Negative value of information in an informed principal problem with independent private values.

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Abstract

This note demonstrates that in a principal-agent environment with independent private values and generic payoffs, the mechanism implemented by an informed principal is *not* ex-ante optimal. This result implies that in (generic) settings where the principal can covertly acquire private information before selecting a mechanism, she will fail to select an ex-ante optimal mechanism. Furthermore, the principal is indifferent whether to become informed before or after selecting a mechanism.

Keywords: ex-ante optimal allocation, informed principal, value of information, information acquisition/gathering.

1 Introduction

This note studies the principal-agent environment with independent private values, in which the principal may have private information at the moment of mechanism selection. It consists of two parts. In the first part (Section 3), I demonstrate that for generic payoff functions, reservation utilities and prior beliefs, the equilibrium allocation implemented by an informed principal is not ex-ante optimal.[1] Because an uninformed principal can implement an ex-ante optimal allocation as an equilibrium in a corresponding (optimal) direct mechanism, the arrival of the principal's private information before mechanism design has negative impact on her ex-ante payoff.

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[1] An ex-ante optimal allocation maximizes the ex-ante payoff of the principal among all interim incentive-compatible and individually rational allocations and can be implemented by an uninformed principal as an equilibrium in an optimal mechanism. A property is generic if it holds on a dense open set of the payoff functions satisfying the assumptions of the model. This genericity concept is the same as in Mas-Colell [10] and Maskin and Tirole [13]. The quasilinear preferences are non-generic in our model and our results will not hold for these preferences.

The second part of this note (Section 4) discusses one implication of this tension between what the informed principal can implement and what the principal would like to implement ex-ante: I consider a setting in which a principal can covertly decide whether to select a mechanism before or after observing her private information.[2] I demonstrate that the equilibrium allocation is equivalent to the allocation implemented by the informed principal and thus generically is not ex-ante optimal. In particular, there is a surprising equilibrium in which the principal stays uninformed until after selecting a mechanism and yet selects a mechanism which would be implemented if she were informed.

In our model, the principal always observes her private information by the time of execution of a mechanism. Therefore, early realization of information may only decrease the principal's ex-ante payoff. One channel through which this can happen is the information leakage through mechanism design: if the agent infers something about the principal's private information from the choice of mechanism, his incentives constraints would be tightened. However, the Inscrutability Principle by Myerson [15] states that the mechanism selected by an informed principal need not signal any information: any allocation which can be achieved by an informed principal can also be achieved without revealing any private information until after the mechanism is played. Moreover, Maskin and Tirole [13] (henceforth, MT) show that generically in equilibrium no principal's private information is revealed to the agent until after mechanism design.

Yet, neither the Inscrutability Principle nor the results in MT imply that an informed principal can achieve an arbitrary allocation and, in particular, an ex-ante optimal allocation. What the informed principal can achieve in equilibrium depends on the deviations available to her different types. In an ex-ante optimal allocation the principal gets insured by increasing payoffs of some types at the expense of other types. This is done to such an extent that these latter types can always construct profitable deviations guaranteed to be accepted by the agent. Hence, the informed principal is unable to implement an ex-ante optimal allocation not because of information leakage to the agent but rather because of the conflict among her own types.

The crucial step in proving this result is to establish a restriction on an ex-ante optimal allocation that must be satisfied if this allocation is implemented by an informed principal. Following MT, I analyze the problem in terms of *indirect* payoff functions of the principal's types defined on the space of slacks in incentive compatibility and individual rationality constraints of the agent. In an ex-ante optimal allocation, the marginal values of these payoff functions are equal across the types of the principal. The allocation selected by the informed principal violates this condition for generic *indirect* payoff functions that are *differentiably strictly concave.* This genericity result is established explicitly through invoking the Transversality and the Implicit Function Theorems. Thus, this note complements the work of MT, who leave the proofs of genericity results out of the scope of their paper.

Under additional assumptions on the primitives of the model, which guarantee that the allocation selected by the informed principal is deterministic, this result implies the equivalent result in the space of *direct* payoff functions defined over allocations. Because of

[2] The assumption that the the principal's decision is covert is in line with the literature on information acquisition in mechanism design (see, e.g., Bergemann and Välimäki [3]). In Section 4, I also discuss how our results would be affected if the principal's decision were observed by the agent.

technical difficulties I have been unable to establish this equivalence for stochastic allocations. However, the result for deterministic allocations is still a valuable observation since due to similar technical reasons the literature almost exclusively restricts attention to deterministic allocations.

Our results do not imply that an informed principal implements an allocation that is worse than an ex-ante optimal allocation for *all* types of the principal. Both allocations are interim incentive-efficient in the sense of Holmström and Myerson [8]: there are no other incentive-compatible and individually rational allocations that generate higher payoffs for all types of the principal. However, (generically) the allocation selected by the informed principal does not generate the highest feasible ex-ante payoff and hence allowing the principal to observe her private information before selecting a mechanism has negative value from the ex-ante perspective. At the same time, as MT has demonstrated (generically) the allocation selected by the informed principal achieves for each type of the principal a payoff strictly higher than would be possible if the principal's information were common knowledge. This reveals that allowing the agent to observe the principal's private information before selecting a mechanism has negative value from both the ex-ante and the interim perspectives.

The consequence of our genericity result is that in (generic) environments where the principal can covertly acquire information before mechanism design she will be unable to implement an ex-ante optimal allocation. In equilibrium, the principal can always deviate and learn her information early. Because the agent anticipates this the only equilibrium outcome is an allocation which would be implemented by the informed principal, regardless of whether the principal actually becomes informed. In fact, in equilibrium the principal is indifferent about when to acquire information.

The question of information acquisition in mechanism design has been recently studied by Bergemann and Välimäki [2]. Their focus is on the existence of mechanisms that simultaneously create efficient incentives for information acquisition by agents and achieve an ex-post efficient allocation. In contrast, in our model the issue is the timing of information acquisition by the principal; information acquisition is costless and the principal always observes her information before execution of the mechanism. This note can also be considered as a counterpart to the analysis in Cremer and Khalil [5], who consider a setting in which the agent has a choice about whether to become informed before or after signing a contract.

The rest of the note is organized as follows. Section 2 presents the environment. Section 3 shows the genericity result. Section 4 considers a setting in which the principal has a choice over when to become informed.

2 Model

Environment. Our environment is identical to the one in MT. There is a principal (she) and an agent (he), who can contract on an observable and verifiable allocation $(y, t) \in \Omega$, where $\Omega = \Omega_y \times \Omega_t$ is a convex and compact subset of $\mathbb{R} \times \mathbb{R}$.[3] (As an example, one might think of the principal and the agent as a buyer and a seller, in which case y is the amount of good

[3] The convexity and compactness of Ω are imposed by MT in order to establish existence of equilibrium in the informed principal game.

exchanged and t is the price paid.)

The agent has a twice continuously differentiable von Neumann - Morgenstern payoff function $U_j(y,t)$, where $j = 1, 2$ is the agent's type.[4] The payoff function $U_{(\cdot)}(\cdot)$ is decreasing in y, increasing in t, and strictly concave in (y,t).[5] It decreases in j,

$$U_1(y,t) > U_2(y,t) \text{ for all } (y,t) \in \Omega,$$

and also satisfies the single-crossing condition,

$$-\frac{\partial U_1/\partial y}{\partial U_1/\partial t} < -\frac{\partial U_2/\partial y}{\partial U_2/\partial t} \text{ for all } (y,t) \in \Omega.$$

The principal has a twice continuously differentiable von Neumann - Morgenstern payoff function $V_i(y,t)$, where $i = 1,\dots,N$, $N \geq 2$, is the principal's type. The payoff function $V_{(\cdot)}(\cdot)$ is increasing in y, decreasing in t, and strictly concave in (y,t).

The agent has interim individual rationality constraint: in any contract *each* type of the agent has to obtain at least his reservation utility $\overline{u}$. There are no individual rationality constraints for the principal.

Let $\underline{t}$ be the minimal transfer in Ω_t. It is assumed that payoff functions are such that (1) given this transfer the agent is worse off than without a contract, regardless of his type or action y:

$$U_j(y,\underline{t}) < \overline{u} \quad \text{for all} \quad (y,\underline{t}) \in \Omega \quad \text{and} \quad j = 1,2.$$

and (2) regardless of the players' types, there exists an allocation such that both players prefer it to no contract.

The types of the players i and j are their private information, independently realized with strictly positive probabilities ϕ_i and p_j.[6] The structure of preferences and rules of the games are common knowledge.

Allocations and mechanisms. Let $\mathcal{P}$ be the set of probability measures over the set of allocations Ω. An allocation rule is a function which maps the types of the players into a distribution over allocations $\nu : \{1,\dots,N\} \times \{1,2\} \to \mathcal{P}$. Let Ψ be the set of allocation rules. For the principal denote the expected payoff of type i, who obtains the allocation of type i', by

$$V_i(i',\nu) = \sum_j p_j \int_\Omega V_i(y,t) d\nu(i',j).$$

Define similarly for the agent

$$U_j(j',\nu) = \sum_i \phi_i \int_\Omega U_j(y,t) d\nu(i,j').$$

[4] MT require payoff functions to be continuously differentiable. However, twice continuous differentiability is needed for our genericity result.

[5] MT require payoff functions to be strictly concave in y and concave in (y,t). The genericity results in MT and this note will not hold if payoffs are linear in t. See Proposition 11 in MT, Yilankaya [21] and Mylovanov [16] for the corresponding results.

[6] MT allow for degenerate probability distributions.

An allocation rule ν is *incentive-compatible* if for $i, i' = 1, \ldots, N$,

$$V_i(i, \nu) \geq V_i(i', \nu) \tag{PIC}$$

and for $j, j' = 1, 2$,

$$U_j(j, \nu) \geq U_j(j', \nu). \tag{AIC}$$

An allocation rule is *individually rational* if for $j = 1, 2$,

$$U_j(j, \nu) \geq \overline{u}. \tag{AIR}$$

A *mechanism* is a game form whose outcome is a probability distribution over allocations.[7] The solution concept for a mechanism is Perfect Bayesian equilibrium. Following MT, the set of allowed mechanisms is determined by two properties: for any mechanism (a) there exists equilibrium regardless of the agent's beliefs about the principal, and (b) the equilibrium payoff correspondence is upper-hemicontinuous and convex-valued in beliefs.[8]

Uninformed and informed principal games. Figure 1 depicts the timing of the game in which the principal selects a mechanism before learning her type. The highest expected payoff for the principal in a Perfect Bayesian equilibrium of this game is obtained by an *ex-ante optimal allocation*, which is a solution of the following program:

$$\max_{\nu \in \Psi} \sum_i \phi_i V_i(i, \nu) \tag{EA}$$

subject to (PIC), (AIC), and (AIR). An ex-ante optimal allocation exists, since the set of allocations rules satisfying the constraints is compact in the weak topology and the objective function is continuous.

Figure 2 depicts the timing of the informed principal game, in which the principal selects a mechanism after she learns her type. The equilibrium allocation in the informed principal problem is characterized in MT. Following MT define an indirect payoff function $V_i(c_i, r_i)$ to be the value function of the program

$$\max_{\nu \in \Psi} V_i(i, \nu) \tag{IU_i}$$

subject to

$$\int_\Omega U_1(y,t) d\nu(i,1) - \int_\Omega U_1(y,t) d\nu(i,2) = -c_i,$$
$$\int_\Omega U_2(y,t) d\nu(i,2) - \overline{u} = -r_i.$$

[7]In our model, the continuation equilibrium in a mechanism offered off the equilibrium path depends on the out-of-equilibrium beliefs of the agent. The Revelation Principle (e.g., Myerson [14] or Dasgupta et al. [6]) does not apply off the equilibrium path and hence we cannot restrict attention to direct mechanisms. However, the Revelation Principle applies on the equilibrium path: any equilibrium outcome can be described by an incentive-compatible and individually rational allocation rule, which is a truth-telling equilibrium outcome of some direct mechanism.

[8]These properties are needed for existence of equilibrium; standard existence results do not apply because the action space includes a choice of a mechanism. Any set of one-stage mechanisms with finite strategy spaces satisfies these properties, given the players can use a public correlation device (needed for convexity of equilibrium correspondence). The requirement (b) could be somewhat weakened. See p. 398 in MT for details.

Figure 1: Uninformed Principal: Ex-ante selection of a mechanism.

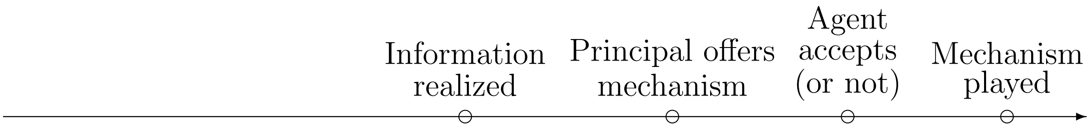

Figure 2: Informed Principal: Interim selection of a mechanism.

Propositions 2 – 4, 6, and 7 in MT prove existence of Perfect Bayesian equilibrium and show that an allocation ν is an equilibrium outcome if and only if there exist strictly positive real numbers μ_r and μ_c such that

1. for $i = 1, \ldots, N$ the allocation $\nu(i, \cdot)$ solves program IU_i given r_i and c_i that are a solution of the program

$$\max_{(r_i,c_i)\in Z_i} V_i(c_i, r_i) \tag{WE_i}$$

$$\text{s.t.} \quad \mu_r r_i + \mu_c c_i = 0, \tag{BC_i}$$

   where $Z_i \subset \mathbb{R} \times \mathbb{R}$ is the set of (r_i, c_i) for which there exists a solution of program IU_i for i with $\nu(i, \cdot)$ that has support on Ω (see Maskin and Tirole [11], the proof of Proposition 2); and

2. ν satisfies AIR for $j = 2$ and AIC for $j = 1$,

$$\sum_i \phi_i r_i = 0, \tag{IIR}$$

$$\sum_i \phi_i c_i = 0. \tag{IIC}$$

MT call these allocations *strong unconstrained Pareto optimum* (henceforth, SUPO).[9] By construction SUPO can be characterized by a vector $(r_1, c_1, \ldots, r_N, c_N)$ which solves program

[9] More precisely, MT (1) define Walrasian equilibrium to be the triple (μ_r, μ_c, ν) that solves programs WE_i and satisfies IIC, IIR and (2) establish equivalence between SUPO and ν of the Walrasian equilibrium for strictly positive beliefs. As MT show SUPO exists and generically there are at most finitely many of them. Under additional assumptions on preferences, Quesada [18] establishes some further properties of SUPO. However, in environments other than with independent private values SUPO may not exist. See Maskin and Tirole [12] and Tisljar [20] for analysis of common values environments and Cella [4] and Kosenok and Severinov [9] for analysis of environments with correlated types.

WE_i, $i = 1, \ldots, N$, for some (μ_r, μ_c) and satisfies IIR and IIC. With some abuse of terminology I call this vector Indirect SUPO.

Interior Solution. I will further assume that the set of payoff functions and the allocations Ω is such that the first order conditions of program WE_i are satisfied with equality, i.e., its solution is interior. Without this restriction the results in this note would not be valid for two reasons. First, Proposition 4 in MT, which is further used for the equilibrium characterization in Proposition 6 in MT, makes use of first order conditions that hold with equality. Second, the first order conditions which hold with equality are a crucial building block in the proof of Lemma 3 in this note.

Properties of indirect payoff function. I also make

Assumption 1. The payoff functions $U_i(y,t)$ and $V_i(y,t)$ are such that the indirect payoff functions $V_i(r_i, c_i)$ are strictly concave and twice continuously differentiable.

These properties are required to prove the regularity of SUPO by showing that the Jacobian of the system of first-order optimality conditions describing SUPO allocations has non-zero determinant. Unfortunately, general conditions under which these properties hold have not been established in the literature and I have also been unable to do so.

On the other hand, these properties are straightforward to obtain if the solution of program IU_i is a deterministic allocation. In the setting with two types of the principal, Quesada [18] imposes the following additional assumptions on preferences:

Assumption 2.

1. The principal has two types.[10]
2. $V_i(y,t) = V(S_i(y) - t)$ and $U_j(y,t) = U(t - \psi_j(y))$, where V and U are increasing and strictly concave, S_i is increasing and concave and $S_2(y) > S_1(y)$, and ψ_j is increasing and convex and $\psi_2(y) > \psi_1(y)$;
3. The function $\Phi(x,y) = U(U^{-1}(x) + \psi_2(y) - \psi_1(y))$ is strictly convex.

Proposition 1 in Quesada states that under Assumption 2 any SUPO allocation is deterministic or, more precisely, that any SUPO allocation has support consisting of one allocation (y,t) for each combination of types of the principal and the agent.[11] However, the proof of Proposition 1 in Quesada does much more: it shows that any solution of program IU_i has support consisting of one allocation (y,t) for each type of the agent. This allows us to obtain

Lemma 1. *If Assumption 2 holds, then the solution of program IU_i is unique and the indirect payoff function $V_i(r_i, c_i)$ is strictly concave and twice continuously differentiable.*

Proof. "Uniqueness of solution". Imagine there are two distinct allocations ν_1 and ν_2 that solve program IU_i. Then, the stochastic allocation ν_S which mixes with equal probabilities

[10] Although MT allow for $n \geq 2$ types of the principal, the proof of the uniqueness of the equilibrium (Proposition 7) in MT is carried out for the case of $n = 2$. MT state that their results are straightforward to generalize for the environments with more than two types.

[11] Quesada shows existence of payoff functions satisfying her assumptions. It can be further shown that the set of payoff functions satisfying Assumption 1 is open in the set of payoff functions satisfying assumptions of our model. The topology for the space of payoff functions on compact Ω is given by C^2 uniform convergence: a sequence of functions f_n converges to a payoff function f, $f_n \to f$, if and only if every derivative of $f_n \to f$ from 0-th to 2-nd order uniformly converges to zero.

between ν_1 and ν_2 is also a solution of program IU_i: it satisfies constraints IIR, IIC and achieves the same value of the objective function as ν_1 and ν_2. However, this allocation has support consisting of two points (y, t) for each agent's type and therefore, as shown in the proof of Proposition 1 in Quesada, cannot be a solution of program IU_i.

"Properties of $V_i(r_i, c_i)$". Strict concavity of $V_i(r_i, c_i)$ follows directly from the uniqueness of the solution of program IU_i over the set of deterministic allocations. An application of the Implicit Function Theorem to the first-order conditions describing the solution of program IU_i over the set of deterministic allocations gives continuous differentiability of the Lagrange multipliers and thus twice continuous differentiability of the value function. □

3 Negative Value of Information

An (indirect) SUPO allocation is computed subject to IIR and IIC and ignoring other incentive compatibility and individual rationality constraints. To facilitate the comparison between the ex-ante optimal allocation and SUPO, define an *unconstrained ex-ante optimum* (UEAO) to be a vector $(r_1, c_1, \ldots, r_N, c_N)$ which maximizes the (indirect) ex-ante expected payoff of the principal $\sum \phi_i V_i(c_i, r_i)$ subject to constraints IIR and IIC. The following lemma establishes restrictions on allocations which are simultaneously Indirect SUPO and UEAO. It states (equation (5)) that marginal values of indirect payoff functions should be equal across the types of the principal.

Lemma 2. *If Indirect SUPO is UEAO, then there exists $\gamma > 0$ such that*

$$\begin{aligned}
\frac{\partial V_i(r_i, c_i)}{\partial r_i} - \gamma_i &= 0 \quad (\textit{for } i = 1, \ldots, N), && (1)\\
\frac{\partial V_i(r_i, c_i)}{\partial c_i} - \gamma_i \mu_c &= 0 \quad (\textit{for } i = 1, \ldots, N), && (2)\\
r_i + \mu_c c_i &= 0 \quad (\textit{for } i = 1, \ldots, N), && (3)\\
\sum_i \phi_i r_i &= 0, && (4)\\
\gamma_i &= \gamma \quad (\textit{for } i = 1, \ldots, N), && (5)
\end{aligned}$$

where $\mu_c, \gamma_i > 0$.

Proof. By definition, Indirect SUPO satisfies equations (1)–(4), with μ_r normalized to 1. Let γ_i be the Lagrange multiplier corresponding to constraint BC_i in program WE_i. Equations (1) and (2) are the first order conditions with respect to r_i and c_i, and (3) is constraint (BC_i) of programs WE_i. Equation (4) is constraint (IIR). The last equation holds if SUPO is UEAO. The first order conditions of the program determining UEAO with respect to r_i and c_i are

$$\frac{\partial V_i}{\partial r_i}(r_i, c_i) - \lambda_r = 0 \quad (\text{for } i = 1, \ldots, N),$$
$$\frac{\partial V_i}{\partial c_i}(r_i, c_i) - \lambda_c = 0 \quad (\text{for } i = 1, \ldots, N).$$

Together with (1) and (2) it implies $\gamma_i = \lambda_r$, $\gamma_i \mu_c = \lambda_c$ and $\gamma_i = \lambda_r =: \gamma$. □

The next Lemma indicates that the conditions under which SUPO is UEAO are limited. We say that a property is generic if it holds on a dense open set of the payoff functions satisfying the assumptions of the model.[12]

Lemma 3. *Indirect SUPO is not UEAO generically on the set of the principal's payoff functions and the agent's prior beliefs about the principal.*

Proof. Let $\mathbb{C}^2_{SC}$ be the set of twice continuously differentiable and strictly concave functions defined on Ω, $\mathbb{V} \subset \times_{i=1}^{N} \mathbb{C}^2_{SC}$ be the set of the principal's payoff functions satisfying the assumptions of our model, and Δ^{N-1} the simplex of dimension $N-1$. I will show that the set the principal's payoff functions and the agent's prior beliefs about the principal for which SUPO is not UEAO is dense in $\mathbb{V} \times \Delta^{N-1}$ by demonstrating that

> (*) if for some principal's payoff functions, $V = (V_1(y,t), \ldots, V_N(y,t)) \in \mathbb{V}$, and the agent's prior beliefs about the principal, $\phi = (\phi_1, \ldots, \phi_N) \in \Delta^{N-1}$, Indirect SUPO is UEAO, then in any open set containing these payoffs and beliefs, $\mathcal{V} \times \mathcal{D} \subseteq \mathbb{V} \times \Delta^{N-1}$, there exist $V' \in \mathcal{V}$ and $\phi' \in \mathcal{D}$ for which Indirect SUPO is not UEAO.

Proposition 1 in MT implies that $r_i \neq 0$ at SUPO generically on $\mathbb{V}$. Let $\mathcal{V}_r$ be the corresponding dense and open subset of $\mathbb{V}$. Hence, because of the Baire property, I only need to prove (*) for $V \in \mathcal{V}_r$. Now, let us assume that Indirect SUPO is UEAO for some $V \in \mathcal{V}_r$ and $\phi \in \mathcal{D}$. Clearly (*) holds if there is no open set $\mathcal{D}_\phi \subset \Delta^{N-1}$ containing ϕ such that Indirect SUPO is UEAO for the principal's payoff functions V and all agent's prior beliefs $\phi \in \mathcal{D}_\phi$.

If this is not the case, let us consider an arbitrary open set $\mathcal{V} \subseteq \mathcal{V}_r$ containing V. Then, there exists $\varepsilon > 0$ such that for all $\epsilon \in (-\varepsilon, \varepsilon)$ the payoff functions $\tilde{V}$ defined as

$$\tilde{V}_i(y,t) = (1 + a_i \epsilon) V_i(y,t), \quad a_i = \begin{cases} 1, & i = 1; \\ -1, & i = 2; \\ 0, & \text{otherwise.} \end{cases}$$

belong to $\mathcal{V}$. Because $\tilde{V}_i(y,t)$ is a monotone transformation of $V_i(y,t)$, the solution of program IU_i is the same for both functions. Hence,

$$\tilde{V}_i(r_i, c_i) = (1 + a_i \epsilon) V_i(r_i, c_i).$$

Since $\tilde{V} \in \mathcal{V} \subset \mathcal{V}_r$ and $r_i \neq 0$, there exists $r_{i'}$, $i' \neq i$, such that $r_1 - r_{i'} \neq 0$. Let D be an arbitrary subset of Δ^{N-1} containing ϕ. To establish (*) I will demonstrate that there exists $\epsilon \in (-\varepsilon, \varepsilon)$ and $\phi' \in D$ such that Indirect SUPO is not UEAO for the principal's payoff functions $\tilde{V}$. In order to do so, by Lemma 2, it is sufficient to show that there exists $\phi' \in \mathcal{D}$ such that the following system is satisfied for the set of ϵ of measure zero,

[12] I do not use a more recent concept of genericity developed in Anderson and Zame [1] because my results rely on genericity results proven in MT using the older concept. See footnote 11 for topology on the space of payoff functions.

$$(1+a_i\epsilon)\frac{\partial V_i(r_i,c_i)}{\partial r_i}-\gamma=0 \quad (\text{for } i=1,\ldots,N), \tag{R_i}$$

$$(1+a_i\epsilon)\frac{\partial V_i(r_i,c_i)}{\partial c_i}-\gamma\mu_c=0 \quad (\text{for } i=1,\ldots,N), \tag{C_i}$$

$$r_i+\mu_c c_i=0, \qquad (\text{for } i=1,\ldots,N), \tag{BE_i}$$

$$\sum_{i\neq i'}\phi_i(r_i-r_{i'})=0. \tag{IIR'}$$

where IIR' is obtained by substituting $\phi_{i'}=1-\sum_{i\neq i'}\phi_i$ in (4).

By our assumption that Indirect SUPO is UEAO for the principal's payoff functions V and beliefs ϕ, the system $(R_i)-(IIR')$ is satisfied for $\epsilon=0$. The Jacobian of this system with respect to $(r_1,c_1,\ldots,r_N,c_N,\gamma,\epsilon,\phi_1,\mu_c)$ evaluated at $\epsilon=0$ is

$$J=\begin{bmatrix}
\frac{\partial^2V_1}{\partial r_1^2} & \frac{\partial^2V_1}{\partial r_1\partial c_1} & 0 & 0 & \ldots & -1 & 1\cdot\gamma & 0 & 0\\
\frac{\partial^2V_1}{\partial r_1\partial c_1} & \frac{\partial^2V_1}{\partial c_1^2} & 0 & 0 & \ldots & -\mu_c & \mu_c\cdot\gamma & 0 & -\gamma\\
0 & 0 & \frac{\partial^2V_2}{\partial r_2^2} & \frac{\partial^2V_2}{\partial r_2\partial c_2} & \ldots & -1 & -1\cdot\gamma & 0 & 0\\
0 & 0 & \frac{\partial^2V_2}{\partial r_2\partial c_2} & \frac{\partial^2V_2}{\partial c_2^2} & \ldots & -\mu_c & -\mu_c\cdot\gamma & 0 & -\gamma\\
\ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots\ldots & \ldots & \ldots\\
1 & \mu_c & 0 & 0 & \ldots & 0 & 0 & 0 & c_1\\
0 & 0 & 1 & \mu_c & \ldots & 0 & 0 & 0 & c_2\\
\ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots\ldots & \ldots & \ldots\\
\phi_1 & 0 & \phi_2 & 0 & \ldots & 0 & 0 & r_1-r_{i'} & 0
\end{bmatrix}$$

where the order of equations is (R_1), (C_1), ..., (R_N), (C_N), (BE_1), ... , (BE_N), (IIR') and it is assumed (without loss of generality) that $i'\neq 2$. Its dimensionality is $2(N+1)+1$ by $2(N+2)$.

Observe that the second last column of Jacobian contains one non-zero element and $2(N+1)$ zeros. This implies that $rank(J)=1+rank(J_1)$, where J_1 is the matrix obtained from J by removing the $2(N+1)+1$-th row and the $2(N+1)+1$-th column. Now, form a matrix by taking first $2(N+1)$ rows with first $2(N+1)$ elements of J. Clearly, $rank(J_1)\geq rank(J_2)$. The matrix J_2 is square and has a determinant, which is equal to

$$2\gamma\cdot\prod_{i=1}^{2}\left(\frac{\partial^2V_i}{\partial r_i^2}\mu_c^2-2\frac{\partial^2V_i}{\partial r_i\partial c_i}\mu_c+\frac{\partial^2V_i}{\partial c_i^2}\right)\prod_{i=3}^{N}\left(\frac{\partial^2V_i}{\partial r_i^2}\frac{\partial^2V_i}{\partial c_i^2}-\left(\frac{\partial^2V_i}{\partial r_i\partial c_i}\right)^2\right). \tag{6}$$

The last product is strictly positive because its each term is a Hessian of V_i. The first product is non-zero because its each term is a quadratic polynomial in μ_c, whose discriminant,

$$D=4\left[\left(\frac{\partial^2V_i}{\partial r_i\partial c_i}\right)^2-\frac{\partial^2V_i}{\partial r_i^2}\frac{\partial^2V_i}{\partial c_i^2}\right],$$

is proportional to the negative of the determinant of the Hessian of $V_i(r_i,c_i)$ and is less than zero. Hence, there is no real μ_c setting any of the terms to zero. This gives $rank(J_2)=2(N+1)$ and $rank(J)=2(N+1)+1$.

Because J has full rank, the Transversality Theorem implies that the system (R_1), (C_1), ..., (R_N), (C_N), (BE_1), (BE_2), (IIR') in variables $(r_1, c_1, \ldots, r_N, c_N, \gamma, \epsilon, \mu_c)$ has a regular solution on an open set $\mathcal{D}' \subseteq \mathcal{D}$ containing ϕ.[13] By the Implicit Function Theorem $(r_1, c_1, \ldots, r_N, c_N, \gamma, \epsilon, \mu_c)$ is a function of ϕ_1 for $\phi \in \mathcal{D}'$.[14] Therefore, for $\phi \in \mathcal{D}'$ system (R_1), (C_1), ..., (R_N), (C_N), (BE_1), (BE_2), (IIR') has a solution only on a set of ϵ of Lebesgue measure zero. This implies that for any $\phi \in \mathcal{D}'$ we can always find ϵ arbitrarily close to 0 such that the system (R_1), (C_1), ..., (R_N), (C_N), (BE_1), (BE_2), (IIR') does not have a solution and hence Indirect SUPO is not UEAO. This establishes claim (*).

The fact that the set of principal's payoffs $\mathcal{V} \subset \mathbb{V}$ and the agent's prior beliefs $\mathcal{D} \subset \Delta^{N-1}$ on which Indirect SUPO is not UEAO is open follows from the upperhemicontinuity of the solutions of programs IU_i and the program corresponding to UEAO in the payoff functions and prior beliefs. □

I now present the main result that generically the informed principal will fail to select an ex-ante optimal allocation.

Proposition 1. *Strong unconstrained Pareto optimum is not ex-ante optimal allocation generically on the set of the principal's payoff functions and the agent's prior beliefs about the principal.*

Proof. Lemma 1 in MT, Proposition 1 in MT, and the proof of Corollary to Proposition 4 in MT imply that the subset of the set of payoff functions and beliefs satisfying the assumptions of our model for which in SUPO the only binding constraints are IIR (AIR for $j = 2$) and IIC (AIC for $j = 1$) is dense and open. By Lemma 3 the subset of payoffs and beliefs on which Indirect SUPO is not UEAO and, hence, the subset on which SUPO differs from an allocation that maximizes the principal's ex-ante payoff subject to AIR for $j = 2$ and AIC for $j = 1$ is dense and open. By the Baire property, the intersection of these two subsets is also dense and open. □

Hirshleifer [7] was first to point out that premature acquisition or release of information may destroy beneficial trading opportunities. Proposition 1 in MT shows that generically SUPO Pareto dominates the allocation implemented under common knowledge of the principal's type, implying that the principal would be strictly hurt if her information were released to the agent. The result that generically SUPO is not ex-ante optimal allocation complements these findings by showing that the principal's ex-ante payoff is decreased even if the information is released only to the principal and not to the agent.

The results obtained in this section will not hold if the players have quasilinear preferences. Proposition 11 in MT and Proposition 1, and Corollary 2 in Mylovanov [16] imply that in this environment the sets of SUPO, UEAO, and ex-ante optimal allocations coincide.

4 Information Acquisition

The results in the previous section could be used to study the outcome of mechanism design in settings where the principal has a choice over when to acquire her private information.

[13] See, for example, Mas-Colell [10], Proposition 8.3.1, p. 320. or Theorem I.2.2, p. 45.

[14] See, for example, Mas-Colell [10], Theorem H.2.2, p. 38.

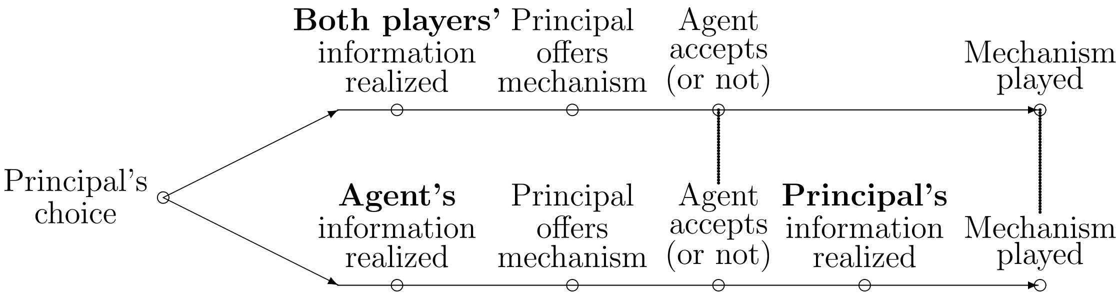

Figure 3: Principal secretly chooses between acquiring information before selecting a mechanism or after selecting but before executing a mechanism. Vertical lines indicate that the agent does not observe when the principal acquires information.

Consider a mechanism selection game with covert information acquisition, in which the principal can privately decide whether to obtain private information before selecting a mechanism or after selecting but before executing a mechanism (Figure 3). In equilibrium, the principal can always secretly learn her type before mechanism design without affecting beliefs of the agent. This allows her to choose between the mechanism expected on the equilibrium path and a deviation to a mechanism that maximizes the payoff of her type given the out-of-equilibrium beliefs of the agent.[15] This implies that any mechanism implemented in an equilibrium of this game can also be implemented in an equilibrium of the informed principal game and therefore it is SUPO. We have,

Proposition 2. *In the mechanism selection game with covert information acquisition, an allocation rule ν is an outcome of a Perfect Bayesian equilibrium if and only if it is SUPO.*

Proof. "Necessity." If in equilibrium, the principal knows her type when she selects a mechanism, she will implement SUPO (Proposition 7 in MT). Suppose now that in equilibrium the principal stays uninformed at the moment of mechanism selection with probability $p > 0$. First, the principal can always deviate and learn her type without affecting the agent's beliefs. Second, the proof of Proposition 7 in MT shows that, regardless of the agents' (out-of-equilibrium) beliefs, the principal can implement an allocation that yields payoffs that are arbitrary close to those of some SUPO. Therefore, her equilibrium payoff should weakly dominate her payoff from this SUPO for each type. However, by definition of SUPO, there is no allocation that strictly dominates SUPO and hence the equilibrium allocation must be SUPO.

"Sufficiency." Choose any $p \in [0, 1]$ and consider the following candidate for equilibrium: (1) with probability p the principal first observes her type and then selects a mechanism and with probability $1-p$ the principal first selects a mechanism and then observes her type; (2) the principal always selects a mechanism whose equilibrium outcome given prior beliefs is a certain SUPO. The proof of Proposition 6 in MT establishes that there exist agents' beliefs for any alternative (out-of-equilibrium) offer of a mechanism such that in the continuation

[15] This is a specific instance of a general observation made by Neyman [17] that in equilibrium a player cannot be made strictly worse off by a unilateral improvement in her information.

equilibrium the principal's payoff is weakly lower than in this SUPO for all types. Hence, assigning these beliefs to the agent guarantee that the principal does not want to deviate and offer a different mechanism. □

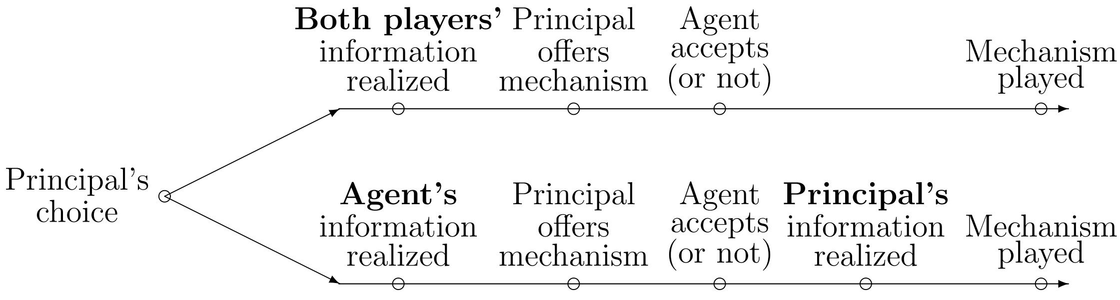

Figure 4: Principal publicly chooses between acquiring information before selecting a mechanism or after selecting but before executing a mechanism.

This result together with Proposition 1 implies that generically the principal who can covertly acquire information before designing a mechanism will fail to implement an ex-ante optimal allocation.

Corollary 1. *In the mechanism selection game with covert information acquisition the equilibrium allocation is not ex-ante optimal generically.*

In our model, there are multiple equilibria that differ in the probability with which the principal gets informed before selecting a mechanism. This is because (1) the principal can design a mechanism which implements SUPO regardless of whether she knows her type at the moment of design and (2) in equilibrium out-of-equilibrium beliefs of the agent are such that any deviation to a different mechanism leads to lower payoffs for each type of the principal (see proof of Proposition 6 in MT). Hence, when the principal acquires her information does not have any effect on the final outcome. In particular, this implies that generically there is an equilibrium in which the *uninformed* principal fails to implement an *ex-ante optimal* allocation.

Corollary 2. *In the mechanism selection game with covert information acquisition there generically exists an equilibrium in which the principal is uninformed at the moment of mechanism design and the equilibrium allocation is not ex-ante optimal.*

It is interesting to contrast these results with the principal's behavior in the environments with other information acquisition structures. In the setting in which the principal's decision about when to become informed is observed by the agent (Figure 4), (generically) the only equilibrium for the principal is to stay uninformed until after mechanism design and select a mechanism which implements an ex-ante optimal allocation:[16] Unlike in the

[16]In the (non-generic) settings where SUPO is an ex-ante optimal allocation, there also exist equilibria in which the principal becomes informed before mechanism design.

setting with covert information acquisition, the principal cannot learn her type without affecting the agent's beliefs. As a result, in equilibrium the principal who deviates and learns her type before selecting a mechanism forgoes the mechanism expected on the equilibrium path and has to restrict her choice to mechanisms that can be implemented by an informed principal. Certainly, if the mechanism expected on the equilibrium path is ex-ante optimal, this deviation does not pay.

The observation that common knowledge of information acquisition may benefit the principal by substituting for the principal's inability to commit not to acquire information early is similar to the result in Shavell [19] that mandatory disclosure is often socially desirable. In Shavell, however, mandatory disclosure limits the cost of information acquisition exhorted by the seller and ensures that socially desirable information is revealed (Shavell, [19], p. 28), whereas in our model it prevents the principal from acquiring information early and does not allow the conflict among different types of the principal to decrease the principal's ex-ante payoff.

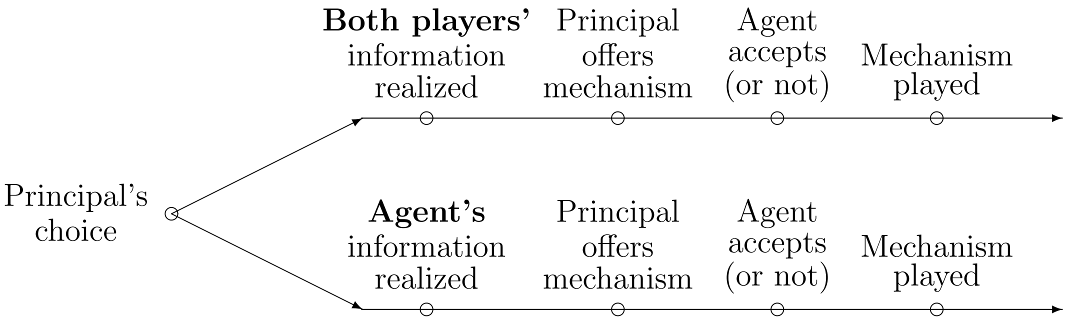

Figure 5: Principal has a choice between acquiring information before selecting a mechanism or never acquiring it.

On the other hand, in the environments in which the principal faces a choice between acquiring information before selecting a mechanism or never acquiring it (Figure 5), the principal will never benefit from postponing information acquisition, regardless of whether the agent observes her decision. In this setting, the uninformed principal has to restrict herself to allocations which do not condition on her type. Clearly, these allocations are weakly Pareto dominated by the allocation that would be implemented if the principal's type were common knowledge, since the latter conditions on the type of the principal. In turn, this latter allocation is (generically) strictly Pareto dominated by the allocation implemented by the informed principal (Proposition 1 in MT).

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