Worker Discretion and Misallocation of Talent within Firms*

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Abstract

We develop a theory of worker discretion over task choice within a firm. Increasing the workers’ discretion has a trade-off between the gains from workers using private information about their abilities, and the costs from adverse selection within the firm due to workers herding into prestigious tasks. The theory leads to the result that, in line with the Peter Principle, misallocation of talent within firms takes the form of too many workers undertaking tasks with a high return to ability. Moreover we find that the degree of misallocation of talent is decreasing in the degree of discretion given to workers.

Keywords: Authority, Bureaucracies, Career Concerns, Discretion, Organizational Design, Misallocation, Peter Principle, Principal-Agent Theory, Sun Hydraulics, Wage Dynamics.

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1 Introduction

A key factor to the success of a company is the extent to which its employees work on tasks, or specialize, in accordance with their abilities. For example, hi-tech companies want their most talented programmers to work with creative tasks like new software development, and their less talented programmers to work with more routine tasks, like the updating of old software, user-support and code documentation.

In principle, a firm has two ways to ensure a good match between employees and tasks – to assign employees to tasks or to let the employees decide which tasks to work on. The traditional way of ensuring a good match between workers and work content, as evidenced by hiring procedures in government bureaucracies, is to first define the tasks contained in job slots and then to hire suitable workers (or to reallocate existing workers) to fill the slots, giving workers limited discretion in defining their work content. In recent years, the bureaucratic, ‘Weberian’, solution has been challenged by firms with innovative management practices. For example, as described by Baron & Kreps (1999), the engineering company Sun Hydraulics gives employees ‘the right and responsibility to choose how they spend their time,’ and Gore & Ass., the producer of Gore-Tex® products, encourages ‘maximum freedom for each employee.’

While these two examples are extreme, they seem to be part of mainstream managerial thinking, as evidenced by the huge popularity of ‘The One-Minute Manager’ (Blanchard & Johnson, 1981), a management ‘bible’, advocating such maxims as [a good manager should not] believe in participating in any of [his] staff’s decisionmaking.1 Reasonably, however, most firms lie somewhere in between government bureaucracies and Sun Hydraulics, in that workers have some, but not complete, discretion over their field of work.

In this paper we construct a theory of worker discretion that in a simple manner accommodates the discretion rules of Sun Hydraulics, bureaucracies, and points in between. Furthermore, we investigate the relation between a firm’s discretion rule and the degree of misallocation of workers within the firm. For example, can we expect firms with a high degree of worker discretion to have a higher degree of misallocation of talent within the

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1Increased worker discretion may also be seen as part of a more general workplace trend emphasizing job rotation, matrices, and self-monitoring groups (Lindbeck & Snower, 1996, 2000, 2001).
Our theory of discretion is based on the firm balancing off the gains from workers exploiting their private information and the cost of career concerns. Worker private information favors giving workers a high degree of discretion in specialization, because such private information means that workers are better equipped to assess their correct task. On the other hand, however, career concerns may lead workers to choose specialization strategically, if given the discretion. For example, if the most-able workers in a firm specialize in developing new products, it may be beneficial for less-able workers (if they were given the choice) to also specialize in product development, and thereby be associated with high ability by the market, even if that would lower their productivity. That firms with a high degree of worker discretion can experience such misallocation inside the firm, away from routine tasks into more prestigious tasks, is indicated by the observation that Sun Hydraulics for an extensive period had problems with personnel not updating the product catalogue (Kaftan, 1984).²

Given these concerns that a high degree of discretion will cause workers to herd into prestigious tasks, one would expect that the degree of discretion given to workers and the degree of misallocation of labor input would be positively related. However, when taking into account the contractual response by firms to the herding motive, i.e., in equilibrium, we find that more discretion is associated with less misallocation. Hence while it may be true that more discretion leads to more misallocation for a given firm, the hypothesis we obtain for a cross-section of firms is that firms with a higher level of discretion have a lower degree of misallocation.

To model a firm’s choice of discretion rule, we construct a model with two types of workers, low and high ability, and two types of tasks, ‘easy’ and ‘difficult’. An efficient allocation of workers occurs when the low ability workers specialize in the easy task, and the high ability workers specialize in the difficult task. Firms can only condition payment to an employee on his choice of task and on his level of effort. There are two periods. In the first period, firms offer contracts to the workers, and workers choose which firm to work for and which task to work on. Before the second period, the inside firm knows with

²Perhaps the herding of scientific activity into ‘promising’ venues of research, where the top researchers go, is an example of a similar type of misallocation.
certainty which task each employee worked on in the first period (but not necessarily his type), while the market receives merely an imperfect, public, signal. In the second period, the firms make offers simultaneously to each employee, and workers accept the highest offer.

There are two types of equilibria: separating and rationing. In a separating equilibrium, employees are given full discretion in task choice and a wage scheme is designed such that workers separate themselves to their efficient tasks. This high-discretion equilibrium resembles play in companies such as Sun Hydraulics and Gore & Ass., where the allocation of employees inside the firm to a large extent is decided by the individual employee. In rationing equilibria, in contrast, only a fraction of employees (which may be equal to zero) are given discretion over task choice, and the remaining employees are assigned to a task. A rationing equilibrium with a high degree of assignment resembles play in bureaucracies, with little or no worker discretion over specialization, while a rationing equilibrium with a moderate degree of assignment resembles play in typical firms, where only a fraction of workers are given discretion in specialization, for example, through trainee programs or work matrices.

The degree of discretion in equilibrium depends on the market observability of the task choices of individual workers. When the market observability is high, due e.g., coordination problems within the firm requiring clear job titles, or due to political regulations promoting transparency, the difficult task becomes more attractive for low workers, due to the herding motive. To counteract this effect, firms assign workers tasks, in order to reduce market information about employee quality.

The assignment to tasks in rationing equilibria implies that workers’ private information is not used efficiently, and a misallocation of workers occurs in equilibrium. Intuitively, one may think that the greatest source of misallocation arising from assignment is able workers that are not permitted to work in the difficult task. It turns out, however, that the ‘mistake’ made by firms in our model is the opposite: too many workers are assigned to the difficult task. This result accords with the Peter principle,\(^3\) in that the prime source of misallocation occurs due to workers being allocated to tasks above

\(^3\)The Peter principle (Peter & Hull, 1969) states that in a hierarchy, employees are promoted to their incompetence level.
their competence level (rather than the source of misallocation being that able workers are occupied below their competence level).

A large literature builds on Akerlof (1970) to consider adverse selection in the labor market (e.g., Greenwald, 1986, and Acemoglu & Pische 1998), which occurs when workers know more about their abilities than firms do. This literature implicitly assumes that the workers ability is revealed to the firm once hired. The novelty of our approach is that firms face two adverse selection problems, when hiring workers, and when allocating these workers within the firm.

The assignment literature, e.g., Rosen (1982) and Gibbons & Waldman (1999a), considers how the firm should allocate workers to tasks, in settings where workers and firms have symmetric information at the hiring stage. Hence there are no adverse selection problems in this literature.\(^4\) \(^5\)

Aghion & Tirole (1997) considers a setting where a principal may delegate the authority over a project choice to an agent. Giving the agent a higher degree of authority increases the agent’s effort, but also increases the probability that a project with high (exogenous) private benefits for the agent – but low principal benefits – is chosen. The present paper complements Aghion & Tirole (1997) by focusing on adverse selection rather than moral hazard (provision of effort) issues related to degree of discretion (or authority). In addition, we consider the issues of career concerns and misallocation, not considered by that paper.\(^6\)

The paper is structured as follows. In Section 2, we construct the model, and in Section 3 we present the main results, and Section 4 concludes.

\(^4\)The same point applies for the literature on career concerns, as in Holmstrom (1982/1999), two exceptions being Prendergast & Stole (1996) and Hvide (2002). Prendergast & Stole (1996) models a situation where managers have private information about investments projects, and Hvide (2002) considers a model where workers learn (more than firms) about their abilities through undertaking education.

\(^5\)Holmstrom & Milgrom (1991) considers which tasks should be included in the description of a job, and how to give incentives such that workers undertake all those tasks, while we ask how the firm should make the workers specialize efficiently.

\(^6\)Prendergast (1996) considers a setting where the manager decides which projects to take on, and which projects to leave to the worker. Due to concerns about building his own human capital base, the manager takes on too many tasks. Due to lack of worker private information, there is no notion of attempting to exploit worker’s competence in designing jobs in this paper.
2 The Model

Here we first describe the technology and contracts of the model, and then the timing.

2.1 Technology and Contracts

There is a continuum of workers and two firms. Each worker privately knows whether he has either low or high ability, while firms know the probability of a worker being high ($\theta$), but individual workers are indistinguishable. In each firm, there are two tasks, ‘Easy’ and ‘Difficult’, denoted by E and D. Task E requires the effort level $e^E$ to be completed for both worker type. Given that $e^E$ is exerted, both workers have the same productivity in the E task, $\pi^0$. Task E requires the effort level $e^D$ to be completed. Given that $e^D$ is exerted, the low type has productivity $\pi^L$ in the D task, and the high type has productivity $\pi^H$, where $\pi^L < \pi^H$. For example, we can think of effort as the time spent on doing a certain task and $\pi$ as the quality of the marginal product of a worker. We assume that the cost of effort is identical across workers. For simplicity, we normalize the cost of low effort to zero, and the cost of high effort to $c$, i.e., $c(e^E) = 0$, and $c(e^D) = c$. We confine attention to the case when it is efficient that high workers are allocated to task D and for low workers to be allocated to task E, which occurs when $\pi^H - c > \pi^0 > \pi^L - c$.

We assume that the only contractible variables are the workers’ choice of effort and their choice of task. Conditional on the correct effort level being exerted, firms offer one wage for the D task and one wage for the E task. If an incorrect level of effort is exerted, it is assumed that the wage to a worker is zero. The case when individual output is contractible is considered in Appendix D, where we show that our basic results (under certain conditions) hold under such a modification.

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7 All the results can be generalized to the case with more than two firms.
8 Foster & Rosenzweig (1996) considers the allocation of workers to different tasks within an agricultural market in Philippines, and find evidence that asymmetric information is present.
9 Differences in the cost of effort for the two types of workers would create qualitatively similar results to those obtained.
10 It may seem odd that an offer by a firm is a vector of wages, rather than just a wage. However, we can interpret the vector as reflecting differences in overtime payment or fringe benefits between the two possible tasks.
All workers and firms are risk neutral, and there is no discounting. We assume that if the incentive scheme is such that a worker is indifferent between doing the E task or the D task (taking into account the implicit incentives), he will choose the task that is efficient for him. This may be due to an (unmodeled) option plan or ownership share, or alternatively due to increased job satisfaction in the efficient task.

2.2 Timing

In the first period, workers are born knowing their ability level and the two firms compete in attracting them. Firms offer contracts lasting one period. Assuming that workers exert the correct level of effort, a firm offers workers $w_1^D$ for the D task and $w_1^E$ for the E task.\textsuperscript{11} Given the offers, workers choose for which firm to work. Before workers choose their task, a firm has the option to raise any of the wages \{w_1^D, w_1^E\} offered. In other words, firms can commit to not lowering wages, but may choose to raise one of them. This is a natural requirement, because both the firm and workers would (weakly) prefer such a reneged contract.\textsuperscript{12} Although such wage raises do not occur in equilibrium, it will turn out to have an impact on equilibrium. Finally, workers choose task, and production takes place.

After the first period, the two firms bid for the workers. The inside firm (a worker’s first period employer) is assumed to be fully informed about the task choice of the worker. The outside firm (the competitor of a worker’s first period employer), however, receives some public, imprecise signal about the task performed (or the wage) by a worker.

To fix ideas, we can think of the signal precision, which is exogenous in the model, as determined by the extent to which job titles and salaries are precise or diffuse. In this respect, Sun Hydraulics lies at one end of the spectrum by not having job titles for its employees, and a very covert pay policy, while bureaucracies, with well-defined job titles, job descriptions, and salary ladders, being at the other end.

We model the public information about task choice of a worker in the first period as an independent realization of a random variable $X$.\textsuperscript{13} For simplicity, it is assumed that

\textsuperscript{11}The workers are assumed to receive a wage equal to zero if an incorrect level of effort is exerted.

\textsuperscript{12}In technical terms, we are imposing the criterion of renegotiation-proofness.

\textsuperscript{13}We would obtain the qualitatively same results from assuming that $X$ is only observable to the outside firm, e.g., that $X$ is a private signal.
$X$ can take just two values, $E$ and $D$. If the worker is in task $E$, then $X = E$ occurs with probability $p$, and $X = D$ occurs with probability $1 - p$. If the worker is in task $D$, then $X = D$ with probability $p$, and $X = E$ occurs with probability $1 - p$, where $\frac{1}{2} \leq p \leq 1$, and the larger $p$ the higher precision of the signal (and the higher outside observability)

When $p = 1$, the inside firm and the outside firm are symmetrically informed after the first period.

Given the informational structure, the inside firm and the outside firm compete for the workers before the second period. We assume that the bidding follows a first-price sealed-bid auction; each firm gives a single offer to a worker, in ignorance of the other firm’s offer, and the worker accepts the highest offer.

3 Results

We first present results on the separating equilibria, where both type of employees work on their appropriate task in period 1. We then examine rationing equilibria, where at least one of the tasks is occupied by both type of workers. Notice that there is no incentive for worker misrepresentation after the contracts are signed in the second period, and firms will simply offer all workers an excess payment of $c$ for choosing the difficult task in that period, and an efficient allocation of workers occur. Hence inefficiencies, if they occur, occur in the first period.\footnote{The model can easily be extended to cover an arbitrary number of periods, in which case there can be inefficiencies in all periods except the last one.}

3.1 Separating equilibria

To study separating equilibria, we start out by analyzing equilibrium bidding for workers in the second stage, given that a separating equilibrium is played in the first stage. Recall that when the sorting is efficient at time 1, the inside firm knows the ability of a worker before the second period, while the outside firm receives a noisy public signal about the task choice of a worker in the first period.

For notational convenience, we derive the following result assuming $c = 0$, in which
case a worker’s wage in the second period will not depend on which task he works on in that period. Let $w^E_2$ and $w^D_2$ denote the expected second-period wage of a worker that chose task E and D, respectively, in the first period. $w^E_2$ and $w^D_2$ are the expected maximum offers before the second period.

**Lemma 1** Given that a separating equilibrium is played,

(i) $\pi^0 \leq w^E_2 < w^D_2 \leq \pi^H$, with strict inequalities for $p < 1$.

(ii) $\frac{\partial(w^D_2 - w^E_2)}{\partial p} > 0$.

**Proof.** For (i), see Appendix A, and for (ii), see Appendix B.

Lemma 1 gives the essential properties of the mixed-strategy Nash Equilibrium of the bidding game between the inside firm and the outside firm, where the inside firm bids conditional on the true type of each worker (since ability is revealed to the inside firm in a separating equilibrium), and the outside firm bids conditional on the imperfect signal $X$.\(^\text{15}\)

Part i) of Lemma 1 says that in a separating equilibrium, the wage in the second period is higher for a worker that chose the difficult task in the first period than for a worker that chose the easy task in the first period. The intuition is that a worker that chooses the difficult task in the first period enjoys better career prospects than a worker that chooses the easy task in the first period, since ability is (partially) revealed. Part ii) says that the wage difference in the second period is increasing in the precision of the noisy signal observed by the outside firm. The intuition for part (ii) is that the more informative signal, the more precise inference can be made from the signal about the ability type of a worker before the second period. Hence the difference in (expected) second period wages for the low and high ability workers will be magnified by a higher informativeness of the signal, as one would expect. When $p = 1$, the firms bid equally aggressively for both types of workers before the second period, and we have that $w^E_2 = \pi^0$ and $w^D_2 = \pi^H$, i.e., wage equals productivity for both types.\(^\text{16}\)

\(^\text{15}\)As derived in Appendix A, the outside firm bids $\pi^0$ for a fraction of workers. We can interpret such a bid as 'no bid'.

\(^\text{16}\)When the signal is completely uninformative ($p = \frac{1}{2}$), the high workers receive a higher wage than the low worker in the second period. In this case, the outside firm must bid equally aggressive for both type of workers. The inside firm, however, bids more aggressively for the high workers than for the low workers, since the former has a higher value to the firm.
firm makes positive information rents in the second period (on the high workers), and that these rents are offset by negative profits in the first period.

The simultaneous bidding structure that underlies the results of Lemma 1 is realistic for situations where firms may bid in turn, but where workers have no way of verifying the offer made by one firm to the other firm. Hence firms make secret or unverifiable offers to workers, so that a worker cannot start a ‘bidding war’ by presenting one firm with the offer from the other firm. As shown in Appendix A, the bidding setup ensures that there will be positive turnover between the two periods (and a higher turnover rate for low workers than for high workers). Hence there will be a ‘lemons problem’ in equilibrium, but not to the extent that trade breaks down, as in the sequential bidding setup of Greenwald (1986).18

The following proposition describes the contracts and the wage dynamics of separating equilibria. The proof and some additional results on wage dynamics are contained in Appendix A.

**Proposition 1** A separating equilibrium has the following properties:

1. Workers are given full discretion over task choice.
2. Low (high) workers get a wage that is higher (lower) than their marginal product in both periods.
3. High workers have a steeper wage profile than low workers.

In a separating equilibrium, the contracts offered by firms are such that each worker expects the same lifetime monetary payment from choosing the easy task as for choosing the difficult task in the first period. Hence $w_1^E + w_2^E = w_1^D + w_2^D$ (for $c > 0$, the right side

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17If firms can choose whether to give verifiable offers to workers or not, Hvide & Kaplan (2001) finds that neither firms would give verifiable offers in equilibrium, in fear of starting a bidding war. Other papers, e.g., Greenwald (1986) and Acemoglu & Pische (1998), have modeled the competition for workers as a sequential auction where the inside firm can always match the offer made from the outside firm. Since it is not obvious what the actual ‘rules’ of the bidding game are, it is comforting that the properties described in Lemma 1 also hold in more general auction models, for example in certain hybrid versions of the auction studied and the auction considered by Greenwald (1986).

18Greenwald (1986) creates turnover by assuming one-sided ‘utility shocks’ in disfavor of the inside firm, i.e., an urge to change employer even if the inside firm offers a higher wage.
is greater than the left side, but still lifetime utilities are equal), and therefore workers have incentive to sort themselves into their efficient task. In a separating equilibrium, the firm sets no limit to entry in any of the tasks, and we can interpret this equilibrium as a situation where workers are hired and then given full discretion over their task choice. In a separating equilibrium, wages are compressed during both periods, which occurs because the identity of high-ability workers is not known. For an intuition for why high ability workers have a steeper wage profile through time than low ability workers, recall that high workers have better career prospects than low workers under separation, due to the (partial) revelation of abilities in the market before the second period. To be willing to separate, low workers must hence be compensated by a relatively high wage in the first period, which implies that the wage profile of high workers is steeper than the wage profile of the low workers.

Several papers have shown that worker (nominal) wages and wage dispersion typically increase over time (see Gibbons & Waldman, 1999b, for an excellent overview of the careers in organization literature). In Appendix C, we show that separating equilibria have these properties given that we accommodate a degree of human capital acquisition between the two periods.

We now explain the conditions for existence of a high-discretion separating equilibrium.

**Proposition 2** A separating equilibrium is more likely to exist for lower $p$.

**Proof.** See Appendix A. ■

In a separating equilibrium, the low workers are paid more than their marginal product in the first period, while the high workers get paid less than their marginal product, as shown in Proposition 1, part ii). This creates a potential incentive for firms to deviate in order to attract only high workers, by holding the offer for the difficult task ($w_1^D$) constant, and lower the offer for entering the easy task ($w_1^E$). However, when it is sufficiently inexpensive for firms to make low ability workers choose the easy task instead of the difficult task once workers have entered the firm, by raising the offer $w_1^E$ at that point, such cream-skimming is not credible, and a separating equilibrium exists. When the signal precision is high, it is cheap to revise the offer $w_1^E$ upwards to the effect that low workers switch from the difficult to the easy task. Hence a separating equilibrium is less likely to
exist the higher $p$.

Since the full discretion in separating equilibria is fundamentally different from the spirit of the assignment literature, and of principal-agent theory, such as expressed by Holmstrom & Milgrom (1991), where firms assign workers to do specific tasks, it is natural to discuss the plausibility of separating equilibria in light of documented management practices. Baron & Kreps (1999) reports on the management practices of Sun Hydraulics Corp., a company founded in 1970 to manufacture fluid power products. The founder of Sun, Robert Koski, deemed standard management tools such as organization charts to be destructive, by restricting employee initiative and information. To deal with such problems, Koski designed the organization to eschew with almost all forms of hierarchy (to accord with State of Florida law, there is a President and a Controller). As Baron & Kreps (1999), p. 87, put it: ‘Work [at Sun] is self-organized. [..] Individual workers retain the right and responsibility to choose how they spend their own time.’\(^{19}\) In 1997, Sun’s products apparently enjoyed a higher margin than competitors, and had a reputation for outstanding quality.\(^{20}\)

Separating equilibria fit to several features of Sun. In particular, Sun is – in addition to having a high degree of worker discretion – characterized by low degree of outside observability. For example, job titles being non-existent at Sun (Baron & Kreps, 1999, p. 295), it is difficult for outside firms to assess the allocation of individual employees. Also, the pay of individual workers is very covert information. Lacking solid empirical evidence, it is hard to assess how wide-spread it is that firms induce a very high degree of worker discretion. However, that high-discretion rules within firms are common beyond the examples cited above is indicated by the study of Osterman (1994), which finds that 45% of employees in a representative sample of US companies have complete or large

\(^{19}\)The degree of discretion given to workers at Sun can be illustrated by a case where an engineer had been hired with a product development function in mind but had ‘become intrigued with the computer in his first days on the job, and since had concentrated entirely on creating new programming applications.’ (Kaftan, 1984).

\(^{20}\)The following statement from W. L. Gore, founder of Gore & Associates, is an echo from Sun: ‘In Gore & Ass., one of our basic principles is to encourage maximum freedom for each employee. There is no need for bosses, assignment of tasks, establishing lines of command, defining channels of permitted communication, and the like’ (Gore, 1990).
discretion over the choice of work method.\footnote{Discretion over work method and discretion over task choice are strictly speaking two separate issues, but it does not seem implausible that they are closely related.}

## 3.2 Rationing equilibria

We now consider the discretion policy of a firm when the degree of outside observability is high, cream-skimming is credible, and separating equilibria consequently do not exist.

**Proposition 3** i) For a high degree of outside observability, there exists a rationing equilibrium, where only a fraction of workers are given the discretion to choose task, and the remaining fraction of workers is assigned to the difficult task. (ii) There does not exist a rationing equilibrium where any workers are assigned to the easy task. (iii) The fraction of workers that are assigned increases in the degree of outside observability of task allocation for individual workers.

**Proof.** See Appendix A. ✷

When the degree of outside observability is high, firms must ration the slots in one of the tasks, in order to make the market learn less about the ability through the work allocation.\footnote{We surpress the mechanism in which the firm rations slots. Implicitly, it is assumed that the identity of those workers that prefer a task, but are not allowed entry, is unknown to the firm. For example, a first-come first-serve principle in determining entry would satisfy this assumption. We can modify this implicit assumption and the same type of results would go through.} We can interpret such rationing as the firm hiring workers and then only give a fraction of them discretion over task choice, and assign the remaining fraction of workers. An alternative interpretation of rationing equilibria is that of job rotation; all interested workers are allowed to do the easy task, but only a certain amount of time.

The intuition for why there cannot be a rationing equilibrium where the number of slots in the difficult task is rationed is that if the D slots were rationed the firm could increase productivity without increasing costs of compensation, by letting more (high) workers do the difficult task.\footnote{If the production technology were such that the simple task must be done (as with the product catalogue of Sun Hydraulics), a high degree of rationing in equilibrium implies that separate workers, without the option to switch to the difficult task, must be hired to do the easy task.}
The degree of rationing in a rationing equilibrium is determined by the degree of outside observability, $p$. A higher $p$ implies a higher degree of rationing, and when $p = 1$, i.e., symmetric information between the inside and the outside firm, there can exist pooling equilibria where all workers are assigned (such equilibria are considered in the numerical example in the next section). Hence rationing equilibria captures both firms with a low degree of worker discretion, as in government bureaucracies, and more typical firms, where a certain fraction of employees are given discretion over their specialization.

Several papers have shown that employers have incentive to hide their best workers, once the ability of these workers are known to the firm. In particular, the literature on strategic promotions, Waldman (1984b), Bernhardt & Scoones (1993) and Bernhardt (1995) show that a firm knowing individual workers’ ability may have incentive to delay promotion for able workers. We show that the incentive to avoid revealing ability also affects the firm's strategy before it learns about the ability of individual workers, through design of the discretion rule. In rationing equilibria, a firm hides its best workers through assignment even if it does not have private information about worker quality, but takes into account the information about worker ability revealed to other firms in equilibrium.

It is somewhat surprising that rationing takes the form of assignment to the difficult task, not to the easy task. We can add plausibility to this result by considering an example. A frequent complaint about bureaucracies is that too many persons are employed in planning, analysis and management tasks, rather than working on more customer-oriented, clerical tasks (the Peter principle). We can interpret management as the difficult task and the clerical task as the easy task. The model then provides the following explanation for having too many employees involved in management: such a practice dilutes the quality of management employees, and makes it more difficult for outside firms to hire away high-quality employees and consequently cheaper for the bureaucracy to retain them. Moreover, since this practice reduces the career advantages of persons involved in management, it reduces the per worker costs of hiring lower level workers.\footnote{A different piece of evidence for rationing equilibria can be found in Lerner and Tirole (2000), which proposes that the career concerns are a major reason that programmers freely contribute to open source software with credit being assigned to each section of code written. Lerner & Tirole (2000) further claim that this strong incentive cannot be utilized by for-profit firms for fear of having their best workers leave for other firms.}
Finally, we can notice that if a firm simply decided not to assign the workers - and set equal wages for the two tasks - low-quality workers would imitate high-quality workers and herd into the more prestigious task (to obtain a higher future compensation), and there would be harmful adverse selection of workers within the firm. The firm could have improved productivity by moving the low ability workers from the difficult task to the easy task; however, only a worker knows if he should move and since this movement has a career cost, he must compensated to make it. It is only worthwhile for the firm to pay a compensation that is equal or less to the gain in productivity. In the rationing equilibrium, the necessary compensation would satisfy this property since only some of its low-ability workers are allowed to.25

3.3 Numerical example

In this section we first summarize our findings so far, and then consider a numerical example.

Separating equilibria are characterized by the firm hiring workers and then giving them full discretion in defining their specialization, while in rationing equilibria, the firm hires workers and only partly let the workers decide. There are two underlying factors that determine the equilibrium played, and hence the degree of worker discretion. On one hand, worker private information favors a high degree of discretion, because a worker is better able to judge his appropriate specialization. On the other hand, career concerns create problems, because a worker may have incentives to choose a specialization that makes him look good to the market rather than in the interest of the firm.

To add realism to the example, we assume that (general) human capital acquisition in the first period results in second-period productivities given by \( g(\cdot) \), where

\[
g(\pi^L, \pi^0, \pi^H) = \text{plucked by other firms}. \]

To avoid this problem, a company can assign extra programmers to a project and try to blur information as to which workers contributed the most. There is some evidence of such bluring by the fact that Apple recently removed credits from its software (see Claymon, 1999), which, from outside the firm, makes the list of possible contributors longer.

25 If all the low-ability workers moved, this compensation would be high, since remaining in the difficult task would be a clear signal of high-ability. If only a few low-ability workers moved, this compensation would be low.
$$(\pi^L + h, \pi^0 + h, \pi^H + h)$$. 

**Example 1** Suppose $\theta = 1/2$, $\pi^0 = 1$, $\pi^H = 6$, $\pi^L = 1$, $c = 1$, $h = 1$.

Notice that a high worker has a higher (net) productivity in the difficult task, and a low worker has a higher productivity in the easy task, since the difficult task requires more effort. We illustrate the equilibria of the example, for varying values of $p$, by the following figure.

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<tr>
<th>Separating Eq.</th>
<th>Rationing Eq.</th>
<th>Pooling Eq.</th>
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<tbody>
<tr>
<td>0.5</td>
<td>0.77</td>
<td>1</td>
</tr>
</tbody>
</table>

Figure 1

The figure depicts the structure of equilibrium for varying $p$. For a low $p$, there exists a high-discretion separating equilibrium, which confirms Proposition 1 and Proposition 2. When $p$ increases to 0.77, there only exists a rationing equilibrium, where slots in the E task are limited, due to the possibility of cream-skimming with separation: for a high $p$, it becomes credible to pay a low wage for the easy task, because it is expensive to make low workers switch tasks in the interim. Therefore, on the interval (.77, 1) there exists a rationing equilibrium where a fraction $(1 - f)$ of those workers that prefer to work in the easy task are assigned to the D task (which confirms Proposition 3 (i)). The fraction $f$ decreases in $p$, due to the increased threat of cream-skimming (which confirms Proposition 3 (iii)). When $f$ increases, the degree of misallocation decreases, and the welfare of workers increases.\(^{26}\) When $p$ goes to 1, the rationing equilibrium becomes a pooling equilibrium, where all workers are assigned to the D task.\(^{27}\)

\(^{26}\)It holds generally that the welfare of both type of worker decreases with $p$, since total wages are equal across workers in both separating and rationing equilibria, and are higher in separating equilibria due to the more efficient allocation of workers.

\(^{27}\)Whether or not to have diffuse or precise job descriptions could be a choice variable for the firm. One cost of decreasing outside visibility through making job descriptions less precise could be increased coordination costs inside the firm due to e.g., duplication of work.
High workers earn more than low workers in both periods, and both type of workers experience a wage increase between the two periods (for sufficiently high values of $p$). Moreover, high workers have a steeper wage schedule than low workers, for any value of $p$, and the turnover rate is higher for low workers than for high workers.

We summarize the findings of the example in the following remark.

**Remark 1** For example 1, there exists a separating equilibrium for $p < .77$. For $p > .77$, there exists a rationing equilibrium where only a fraction $f$ of the low workers are allowed into the $E$ task. The fraction $f$ decreases in $p$, and for $p = 1$ there only exists a pooling equilibrium, where no workers are allowed into the $E$ task. High workers earn more than low workers in both periods, and for sufficiently high values of $p$, both type of workers experience a wage increase between the two periods. High workers have a steeper wage dynamics than low workers.

### 3.4 Misallocation of workers

We have provided a theory of discretion within firms, that in a tractable manner accommodates the discretion rules of hi-tech firms such as Sun Hydraulics, giving a lot of discretion, and (government) bureaucracies, giving a small amount of discretion to individual workers, and points in between. We now wish to analyze the implications of this theory of discretion to the issue of equilibrium misallocation of workers within firms.

Let us first define the misallocation of a worker as the difference between his productivity in equilibrium and that under a full information equilibrium. Then we have the following.

**Proposition 4** i) Misallocation of workers can occur in equilibrium, and is lower the higher degree of discretion. ii) Misallocation occurs due to low ability workers performing the difficult task.

**Proof.** Follows directly from Proposition 3. ■

In a one-period problem allocation problem, firms would simply pay a wage difference equal to the difference in the cost of effort between the easy task and the difficult task, give the workers full discretion, and an efficient allocation of workers would follow. The
reason why misallocation takes place in our two-period economy is that firms realize that
by an efficient allocation, too much information would become public about the ability of
individual workers.

On the form of misallocation, we find that too many workers are performing the
difficult task in low-discretion firms. In principle, such firms could improve productivity
by inducing (low ability) workers to move from the difficult task to the easy task, but the
necessary compensation to a worker for the career damage of being identified as a low
worker would exceed the gain in productivity from the movement.

A natural question is what hypothesis we can derive on misallocation within firms for
a cross-section of firms from different industries, where the firms in each industry have
the same $p$, but where $p$ differ between industries. For example, for public bureaucracies
there are political regulations, and economic reasons such as facilitating coordination
inside the organization, promoting transparency of job titles (and individual salaries).
On the other hand, we expect hi-tech firms, with a competitive environment and perhaps
less coordination problems inside the firm, to have a low $p$.

**Proposition 5** The degree of misallocation and the degree of worker discretion is in-
versely related for a cross-section of firms

**Proof.** This result follows directly from Proposition 3. ■

Since increased outside observability gives less discretion and more misallocation, for a
cross-section of firms from different industries, the degree of misallocation and the degree
of discretion given to workers is inversely related in equilibrium. From this result, we
expect a higher degree of misallocation of workers in public bureaucracies, than in firms
with a lower outside observability, such as in hi-tech firms.

This argument can provide a limit to the efficiency of reforms within the public sector,
an issue continuously debated in many countries. In the short term, bureaucracies could
keep the same level of production by downsizing and giving the retained workers a higher

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28 We can construct stories that could come up with a different conclusion. One simple theory could
say that workers get a lot of discretion if managers are poorly informed (and output can be conditioned
on), and that workers do not get discretion if the principal is well informed. From such a simple theory
we would expect the degree of misallocation to be positively related to degree of discretion.
level of discretion. However, with such a policy too much would be revealed about the ability of individual workers, and in the long run the public sector could be drained of its talent. Hence it might be that a certain amount of misallocation in the public sector is desirable.\textsuperscript{29}

4 Conclusion

We have considered a dynamic model of allocation of workers within firms with the distinguishing feature that firms face an adverse selection problem \textit{twice}; when hiring workers and when allocating workers within the firm. The second adverse selection problem is created by the fact that workers have career concerns: they may have incentives to choose tasks that are prestigious, but unproductive. To solve this problem, firms assign workers to tasks, even though workers are better equipped to assess their correct task.

When the outside observability is low, the career motives are weak, and the firm can construct a scheme that exploits the worker private information and ensures an efficient allocation of workers. This scheme involves giving the workers full discretion over which tasks to choose, i.e., a total absence of assignment. This type of equilibrium resembles play in hi-tech firms like Sun Hydraulics and Gore & Ass., where employees are given an extreme degree of discretion over which tasks to perform.

\textsuperscript{29}Let us briefly discuss alternative methods to test the hypothesis that degree of discretion and misallocation are negatively related. One testing strategy would be to obtain productivity and profit data from a company performing a ‘natural’ experiment, changing its practice from low worker discretion to high worker discretion, and to test whether the productivity and profit of the firm goes up. A different testing strategy would be to combine techniques from stochastic front analysis (e.g., Kumbhakar & Lovell, 2000), which measures the distance from the efficient production frontier to actual production of firms, with either survey data on discretion (such as those obtained by Osterman 1994), or with data on variables that are likely to be correlated with degree of discretion, such as the number of employees per supervisor, the extent of trainee programs, or the number of levels in the hierarchy in the firm. A third testing strategy is more indirect and based on differences in mobility costs. For example can older workers be expected to have higher moving costs than younger workers. This should make firms less anxious about older workers being bid away, and hence we can – controlling for the fact that more may be known about the ability of older workers – expect a higher discretion for older workers than for younger workers, and hence a more efficient allocation within the firm for older workers than for younger workers.
When the outside observability is higher, however, the career motives are stronger, and the firm assigns workers to task, to avoid herding to the prestigious task. Equilibria with a high degree of assignment rationing mirrors play in bureaucracies. Under such assignment, too many workers are from an efficiency standpoint directed to the tasks with high return to ability, like management, and too few perform 'simple’ tasks, such as customer service or catalogue revision, from an efficiency standpoint. This is the same type of inefficiency as implied by the Peter principle.

Furthermore, a higher level of worker discretion is associated with a lower degree of wasted human resources due to misallocation within the firm. Hence we expect that the Peter principle applies more in industries with a higher degree of outside observability, such as in public bureaucracies, than in firms with a lower outside observability, such as in hi-tech firms.

5 Appendix A: Proofs

Proof of Lemma 1. Clearly there cannot exist a pure strategy auction equilibrium. We here derive the mixed-strategy equilibrium. Consider first the equilibrium offers for a worker who receives a good signal. Let us say that the outside firm uses a mixed strategy with cumulative distribution $G^g(x)$ and support $S^g_{outside} = [\pi^0, \bar{\pi}^g]$, where $\bar{\pi}^g < \pi^H$. The inside firm will also use a mixed strategy with cumulative distribution of $F_l$ for the low worker and $F_h$ for high worker. For a low worker, $F_l$ will simply be the distribution degenerate at $\pi^0$. As can easily be shown, $S^g_{inside} = S^g_{outside} = S^g$. Given that the inside firm offers $x$ to a high worker with a good signal ($D$), the inside firm will get,

$$G^g(x)(\pi^H - x), \quad x \in S^g$$

(A1)

where $G^g(x)$ is the probability that the inside firm will win the auction, and $(\pi^H - x)$ is the surplus he gets in the case he wins. Since the inside firm must be indifferent at all points in his support, we have that,

$$G^g(x)(\pi^H - x) = k^g_{inside h}, \quad x \in S^g$$

(A2)

where $k^g_{inside h}$ is a constant. By integration, this constant equals the profits the inside firm makes on high workers that get a good signal. Now define the probability of a worker
being a high type conditional on a good signal as $\theta^g$ and the bad ($E$) signal as $\theta^b$. As can easily be verified,

$$\theta^g = \frac{p \theta}{p \theta + (1 - p)(1 - \theta)}$$

$$\theta^b = \frac{(1 - p) \theta}{p(1 - \theta) + (1 - p) \theta}$$

(A3)

Given that the outside firm offers $y$ to a worker with a signal $i$, the outside firm will get,

$$\theta^g F^g_h(y)(\pi^H - y) + (1 - \theta^g) F^g_l(y)(\pi^0 - y), \ y \in S^g$$

(A4)

By the same argument as for the inside firm, the outside firm must be indifferent at all points in his support. From Milgrom (1981), we know that the profits of the outside firm must be zero. Hence the above expression must be zero. By inserting for $y = \bar{\pi}^g$, we can determine the upper end of the support, $\bar{\pi}^g$, as,

$$\bar{\pi}^g = \theta^g \pi^H + (1 - \theta^g) \pi^0$$

(A5)

We now determine the cdf’s. From (A2) and inserting for $x = \bar{\pi}^g$ in (A1) to get $k^g_{insideh} = \pi^H - \bar{\pi}^g$, we get that,

$$G^g(x) = \frac{\pi^H - \bar{\pi}^g}{\pi^H - x}, \ \text{where} \ x \in S^g$$

(A6)

Notice that this cdf places an atom at $x = \pi^0$, where the magnitude of the atom equals $\frac{\pi^H - \bar{\pi}^g}{\pi^H - \pi^0}$. To determine $F^g_h$, insert for $F^g_l = 1$ in (A4) to get,

$$F^g_h(y) = \frac{1 - \theta^g}{\theta^g} \frac{y - \pi^0}{\pi^H - y}, \ y \in S^g$$

(A7)

Notice that this distribution does not place an atom at the lower end of the support. For a worker with a bad signal, we use exactly the same procedure to get,

$$G^b_b(x) = \frac{\pi^H - \bar{\pi}^b}{\pi^H - x}, \ \text{where} \ x \in S^b$$

(A8)

where the magnitude of the atom at $x = \pi^0$ equals $\frac{k^g_{insideh}}{\pi^H - \pi^0}$. Notice that we have that,

$$k^b_{insideh} > k^g_{insideh} > 0,$$

(A9)
since $\tilde{\pi}^g > \tilde{\pi}^b = \theta^b \pi^H + (1 - \theta^b)\pi^0$. Hence, as expected, the informed firm makes a higher profit on a (high) worker that receives a bad signal than a (high) worker that receives a good signal. Finally, we get,

$$F^b_H(y) = \frac{1 - \theta^b y - \pi^0}{\pi^H - y}, \quad y \in S^b$$  \hspace{1cm} (A10)

which is an atomless distribution. Now the equilibrium (expected) wage for an agent of type $i$. Clearly his expected wage just equals the expectation of the maximum offer conditional on the signal. First a low ability person. His expected wage equals,

$$w_2^E = \pi^0[p(1 - \theta^b) + (1 - p)(1 - \theta^g)] + p\int_{\pi^0}^{\pi^g} zg^b(z)dz + (1 - p)\int_{\pi^0}^{\pi^g} zg^g(z)dz > \pi^0$$  \hspace{1cm} (A11)

since the offer from the outside firm fully determines his wage. On the other hand, the expected wage for a high ability person equals,

$$w_2^D = p\int_{\pi^0}^{\pi^g} zdG^g(z)F^g(z) + (1 - p)\int_{\pi^0}^{\pi^b} zdG^b(z)F^b(z) < \pi^H$$  \hspace{1cm} (A12)

Uniqueness follows directly from the argument.

The turnover rate for the low workers equals the fraction of low workers that receives a (strictly) higher bid from the outside firm than from the inside firm at time 2, and half of the workers that receive the same offer from the two firms. Recall that the inside firm always bids $\pi^0$ for the low workers. Hence $T_L$ equals,

$$T_L = p[1 - \frac{1}{2}G^b(\pi^0)] + (1 - p)[1 - \frac{1}{2}G^g(\pi^0)]$$

\[= \frac{1}{2} + \frac{\theta^b p + (1 - p)\theta^g}{2} \hspace{1cm} (A13)\]

where $\theta^b$ and $\theta^g$ are defined as in the previous proof. It follows immediately from (A13) that this expression is positive. Now to the turnover rate of the high workers, which equals,

$$T_H = p\int_{\pi^0}^{\pi^g} F^g(z)dG^g(z) + (1 - p)\int_{\pi^0}^{\pi^b} F^b(z)dG^b(z)$$

\[= \frac{\theta^g p + (1 - p)\theta^b}{2} \hspace{1cm} (A14)\]

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As with the low workers, it follows immediately from the expression that the turnover rate for the high agents is positive. Notice that both $T_L$ and $T_H$, and hence total turnover, are increasing in $\theta$, since both $\theta^b$ and $\theta^g$ are increasing in $\theta$. Expressing the difference, we find that,

$$T_L - T_H = \frac{1}{2} + \frac{\theta^b p + (1 - p)\theta^g}{2} - \frac{\theta^g p + (1 - p)\theta^b}{2}$$

$$= \frac{1}{2} + \theta^b p + \frac{1}{2}\theta^g - \theta^g p - \frac{1}{2}\theta^b$$

$$= \frac{1}{2} + (p - \frac{1}{2})(\theta^b - \theta^g) \quad (A15)$$

Since the second term on the right hand side always exceeds $(-\frac{1}{2})$, the turnover is always higher for the low type workers than for the high type workers. Intuitively, if the realization of the signal is $E$, the inside firm bids $\pi^0$ if the worker is low, and $F^g_i(y)$ if the worker is high. So, conditional on the signal being $E$, the turnover is higher for low workers than for high workers. The same type of argument applies if the value of the signal is $D$. 

**Proof of Proposition 1.** Since $\pi^L$ will play no role in this proof, we can normalize by setting $\pi^0 = 0$ and $\pi^H = 1$. In order for a low worker to choose the right task in the first period, the wage over a low worker’s career for choosing the $E$ task must be at least as large as the wage over the career for choosing the $D$ task,

$$w_1^E + w_2^E \geq w_1^D + w_2^D \quad (1)$$

Applying the same argument for a high worker, such a worker chooses the right task if and only if,

$$w_1^D + w_2^D \geq w_1^E + w_2^E \quad (2)$$

Combining (2) and (3), we get that a separating equilibrium implies that,

$$w_1^E + w_2^E = w_1^D + w_2^D \quad (3)$$

If this condition does not hold, either a low worker or a high worker has incentive to allocate himself *inefficiently*. The only way to ensure an efficient allocation of workers is to set wages such that (4) holds, and allow workers to choose their task. Hence workers are given full discretion over task choice in a separating equilibrium.
That \( w^E_2 > \pi^0 \) and \( w^D_2 < \pi^H \) are shown in Lemma 1. We now show that \( w^D_1 < \pi^H \) and that \( w^E_1 > \pi^0 \). As can be seen from the auction equilibrium described in Appendix A, the maximum average profit per worker made by a firm in the second period (which occurs for \( p = \frac{1}{2} \)) is equal to \( \theta(1 - \theta) \). It follows that the maximum average wage in the first period equals \( \theta + \theta(1 - \theta) \), due to the zero profit condition.\(^{30} \) As can easily be seen from this expression, the maximum average wage in the first period cannot exceed 1. Furthermore, from Lemma 1 it follows that \( w^E_1 > w^D_1 \) in a separating equilibrium, and hence \( w^D_1 < 1 \). Second, the maximum average profit per worker in the first period is 0 (which occurs for \( p = 1 \)), and hence average wages must exceed \( \theta \) in the first period. Since \( w^E_1 > w^D_1 \), it follows that \( w^E_1 > 0 \). Hence low (high) workers are paid more (less) than their marginal productivity in both periods. In the proof of the third part of the proposition, we also include the possibility of human capital acquisition, and this proof is relegated to Appendix C.

**Proof of Proposition 2.** We start out by comparing the case \( p = 1/2 \) with the case \( p = 1 \), and show that the conditions for existence of a separating equilibrium is more restrictive in the latter case. For simplicity of exposition, we assume that \( c = h = 0 \) and \( \theta = 1/2 \).

For \( p = 1 \), it follows that in a separating equilibrium, we must have that

\[
\begin{align*}
  w^E_2 &= \pi^0 \\
  w^D_2 &= \pi^H,
\end{align*}
\]

(4)

By the zero-profit condition of firms and the incentive condition of workers to reveal their type, we have

\[
\begin{align*}
  w^E_1 &= \pi^H \\
  w^D_1 &= \pi^0
\end{align*}
\]

(5)

\(^{30}\)Zero profits across periods imply that,

\[
2\theta = (1 - \theta)(w^N_1 + w^S_2) + \theta(w^S_1 + w^S_1)
\]

where \( 2\theta \) is just the total productivity across periods, and the expression on the right hand side is the total wage bill.

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We now check under which circumstances these wage offers are consistent with equilibrium in the game between firms. Suppose that firm 1 sticks to the wage schedule \((w_{1E}, w_{1D})\) and firm 2 deviates by offering the wage schedule \((w_{1E}', w_{1D}')\), where \(w_{1D}' = w_{1D}\) and \(w_{1E}' < w_{1E}\). In that case, firm 2 would attract a share of the high workers while all the low workers choose firm 1. Since \(w_{1D}'\) is less than the marginal productivity of the high worker, firm 2 would run a profit, and hence the deviation \((w_{1E}', w_{1D}')\) would be profitable. However, suppose a low worker also chooses to work for firm 2. Taking this possibility into account, firm 2 may wish to revise \(w_{1E}'\). Denote this revised offer for \(w_{1E}'\). The point with offering \(w_{1E}'\) instead of \(w_{1E}\) would be to give incentives for low workers to self allocate themselves efficiently. The productivity gain from making a low worker choose the E task instead of the D task would be \(\pi^0 - \pi^L\). The wage increase required to make this low worker prefer the E task to the D task would be \(w_{2D}' - w_{2E}' = \pi^H - \pi^0\). Hence, a firm would prefer to set \(w_{1E}' = w_{1D}' + (\pi^H - \pi^0) = \pi^H\) if

\[
\pi^H - \pi^0 < \pi^0 - \pi^L
\]  

But in that case, \((w_{1E}', w_{1D}') = (w_{1E}, w_{1D})\), and the deviation by firm 2 is not credible. Hence, firm 2 cannot only attract high ability workers and does not have additional profits, and there exists a separating equilibrium when equation (7) holds. On the other hand, when \(2\pi^0 < \pi^L + \pi^H\), the firm can commit to setting \(w_{1E}' < w_{1E}\) and hence only attract high workers.

We now use the same type of argument as above to show that the conditions for existence of a separating equilibrium is less restrictive when \(p = \frac{1}{2}\) than when \(p = 1\). Suppose that a separating equilibrium exists, and label the corresponding wages for \((\hat{w}_1^E, \hat{w}_1^D, \hat{w}_2^E, \hat{w}_2^D)\). Then, since there is asymmetric information in the bidding before the second stage,

\[
\hat{w}_2^E > \pi^0 \\
\hat{w}_2^D < \pi^H
\]  

For a separating equilibrium to be played, zero profits (across the two periods) imply,

\[
\hat{w}_1^E < \pi^H \\
\hat{w}_2^D > \pi^0
\]
Suppose that firm 2 deviates by offering the wage schedule \((\hat{w}_1^E, \hat{w}_1^D)\), where \(\hat{w}_1^D = \hat{w}_1^E\) and \(\hat{w}_1^E < \hat{w}_1^F\). The productivity gain from making a low worker choose the E task instead of the D task would, as before, be \(\pi^0 - \pi^L\). The wage increase required to make a low worker prefer the E task to the D task would, however, be \(\hat{w}_2^D - \hat{w}_2^E < \pi^H - \pi^0\). Hence, a firm would prefer to set \(\hat{w}_1^E = \hat{w}_1^D + (\hat{w}_2^D - \hat{w}_2^E) = \hat{w}_1^E < \pi^H\) if

\[
\hat{w}_2^D - \hat{w}_2^E < \pi^0 - \pi^L,
\]

in which case a separating equilibrium exists. Since \(\hat{w}_2^D - \hat{w}_2^E < \pi^H - \pi^0\), the condition for existence of a separating equilibrium is less restrictive for \(p = \frac{1}{2}\) than for \(p = 1\).

For general \(p\), to prove the result it is necessary that \(w_2^D - w_2^E\) increases with \(p\) in a separating equilibrium, which is shown in Lemma 1.31

Proof of Proposition 3. We start out by proving the existence of a rationing equilibrium where the number of slots in the E task is restricted, and then prove the impossibility of a rationing equilibrium where the slots in the D task is rationed. Finally, we prove that the degree of rationing is increasing in \(p\). We start out by assuming \(p = 1\) and then consider the case \(p = \frac{1}{2}\). The case \(p \in (\frac{1}{2}, 1)\) is considered in Appendix B.

For \(p = 1\), when \(\pi^0 < (\pi^L + \pi^H)/2\) then a deviating firm will have incentive to higher the wage of task E once the workers have chosen that firm, and hence there does not exist a separating equilibrium. Suppose that a firm chooses a schedule so that the high workers prefer to work in task D, and the low workers prefer to work in task E. However, the firm allows only a fraction \(f\) of the workers that prefer task E to enter task E. The complementary fraction of workers, \((1-f)\), is forced to work in task D (the admission to

---

31One may notice that an inside firm generates profits from a worker switching to the E task by both the increase of efficiency and the usefulness of the knowledge gained. Why do we only take into account the former and not the latter in the deviation condition? The answer rests in that the gain from the knowledge is solely from the outside firm’s beliefs about \(f\). The outside firm’s strategy is a mixed strategy with support starting from \(\pi^0\). An informed inside firm can extract all the surplus of his knowledge for a D task worker by placing a bid at \(\pi^0 + \epsilon\). Likewise, an uninformed inside firm can also make this bid and lose \(\epsilon\) for all the E task workers he would have avoided. As one can see, this extra cost is negligible for small \(\epsilon\). Thus, the inside firm’s value of information is actually worthless. All that matters is that the outside firm thinks he has such information.
task E is allocated in a way such that the firm does not learn the type of those workers that are not admitted to task E). In that case, we still have that
\[ w_1^E + w_2^E = w_1^D + w_2^D \]
Moreover, second period wages must satisfy,
\[ w_2^E = \pi^0 \]
\[ w_2^D = \frac{\theta \pi^H + (1 - \theta)(1 - f)\pi^0}{1 - f(1 - \theta)} \] (10)
Any value of \( f \) makes the equations consistent, and we now put restrictions on \( f \). If \( f \) is high, then a deviating firm can make a profit by the procedure described in the previous result. On the other hand, if \( f \) is low, the firm will lose money on mis-allocation. So, equilibrium is a situation where \( f \) is the maximal value that is consistent with there not existing a profitable deviation. A deviating firm can only make a profitable deviation if,
\[ w_2^D - w_2^E \geq \pi^0 - \pi^L \] (11)
Hence \( f^* \) is the value of \( f \) such that this condition holds with equality. Simplifying, we get that,
\[ f^* = \frac{(1 + \theta)\pi^0 - \theta \pi^H - \pi^L}{(\pi^0 - \pi^L)(1 - \theta)} \] (12)
Notice that when \( \pi^0 > \frac{\pi^L + \pi^H}{2} \), then \( f^* > 1 \), and we get a separating equilibrium. The case \( f^* \leq 0 \) is considered in a remark below.

We now prove (ii), that there cannot be rationing equilibrium where the number of slots in task D is restricted. If the number of slots in task D is restricted, there are two possibilities. First, it can be the case that both types wish to work in task D. In that case, the proportion of workers should be the same in both jobs. If this happens, there are no career concerns since no information inferred by task choice. Because of this, the firm can induce a high worker switch from task E to task D, by paying the same wage in task D as in task E. Such a scheme would increase productivity without increasing costs. So in equilibrium, it cannot be the case that both types of workers wish to work in task D. The second possibility is that the low type wishes to work in task E, while the high
type workers wish to work in task D. In that case, total wages must be equalized across tasks. But then, the firm can increase profits by allowing a higher fraction of workers in task D, by allowing workers to move from task E to task D (since only high workers would wish to move). This occurs since both the wage in task D is lower than in task E (since the fraction of high workers in task D is higher than in task E) and productivity of high workers is higher in task D. Hence a situation where the slots in task E is rationed cannot be an equilibrium.

That the degree of rationing is higher for \( p = 1 \) than for \( p = \frac{1}{2} \) follows from a very similar argument to why \( w_D^2 - w_E^2 \) is higher for \( p = 1 \) than for \( p = \frac{1}{2} \) (Proposition 2).32 The case with general \( p \) numerically yields the same type of results, and is considered in Appendix B. □

6 Appendix B: Numerical Analysis

In this appendix, we perform a numerical analysis of the claims made in Lemma 1, part (ii), and Proposition 3, part (iii).33 First, for the claim of Lemma 1, we show that \( \Delta w_2 := w_D^2 - w_E^2 \) is increasing in \( p \) (from which Proposition 2 follows). Second, for the claim of Proposition 3, we show that the degree of rationing is increasing in \( p \).

6.1 Proof of Lemma 1 and Proposition 2

To show that a separating equilibrium is more likely to exist the lower \( p \) (Proposition 2), we need to check that \( \Delta w_2 := w_D^2 - w_E^2 \) is increasing in \( p \) (Lemma 1).34

We start out by simplifying \( \Delta w_2 \), then prove analytically that \( \frac{\partial \Delta w_2}{\partial p}_{\theta=\frac{1}{2}} \) is increasing in \( p \), and finally consider numerical analysis for general \( \theta \). From Appendix A, we know

32 The outline of the proof goes as follows. Given a certain degree of rationing, \( f \), the wage difference \( w_D^2 - w_E^2 \) is greater at \( p = 1 \) than at \( p = \frac{1}{2} \). The wage difference \( w_D^2 - w_E^2 \) at \( p = 1 \) is also increasing in \( f \). Since the equilibrium \( f \) is the \( f \) such that \( w_D^2 - w_E^2 = \pi^0 - \pi^L \), the equilibrium \( f \) has to be decreasing from \( p = \frac{1}{2} \) to \( p = 1 \).

33 All calculations and graphs are generated in Maple V. The worksheets are available from the authors.

34 The reason why it is difficult to prove analytically that \( \Delta w_2 \) is increasing in \( p \) is that while \( w_D^2 \) is always increasing in \( p \), surprisingly \( w_E^2 \) is not always monotonic in \( p \).
Hence, \( w^E_2 = \pi^0[p(1-\theta^b) + (1-p)(1-\theta^g)] + p \int_{\pi^0}^{\pi^b} zg^b(z)dz + (1-p) \int_{\pi^0}^{\pi^g} zg^g(z)dz > \pi^0 \)  

(B1)

and that, 
\( w^D_2 = p \int_{\pi^0}^{\pi^g} zdG^g(z)F^g(z) + (1-p) \int_{\pi^b}^{\pi^g} zdG^b(z)F^b(z) < \pi^H \)  

(B2)

Observe that, 
\[
\int_{\pi^0}^{\pi^i} zdG^i(z) = (\pi^H - \pi^0)(1-\theta^i) \int_{\pi^0}^{\pi^i} zd(\frac{1}{\pi^H - z}) \\
= (\pi^H - \pi^0)(1-\theta^i) \left[ \frac{z}{\pi^H - z} + \ln(\pi^H - z) \right]_{\pi^0}^{\pi^i} \\
= \pi^H \theta^i + (\pi^H - \pi^0)(1-\theta^i) \ln(1-\theta^i) 
\]

(B3)

where \( i = b, g \), and where \( \theta^b, \theta^g \) are as in equation (A3). We can use (B3) to simplify \( w^E_2 \) into,
\[
w^E_2 = \pi^0 + (\pi^H - \pi^0) [p(\theta^b + (1-\theta^b) \ln(1-\theta^b)) + (1-p)(\theta^g + (1-\theta^g) \ln(1-\theta^g))] \\
= \pi^0 + (\pi^H - \pi^0) [p\theta^b + (1-p)\theta^g] + \\
(\pi^H - \pi^0)[p(1-\theta^b)\ln(1-\theta^b) + (1-p)(1-\theta^g)\ln(1-\theta^g)]  
\]

(B4)

Moreover notice that, 
\[
\int_{\pi^0}^{\pi^i} zdG^i(z)F^i(z) = \pi^H (2\theta^i - 1) + 2\pi^0 (1-\theta^i) - \frac{(1-\theta^i)^2}{\theta^i}(\pi^H - \pi^0) \ln(1-\theta^i) 
\]

(B5)

Hence, 
\[
w^D_2 = 2\pi^0 - \pi^H + 2(\pi^H - \pi^0)(p\theta^g + (1-p)\theta^b) - \\
(\pi^H - \pi^0)[p(1-\theta^g)^2\ln(1-\theta^g) + (1-p)(1-\theta^b)^2\ln(1-\theta^b)]  
\]

(B6)

We then have that, 
\[
\Delta w_2 = w^D_2 - w^E_2 = \pi^0 - \pi^H + (\pi^H - \pi^0)[(3p - 1)\theta^g + (2 - 3p)\theta^b] - \\
(\pi^H - \pi^0)[(1-\theta^g)(\frac{p}{\theta^g} + 1 - 2p)\ln(1-\theta^g) + (1-\theta^b)(1 - \theta^b)(\frac{1-p}{\theta^b} + 2p - 1)\ln(1-\theta^b)] \\
= \pi^0 - \pi^H + (\pi^H - \pi^0)[(3p - 1)\theta^g + (2 - 3p)\theta^b] - \\
\frac{\pi^H - \pi^0}{\theta}[(1-\theta^g)(1 - p)\ln(1-\theta^g) + (1-\theta^b)p\ln(1-\theta^b)]  
\]

(B7)
Notice that for $\theta = \frac{1}{2}$, we have that $\theta^o = p$ and $\theta^b = 1 - p$, and hence (B7) reduces to (we normalize by setting $\pi^0 = 0$ and $\pi^H = 1$),

$$\Delta w_{\theta=\frac{1}{2}}^2 = -1 + (3p - 1)p + (2 - 3p)(1 - p) - 2[(1 - p)^2 \ln(1 - p) + p^2 \ln p]$$  \hfill (B8)

Differentiating this expression with respect to $p$ we obtain,

$$\frac{\partial \Delta w_2}{\partial p}_{\theta=\frac{1}{2}} = 4[(1 - p) \ln(1 - p) - (1 - p) - (p \ln(p) - p)] > 0$$  \hfill (B9)

This expression is greater than zero because $x \ln(x) - x$ is decreasing in $x$ for $x \in (0, 1)$. Hence we have shown that $\Delta w_2$ is increasing in $p$ for $\theta = \frac{1}{2}$. We expect a proof of the case with general $\theta$ to be along the same lines, but significantly more cumbersome. In absence of an analytical proof, we now plot $\Delta w_2$ for other values of $\theta$ (still using the normalization $\pi^0 = 0$ and $\pi^H = 1$),

![Figure 2](image)

The figure plots $\Delta w_2$ as a function of $p$ for $\theta = .1$ (bottom line), $\theta = .3$, $\theta = .5$, $\theta = .7$, $\theta = .9$ (top line). As can be seen from the figure, $\Delta w_2$ is increasing in $p$ for all the values of $\theta$. This finding has been confirmed by extensive numerical analysis.

### 6.2 Proof of Proposition 3

We now show that the degree of rationing is increasing in $p$, or in other words that $f^*$ is decreasing in $p$ (Proposition 3 (iii)).
Denote the wages in the second period of a rationing equilibrium as $\hat{w}_D^2$ and $\hat{w}_E^2$. To determine $\hat{w}_D^2$ and $\hat{w}_E^2$, we work with the same equations as before, except that $\theta$ is replaced by $\hat{\theta}$, and $\pi^H$ is replaced by $\hat{\pi}^H$, where

$$\hat{\theta} = \theta + (1 - f)(1 - \theta)$$
$$\hat{\pi}^H = \frac{\theta \pi^H + (1 - f)(1 - \theta)\pi^0}{\theta}$$

(B10)

where $\hat{\theta}$ is the expected share of workers that choose the D task in the first period, and where $\pi^H$ is the expected productivity of those workers in the second period (since workers choose their efficient task in the second period, $\pi^L$ does not enter the expression). Once workers have chosen a firm, this firm can alter its profits by inducing low workers to switch to the E task. To induce a low worker to switch, a firm must pay him an extra $\hat{w}_D^2 - \hat{w}_E^2$. The productivity improvement from him switching is $\pi^0 - \pi^L$. There is also gains from knowledge by having him switch. The overall gain from knowledge is equal to the expected profits in period 2 from being the inside firm. We can derive this expression by realizing that the inside firm is playing a mixed strategy and thus indifferent to every strategy in his support. Therefore, we can look at his expected profit if he chose the highest wage and hired all task D workers. This is equal to $\hat{\theta} \cdot [\hat{\pi}^H - (p \cdot (G^g)^{-1}(1) + (1 - p) \cdot (G^b)^{-1}(1))].$ Note that this expression is zero when either $f = 0$ or $p = 1$. Since a firm wants to choose an $f$ to maximize the joint gains from switching (knowledge and productivity) minus increased wage costs. Hence,

$$f^* = \arg \max_{f \in [0,1]} \{\hat{\theta} \cdot [\hat{\pi}^H - (p \cdot (G^g)^{-1}(1) + (1 - p) \cdot (G^b)^{-1}(1))]) + \theta \cdot f \cdot [\pi^0 - \pi^L - (\hat{w}_D^2 - \hat{w}_E^2)]\}$$

(B11)

For the expressions we have checked, $f^*$ is unique, and we expect this to hold generally. We now plot $f^*$ against $p$, using (B8), (B10), and (B11), and insert the parameter values used in Example 1, except that we let $\theta$ be a free parameter (in addition to $p$). The figure plots $f^*$ against $p$, for varying values of $\theta$ [$\theta = .3$ (top line), $\theta = .5$, and $\theta = .7$ (bottom line)]. The figure shows that $f^*$ is decreasing in $p$ for all values of $\theta$. Extensive numerical analysis confirms that point. Hence we have substantiated that the degree of rationing is increasing in $p$. 

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7 Appendix C: More on Wage Dynamics in Separating Equilibria

In this appendix, we consider the wage dynamics of separating equilibria. In addition to a cost of effort, $c$, we assume throughout the appendix that (general) human capital acquisition results in second period productivities $g(.,)$, where $g(\pi^L, \pi^0, \pi^H) = (\pi^L + h, \pi^0 + h, \pi^H + h)$, i.e., that the increase in productivity between the two periods is uniform across workers (and tasks).\textsuperscript{35}

7.1 Proof of the third part of Proposition 1

We first show that high workers have a steeper wage dynamics than a low worker in a separating equilibrium, provided $c$ or $h$ not too high. Define the slope of the wage dynamics

\textsuperscript{35}Specific human capital acquisition has a similar effect to introducing switching costs, in that any positive level of turnover would be inefficient. Proportional human capital acquisition, of the form $g(\pi^L, \pi^0, \pi^H) = h(\pi^L, \pi^0, \pi^H)$, where $h > 1$, would yield the same type of results as the specification chosen.
of a low worker as,
\[ \Psi^E = \frac{w^E_2 - w^E_1}{w^E_1} \] (13)
and for a high worker as,
\[ \Psi^D = \frac{w^D_2 - w^D_1}{w^D_1} \] (14)
We show that \( \Psi^E < \Psi^D \) for \( c \) or \( h \) not too high. Clearly, for \( c = h = 0 \), the denominator of \( \Psi^E \) is higher than the denominator of \( \Psi^D \), since \( w^E_1 > w^D_1 \) in that case. Also, from \( w^E_1 > w^D_1 \) and the fact that \( w^E_2 < w^D_2 \) it follows that the numerator of \( \Psi^E \) is smaller than the numerator of \( \Psi^D \). Hence it follows that \( \Psi^E < \Psi^D \) for \( c = h = 0 \). We now consider the effect of introducing \( c, h > 0 \) on \( \Psi^i \). Assuming that a separating equilibrium exists for \( c, h > 0 \), we have that
\[ \Psi'^E = \frac{w^E_2 + \frac{h}{1-c} - w^E_1}{w^E_1} \] (15)
and,
\[ \Psi'^D = \frac{w^D_2 + \frac{h}{1-c} - w^D_1}{w^D_1 + \frac{c}{1-c}} \] (16)
As can easily be seen from these expressions, \( \Psi'^D > \Psi'^E \) for any \( c \) given that \( h \) is zero, and \( \Psi'^D > \Psi'^E \) for any \( h \) given that \( c \) is zero. \( \Psi'^D < \Psi'^E \) requires that both \( c \) and \( h \) are larger than zero. We assume that \( c, h > 0 \). Briefly, \( c > 0 \) plays the role of ensuring that high workers are paid more than low workers in the first period in a separating equilibrium, and \( h > 0 \) plays the role of ensuring that wages are increasing through time for both type of workers.

### 7.2 Further results on wage dynamics of separating equilibria

We now show that for sufficiently high \( c \) and \( h \), a separating equilibrium satisfies:

- High workers earn more than low workers in both periods.
- Wages increase over time for both types of workers.
Assume that there exists a separating equilibrium for the exogenous parameters \( \{c = 0, h = 0, \pi^H, \pi^0, \pi^L, \theta\} = \Pi_1 \), given by the equilibrium wage vector \( \{w_E^1, w_D^1, w_E^2, w_D^2\} = \Omega_1 \). Further suppose that there exists a separating equilibrium for the exogenous parameters \( \{\hat{c} > 0, \hat{h} > 0, \pi^H + \hat{c}, \pi^0, \pi^L, \theta\} = \Pi_2 \), with equilibrium wages given by \( \Omega_2 \). Notice that with \( \hat{c} > 0 \), firms must condition period 2 wages on task choice in period 2 (in addition to the information about task choice in period 1) to obtain efficient allocation, in contrast to the case when \( c = 0 \). Specifically, to obtain an efficient allocation of workers at time 2, firms will offer the workers that choose the D task an ‘overtime payment’, or bonus, of \( \hat{c} \). The wage vector \( \Omega_2 \) is characterized by four elements, \( \{\hat{w}_1^E, \hat{w}_1^D, \hat{w}_2^E, \hat{w}_2^D\} \), where \( \hat{w}_1^E (\hat{w}_1^D) \) is the period 1 equilibrium wage for a worker that chooses the E (D) task in period 1, and where \( \hat{w}_2^E (\hat{w}_2^D) \) is the expected wage in period 2 when choosing the E (D) task in period 1, conditional on choosing the E (D) task in period 2. We then have that \( \Omega_2 = \{w_1^E, w_1^D + \hat{c}, w_2^E + \hat{h}, w_2^D + \hat{c} + \hat{h}\} \). The reason for this is twofold. First consider the effect of the human capital acquisition factor \( h \). As can easily be confirmed from the auction equilibrium of Proposition 1, the effect of introducing \( h \) to second period wages is simply to increase wages by \( h \), independently of ability and independently of the task choice. Moreover, wages in the first period are not affected by \( h \), because the wage difference in the second period is not affected by \( h \). Now consider the effect of the positive cost of effort in the D task, \( \hat{c} \). Taking into account the effect of \( h \), the productivities in \( \Pi_2 \) net of effort is the same as the productivities in \( \Pi_1 \). Therefore, taking into account \( h \), the equilibrium wages net of effort must be the same. It can easily be shown, and is hence omitted, that given that a separating equilibrium exists for \( \Pi_1 \), there must exist a separating equilibrium for \( \Pi_2 \).

**Proof.** To show that \( \hat{w}_1^D \) can be higher than \( \hat{w}_1^E \), provided \( c \) large enough, notice that for a separating equilibrium it must be the case that

\[
\hat{w}_1^E + \hat{w}_2^E = \hat{w}_1^D + \hat{w}_2^D - 2c
\]  

which holds if \( c > \frac{\hat{w}_2^D - \hat{w}_2^E}{2} \). However, since \( \hat{w}_2^D - \hat{w}_2^E < \pi^H - \pi^0 \), there must exist a range of \( c \) such that a separating equilibrium exists (see Proposition 2), and moreover where \( \hat{w}_1^D > \hat{w}_1^E \). To show that \( \hat{w}_2^E (\hat{w}_2^D) \) can be larger than \( \hat{w}_1^E (\hat{w}_1^D) \) for high enough \( h \) is trivial and hence omitted. ■
8 Appendix D: Performance Contracts

Our justification for not having (a measure of) individual performance as a contractible variable is that for many production processes, measuring individual contribution to profits that go beyond the measurement of effort can be very costly and noisy task.\textsuperscript{36} Moreover, the assumption is consistent with a large empirical literature that shows that real-life payment schemes to a little extent depends on such measures (see Prendergast, 2000). The purpose of the appendix is to show that even when individual output is contractible, the equilibrium contracts can be similar or identical to the (fixed-wage) contracts analyzed in the main text. We illustration, we consider the case when effort is supplied inelastically, and where the wage to a worker can only be made conditional on a measure of his individual output.

For simplicity, we assume that there is only one period, and moreover that $c = 0$. To make our task harder, we assume a very weak form of risk aversion: workers maximize expected (total) wages, but for a given level of (expected) wages, workers prefer a lower-risk scheme to a higher-risk scheme.\textsuperscript{37} If workers are indifferent given this criterion, they choose their efficient task.\textsuperscript{38}

From the assumptions made on risk preferences, we can confine attention to contracts of the following (linear) form,

$$ w = \beta_0 + \beta_Y Y $$  \hfill (C1)

where $Y$ is the (observed) output of an agent, $\beta_0$ is the salary, and $\beta_Y$ is the bonus. We assume that in the E task, the output $\pi^0$ is certain, while the output in the D task is not perfectly observable. $Y_i$ has two possible levels, $\pi_g$ and $\pi_b$, where $\pi_b < \pi^H < \pi_g$. An $H$ worker in the difficult task has a $p_H$ probability of $\pi_g$, while an $L$ worker in the difficult

\textsuperscript{36}For example, several papers within human resource management report how middle managers are reluctant to give individual performance reports for their subordinates. And, as described in the motivating example, the measurement of individual output can be very complex with team processes.

\textsuperscript{37}More general risk preferences give qualitatively the same type of result, but would add technical problems with existence of equilibrium (similar to in Rothschild & Stiglitz, 1975)

\textsuperscript{38}This assumption is made solely for convenience.
task has a \( p_L \) probability of obtaining \( \pi_g \), where \( p_L < p_H \). We assume that,
\[
\begin{align*}
    p_H \pi_g + (1 - p_H) \pi_b &= \pi^H \\
    p_L \pi_g + (1 - p_L) \pi_b &= \pi^L
\end{align*}
\]  
(C2)

As can readily be seen, the deterministic technology studied in the previous sections is obtained for the special case \( p_H = 1, \pi_g = \pi^H, p_L = 0, \) and \( \pi_b = \pi^L \). We have the following result.

**Proposition 6** For sufficiently high \( \pi_0 \) equilibrium contracts will be arbitrarily close fixed-wage (as in the main part of the paper).

**Proof.** We assume that there exists an equilibrium where the L workers choose the E task and get the wage \( \pi_0 \), while the H workers choose the D task and get the scheme \( w = \beta_0 + \beta_Y Y \). For \((\beta_0, \beta_Y)\) to be consistent with equilibrium, it must maximize the utility of the high type, given zero profits and given that the low type prefers to work in the E task. Self-selection of low workers implies that,
\[
\beta_0 + \beta_Y (p_L \pi_g + (1 - p_L) \pi_b) \leq \pi_0 \tag{C3}
\]
while zero profit in the D task implies that,
\[
\beta_0 + \beta_Y (p_H \pi_g + (1 - p_H) \pi_b) = \pi^H \tag{C4}
\]
The second condition determines the salary as,
\[
\beta_0 = \pi^H (1 - \beta_Y) \tag{C5}
\]
Since high workers prefer a lower risk to a higher risk, the self-selection constraint is binding, and we get that,
\[
\beta_Y = \frac{\pi_0 - \beta_0}{p_L \pi_g + (1 - p_L) \pi_b} \tag{C6}
\]
Solving the system, we get that equilibrium contracts \((\beta_0^*, \beta_Y^*)\) must satisfy,
\[
\begin{align*}
    \beta_0^* &= \frac{\pi^H (\pi_b + p_L \pi_g - \pi_b p_L - \pi_0)}{p_L \pi_g + \pi_b - \pi^H - \pi_b p_L} \\
    \beta_Y^* &= \frac{\pi_0 - \pi^H}{p_L \pi_g + \pi_b - \pi^H - \pi_b p_L}
\end{align*}
\]  
(C7)
From those expressions, it can easily be seen that \((\beta^*_0, \beta^*_Y)\) converges to \((\pi^0, 0)\), as \(\pi^0\) approaches \(\pi^H\). ■

Intuitively, when \(\pi^0\) is high, the self-selection constraint becomes easier to satisfy, and hence \(\beta_Y\) can be lowered and still self-selection occurs. When \(\pi_0\) approaches \(\pi^H\), we get that \(\beta_Y\) can be close to zero without self-selection being violated, and since workers are risk averse, the equilibrium \(\beta_Y\) will in fact be close to zero as \(\pi^0\) increases.

Since there is no intrinsic reason why \(\pi^0\) should be close to \(\pi^H\), the result does not seem too strong. However, by adding a cost of monitoring, \(m\), where \(0 < m < \pi^H - \pi^0\), for obtaining a performance measure, one can add realism to the result. As can easily be verified, the result is that when \(m\) is sufficiently high (so that \(\pi^H - \pi^0\) is close to zero), the equilibrium performance contracts are close to the (fixed-wage) contracts considered in the previous sections.

Even if the sufficient condition outlined in Proposition 5 does not hold, there are circumstances under which performance contracts will not be used in equilibrium even if individual output is contractible. One set of circumstances is when workers can commit ex-ante to contractual form (e.g., through labor unions), as the following result shows.

**Proposition 7** If workers can commit to contractual form ex-ante to discovering their ability, equilibrium contracts may consist of fixed wages even if performance contracts were available.

**Proof.** Assume that \((0 < p_l < (\pi_E - \pi_b)/(\pi_g - \pi_b) < p_h < 1)\), that is, for efficiency \(H\) workers should be in the \(D\) task, while \(L\) workers should be in the \(E\) task. In addition, assume that the proportion of \(H\) workers \(\theta\) is such that \(\theta(p_l\pi_g + (1 - p_l)\pi_b) + (1 - \theta)(p_l\pi_g + (1 - p_l)\pi_b) \leq \pi_E\); that is, it is better that all the workers are allocated to the \(E\) task than that all the workers is allocated to the \(D\). (This helps guarantee the existence of a separating equilibrium.) When firms compete for workers by offering contingent contracts, workers choosing the \(E\) task would be paid \(\pi_E\), and workers in \(D\) task would
be paid the \((w^*_g, w^*_b)\) that solves
\[
\max_{w_g, w_b} Eu_h(w_g, w_b) \\
\text{s.t. } p_h w_g + (1 - p_h) w_b \leq p_h \pi_g + (1 - p_h) \pi_b \\
Eu_i(w_g, w_b) \leq u(\pi_E) \tag{C8}
\]
This type of equilibrium is similar to Rothschild & Stiglitz (1975) except we include the moral-hazard of job selection as well. If one cannot write contracts contingent upon outcome, then the equilibrium contract would be \(w^*\) such that \(w^* = \theta(p_h \pi_g + (1 - p_h) \pi_b) + (1 - \theta) \pi_E\). Notice that when workers are risk-averse \(u(w^*) > \theta Eu(w_g, w_b) + (1 - \theta) u(\pi_E)\). Such a contract cannot occur when contingent contracts are available, since one would be able to skim the good workers. This implies that it is best for the ex-ante for the workers to prevent contingent contracts.

9 References


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