p}')$. By convexity of preferences, this point is preferred to $(x^{i\prime}, y^{i\prime})$. By Lemma 1 and continuity of preferences, this contradicts that individual i is playing a Nash equilibrium strategy. ■

By Lemmata 4 and 5, any equilibrium is a competitive solution and by Proposition 1, this solution yields a Pareto optimal allocation. ■

The result of Proposition 2 may be vacuously satisfied if the mechanism suggested does not possess any Nash equilibria. We show that this is not the case. Given our assumptions, a competitive solution always exists. Proposition 3 shows that any competitive solution is a Nash equilibrium. Hence, the mechanism possesses a Nash equilibrium.

Proposition 3. *For any cost-allocation problem in D, the set of competitive solutions is contained in the set of Nash equilibria outcomes of mechanism A.*

Proof. Let $A' = ((x^{i\prime}, y^{i\prime})_{i=1}^N, x'_p, y'_p)$ and price p' constitute a competitive solution. A set of strategies realizing it is: $p^i = p'$; $c^i = (x^{i\prime} - w^i, y^{i\prime})$; $t^i = x'_p/N$; $r^i = 1$ for all i. This N-tuple of strategies yields the average price p' and the consumption–production allocation A'. We now show that these strategies form a Nash equilibrium, since they are best responses. First, we note that an individual is unable to change the price constructed, p'. Second, the production plan in A' maximizes profits given p'; thereby, an individual's choice of t^i gives him the largest budget set. Finally, the choice of c^i and r^i leads to the most preferred consumption bundle in the budget set. Therefore, changes in c^i, t^i or r^i will not improve upon the $(x^{i\prime}, y^{i\prime})$ outcome for individual i. ■

The main features distinguishing mechanism A from other cost-allocation methods is the Pareto optimality of the outcome reached and the relaxation of informational assumptions. In contrast to other cost-sharing methods, our mechanism by virtue of coinciding with competitive solutions yields Pareto optimal levels of y. Furthermore, its operation does not require the designer to know the technology, as assumed with serial cost sharing. Mechanism A, on the other hand, is not immune to coalitional deviations like the serial cost-sharing method and is more complex than the previous methods suggested.

The mechanism's optimality of outcomes and existence of a solution critically depend upon the assumption of convexity of the technology. This phenomena is parallel to the one encountered in a competitive economy. Section 4 specifies solution concepts appropriate for environments with increasing returns (non-convexities) and discusses their implementation.

4. Increasing returns to scale

In this section, we consider the allocation of costs in environments with increasing returns to scale. We use the analogy of these cost allocation problems to economies with increasing-returns-to-scale production to suggest a solution. A common construct for such production economies is a marginal-cost-pricing equilibrium (MCPE). This consists of dictating the production plan of the firm and allowing the individuals to purchase goods at marginal cost after paying for their share of the firm's losses. Existence of such equilibria under certain conditions has been shown in a series of papers (Mantel, 1979; Beato, 1982; Kamiya, 1988a; Bonnisseau and Cornet, 1990). Also, the optimality of these equilibria is guaranteed with stringent enough conditions on the curvature of the indifference curves and production possibility frontiers (Dierker, 1986; Quinzii, 1991). Further results can be found in the work of Cornet (1990). Hence, a device that leads to marginal-cost-pricing equilibria would be an interesting solution to cost-sharing problems.

As before, the standard construction ignores the possibility of manipulation by the individuals. To address this issue, we provide a continuous, feasible and finite-dimensional mechanism that realizes the MCPE solution. Calsamiglia (1977) demonstrates that in the presence of increasing returns to scale it is impossible to obtain Pareto optimal outcomes via a finite-dimensional mechanism. Our mechanism is compatible with this result, since the MCPE that it yields is not always Pareto optimal.

In order to define an MCPE solution, we create a fictitious firm and endow individuals with strictly positive ownership shares given by $a^i = \frac{w^i}{\sum_{j=1}^N w^j}$.[3] Alternatively, we can choose a desired share structure $\tilde{a}^i$ and redistribute endowments as $\tilde{w}^i = \tilde{a}^i \sum_{j=1}^N w^j$. An MCPE solution is a feasible allocation and a price of y where the price equals the marginal cost of production, the firm carries out the prescribed production plan and the individuals maximize utility subject to their budget constraints. These constraints incorporate both the price and their share of the (negative) profits. Formally, the MCPE solution is a feasible allocation $((x^{i\prime}, y^{i\prime})_{i=1}^N, x'_p, y'_p)$ and price p' where $p' = c'(y'_p)$ and $(x^{i\prime}, y^{i\prime})$ solves:

$$\max_{x^i, y^i} u^i(x^i, y^i)$$

$$\text{s.t. } x^i + p' y^i \leq w^i + a^i \pi_p$$

where π_p denotes the profits of the 'firm'.

[3] The creation of shares in this manner prevents individuals from going bankrupt when held responsible for the firm's losses. This is a version of the survival assumption, which appears in the existence proofs for MCPE (Mantel, 1979; Beato, 1982; Kamiya, 1988a; Bonnisseau and Cornet, 1990). A counter-example for nonexistence when the survival assumption does not hold is provided by Kamiya (1988b).

The cost of production imposed on individual i by this solution is the sum of i's expenditure on y and i's share in the losses.

We cannot achieve these outcomes in a straightforward manner due to possible misrepresentation of preferences by the individuals. In order to prevent these problems, we offer a continuous and feasible mechanism implementing MCPE solutions.

4.1. Mechanism B

Except for the construction of the budget sets, mechanism B is defined just like the mechanism implementing the competitive solution. The first constraint in the construction of $B^i(\bar{p}')$ in Section 2 is replaced with $z_1 + \bar{p}z_2 \leq 2a^i(\bar{p}y_p^{-i} - x_p^{-i}) - a^i(\bar{p}y_p - x_p)$, where:

$$\left(x_p^{-i}, y_p^{-i}\right) = \left(\min\left\{\sum_{i=1}^{N} w^i, \max\left(0, \sum_{j\neq 1} t^j\right)\right\},\right.$$

$$\left. c^{-1}\left(\min\left\{\sum_{i=1}^{N} w^i, \max\left(0, \sum_{j\neq 1} t^j\right)\right\}\right)\right)$$

The RHS is twice the profits of the firm at the production plan determined by the announcements of all individuals other than i minus the profits of the firm at the production plan determined by the announcements of all individuals, with all terms adjusted for feasibility.

In equilibrium, all individuals announce the same production plan. When this occurs, the designer can construct the budget sets with only the knowledge gained from producing the announced plan and does not need to know the whole technology (as before, the designer should be able to measure the output that is eventually produced). Outside of equilibrium, individuals may not announce the same production plan. In this case, the designer needs to know the technology at several points, that is, he should be able to discover the output that would be produced for several different levels of inputs in order to construct the outcome function. For some finite cost he should be able to obtain this information, for instance, by either rerunning the technology for several points or stopping the technology at several levels of input. Doing so for the entire curve may entail infinite costs, which would not be a credible option even outside of equilibrium. These requirements (both inside and outside equilibrium) are noticeably weaker than having to know the entire production technology. [4]

[4] An alternate route to revealing the entire technology can be achieved by appending to the mechanism a game similar to the Hurwicz–Maskin–Postlewaite (Hurwicz et al., 1995) construct.

The Nash equilibria resulting from mechanism B are shown to be MCPE solutions in Proposition 4. In Proposition 5, we show that any MCPE solution for the cost allocation problem can be realized as a Nash equilibrium of the mechanism.

Proposition 4. *For any cost-allocation problem in I, the Nash equilibria of mechanism B yield an MCPE solution.*

Proof. The proof coincides with the proof of Proposition 2 except for Lemmata 2 and 4.

Lemma 2. *$B^i(\bar{p}')$ contains net trades leading to strictly positive consumption bundles for individual i.*

Proof. We consider three distinct cases:

(i) Case 1: $0 < x_p^{-i} < \Sigma_{i=1}^N w^i$

Individual i can, by adjusting the t^i announcement, set $y_p = y_p^{-i}$ and $x_p = x_p^{-i}$, hence both x_p and y_p are strictly positive. Letting $z_1 = -w^i$ and $z_2 = 0$ turns the first inequality in the definition of $B^i(\bar{p}')$ into $-w^i \leq \frac{w^i}{\Sigma_{i=1}^N w^i}(\bar{p}' y_p - x_p)$ or $0 \leq \frac{w^i}{\Sigma_{i=1}^N w^i}(\Sigma_{i=1}^N w^i - x_p + \bar{p}' y_p)$. The RHS is strictly positive since $x_p < \Sigma_{i=1}^N w^i$. Hence, $B^i(\bar{p}')$ contains net trades where $z_1 > -w^i$ and $z_2 > 0$. These net trades lead to strictly positive consumption bundles for individual i.

(ii) Case 2: $x_p^{-i} = \Sigma_{i=1}^N w^i$

Once more, individual i can by adjusting the t^i announcement set $y_p = y_p^{-i}$ and $x_p = x_p^{-i}$. Individual i would then have an individual budget constraint for $B^i(\bar{p}')$ (mentioned above) that would allow for positive consumption of both goods. However, the aggregate constraints in $B^i(\bar{p}')$ would restrict the individual to receive zero consumption of the x good. By submitting in a smaller t^i such that $x_p < x_p^{-i}$, the individual can relax the aggregate constraints while still keeping the individual budget constraint not binding. This would allow strictly positive consumption of both goods.

(iii) Case 3: $x_p^{-i} = 0$

The first term on the RHS is zero leaving $-a^i(\bar{p}y_p - x_p)$. Since $w^i > 0$ and $\bar{p}' > 0$ individual i can, just as before, choose a strictly positive production plan that leaves him with strictly positive income (including w^i). Thus, $B^i(\bar{p}')$ contains net trades leading to strictly positive consumption bundles. ■

Lemma 4. *The production plan (x'_p, y'_p) is such that $c'(y'_p) = \bar{p}'$.*

Proof. The negative profit of the firm is in the individual budget constraint for $B^i(\bar{p}')$. By the same argument as in Lemma 4 of Section 3, the individual would be able to expand his budget set if losses were not maximized. Maximization of losses implies the condition $c'(y'_p) = \bar{p}'$. ■

This demonstrates that the Nash equilibria outcomes of mechanism B constitute MCPE solutions. ■

Proposition 5. *For any cost-allocation problem in I, the set of MCPE solutions is contained in the set of Nash equilibria outcomes of mechanism B.*

Proof. Similar to previous proofs. ■

5. Conclusions

In this paper, we study the allocation of costs for environments with both decreasing and increasing returns to scale. In the decreasing-returns-to-scale case, we construct a mechanism that leads to Pareto optimal outcomes, correctly recognizing the incentives of individuals. In the increasing-returns-to-scale case, Pareto optimality is harder to achieve. We construct a mechanism that leads to marginal-cost-pricing equilibria that generate Pareto optimal outcomes under certain conditions. The existence of such a construction further justifies the MCPE concept.

The outcomes of previously suggested mechanisms are not guaranteed to be Pareto optimal even in the decreasing-returns-to-scale case. Furthermore, in contrast to previous mechanisms, our cost-sharing mechanisms do not require the designer to know either the technology or individual preferences. Whether or not there exists a mechanism that is superior to ours for environments where MCPE outcomes fail to be Pareto optimal is a question of interest.

One also may be interested in the equity properties of our mechanism. Such issues can be used to determine share ownership with decreasing-returns-to-scale. This is not an option in our mechanism for increasing-returns-to-scale where share ownership is proportional to endowments; however, since profits are negative, individuals with higher endowments bear a larger cost.

The construction technique we use is not limited to the specified environment. Extending the mechanism to environments with more than two goods or multiple technologies is straightforward. Creating a similar mechanism to allocate resources when externalities are present is also possible. Modifying the mechanism to handle asymmetrically informed individuals involves major changes as well as moving to the Bayes–Nash equilibrium concept and remains a topic of further research.

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