Corporate Control and the Stock Market

Stefano Demichelis* and Klaus Ritzberger†

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Abstract

This paper studies a general equilibrium model with an investor controlled firm. Shareholders can vote on the firm's production plan in an assembly. Prior to that they can trade shares on the stock market. There is always an equilibrium, where share trades lead to owners deciding for competitive behavior, but there may also be equilibria, where monoplistic behavior prevails.

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*Università degli Studi di Pavia, Dipartimento di matematica F. Casorati, Via Ferrata 1, 27100 Pavia, Italy. E-mail. stefanod@netcomp.it

†Institute for Advanced Studies, Vienna, and Vienna Graduate School of Finance, Stumpergasse 56, A-1060 Vienna, Austria. Tel. (+43-1) 599 91-153, Fax. (+43-1) 599 91-555, E-mail. ritzbe@ihs.ac.at

1 Introduction

Economic agents are affected by the activity of firms in two ways. First, as investors they may receive dividends derived from the firms' revenues. Second, as consumers they are affected by externalities generated by firms. Those may be externalities in the narrow sense (pollution, innovation, etc.) or simply the fact that firms' production decisions affect market prices.

Under perfect competition agents are not aware of how firms' production activities affect prices, because they take prices as given. But under imperfectly competitive conditions agents, who understand how firms affect prices, will also understand how they themselves are affected by the firms' decisions. This leads to a failure of shareholder unanimity and constitutes the simplest case, where economic agents hold heterogenous views about what firms ought to do.

In such cases *control* over firms becomes an issue. In modern industrialized societies investors' control over firms is institutionalized through property and control rights. While a variety of securities separate property from control rights (e.g. bonds or preferred stocks), the core institution of *equity* combines them. This creates an industrial democracy distinct from political democracy in two ways. First, a firm's equity owners have a direct saying on the company's operation *in proportion* to their property rights, through voting in shareholder assemblies ("one-share-one-vote"). Second, and again unlike political democracy, voting shares can be *traded* at the stock market.

Despite a wide recognition that *empirically* control commands value (e.g. Zingales (1994), Rydqvist (1996), Modigliani and Perotti (1997), Nenova (2003)) little seems to be known about the interaction of these two aspects on the theory side. The literature on takeover bids has focussed on a partial equilibrium framework, where, first, the control-threshold and the identity of the raider are exogenous and, second, takeovers only affect the value of the firm about which investors otherwise agree (e.g. Grossman and Hart (1980), Grossman and Hart (1988), Harris and Raviv (1988), Bagnoli and Lipman (1988), Hirshleifer and Titman (1990), Burkart, Gromb, and Panunzi (1998)).

General equilibrium treatments have concentrated on the objective function of the firm, ignoring shareholder voting and the conflict that the desire to control creates at the stock market (e.g. Milne (1981), Haller (1986), Grodal (1996), Dierker and Grodal (1996), Dierker and Grodal (1999)). Very few papers have tried to integrate shareholder voting and the interaction on the stock market in a general equilibrium framework. The few that we are aware of include Renström and Yalçin (2003) and Bejan (2003). (The model by Maug and Yilmaz (2002) is a partial equilibrium framework.)

This paper studies a general equilibrium model with a firm that is controlled by investors through shareholder voting. Prior to the shareholder assembly investors can trade their shares at the stock market. We find that there is always an equilibrium, where after trade at the stock market shareholders decide for the efficient production plan, that is, they make the firm behave competitively. On the other hand, there may also be other equilibria, where shareholders decide in favor of monopolistic behavior. The latter tend to be associated with a concentrated ownership structure, while the former are fostered by a dispersed ownership distribution. Only when stock market trades are coordinated by some central agency (in a transferable utility model) monopolistic firms disappear.

The model is admittedly stylized in order to focus on the key issues. In a more general model many feedback effects would appear that are shut down in the present model. But it turns out that the interactions are sufficiently complex even in the present model, so that in order to isolate intuition the simplifications are justified.

The rest of the paper is organized as follows. Section 2 sets out the basic model and analyses the consumption decisions after the firm's production plan has been decided. Section 3 considers shareholder voting in an assembly which determines the firm's production plan. Section 4 provides a model of the stock market, first purely for tender offers, and then for general control trades. Section 5 concludes.

2 The Economy

The model encompasses an arbitrary (but finite) number of economic agents and a single firm. Agents come endowed with a composite commodity ("leisure"), that is perfectly divisible, and with ownership shares in the firm. They wish to consume the composite commodity and a second commodity, that is exclusively produced by the firm and is indivisible.

The interaction extends over three stages. At the initial stage agents can trade shares at a stock market. When the stock market closes, ownership shares are "frozen" and a shareholder assembly may be called. This is the second stage: At the assembly shareholders can decide to participate and vote on the firm's production plan under simple majority rule. Given the outcome of shareholder voting, at the final stage commodity markets open, consumers spend their incomes, and consumption goods prices are determined. This last stage is a standard Cournot-Walras model, as in Gabszewicz and Vial (1972). The three stages are now explained in detail, working backwards from the last to the first.

2.1 Commodity Markets

Consider an economy with n agents ($i \in I = \{1, ..., n\}$) and two goods $l = 0, 1$. Good $l = 0$ ("leisure") is perfectly divisible and can serve both as a consumption good and as a factor of production. Every agent has initially (at the first stage) one unit of endowment with good $l = 0$, i.e. $e_i = (1, 0)$ for all $i \in I$. Commodity $l = 1$ is indivisible and is produced by the only firm in the economy from good $l = 0$ via a technology that converts $c > 0$ units of good $l = 0$ into one unit of good $l = 1$.[1]

Agents' utilities are quasi-linear in good $l = 0$. Each agent i has a valuation ("willingness to pay") $v_i > 0$ for one unit of commodity $l = 1$ (and valuation zero for more than one unit). Agent i's indirect utility function at income $w_i = w$ and relative price $p = p_1/p_0$ is, therefore,

$$U_i(p, w) = w + \max\{0, v_i - p\} \tag{1}$$

For the sake of tractability it is assumed that there are only two types of agents, those with high valuation $v_i = \overline{v}$ and those with low valuation $v_i = \underline{v}$, where $c \leq \underline{v} < \overline{v} \leq 1$. Let $H = \{i \in I \,|v_i = \overline{v}\}$ and $L = \{i \in I \,|v_i = \underline{v}\}$ and denote by m the number of agents with high valuation, $1 \leq m < n$. Then, the aggregate demand function for commodity $l = 1$ is a step function that gives market clearing prices

$$p(y) = \begin{cases} \overline{v} & \text{if} \quad 0 \leq y \leq m \\ \underline{v} & \text{if} \quad m < y \leq n \\ 0 & \text{if} \quad n < y \end{cases} \tag{2}$$

when y is the firm's output.

Agents are also shareholders in the firm, with i owning share $\theta_i \geq 0$, where $\sum_{i \in I} \theta_i = 1$. Therefore, at relative price p and output y agent i's wealth w_i

[1] Under constant returns to scale it is difficult to see why there is a single firm even though each agent could run his own firm and produce as efficiently. To avoid this, a small but positive fixed cost could be assumed without changing the analysis.

(the value of i's interim endowments) is given by $w_i = \omega_i + \theta_i (p - c) y$, where $\omega_i - \theta_i c y$ is agent i's interim endowment with good $l = 0$ and $\theta_i y$ is his interim endowment with good $l = 1$. The part ω_i of i's interim endowment with good $l = 0$ derives from his initial endowment $e_{i0} = 1$ minus what i may have spent on share purchases, or plus what he may have earned from share sales at the stock market. Hence, i's indirect utility from the firm's decision y is given by

$$\begin{aligned} V_i(\theta_i, y) &= \omega_i + \theta_i [p(y) - c] y + \max\{0, v_i - p\} \\ &= \begin{cases} \omega_i + \theta_i [\overline{v} - c] y & \text{if} \quad 0 \leq y \leq m \\ \omega_i + \theta_i [\underline{v} - c] y & \text{if} \quad m < y \leq n \text{ and } v_i = \underline{v} \\ \omega_i + \theta_i [\underline{v} - c] y + \overline{v} - \underline{v} & \text{if} \quad m < y \leq n \text{ and } v_i = \overline{v} \\ \omega_i - \theta_i c y + v_i & \text{if} \quad n < y \end{cases} \end{aligned} \tag{3}$$

Clearly, the maximum of the first line under curly brackets (under the constraint $0 \leq y \leq m$) obtains at $y = m$, the maximum of the second and third line under curly brackets (under the constraint $m < y \leq n$) obtains at $y = n$ (uniquely so if $c < \underline{v}$), and the last line under curly brackets is strictly decreasing in y with a maximum at $y = n + 1$. Denote by $\overline{\pi} = [\overline{v} - c] m > 0$ the maximum profit at the high price $p = \overline{v}$ (low output, $y = m$) and by $\underline{\pi} = [\underline{v} - c] n \geq 0$ the maximum profit at the low price $p = \underline{v}$ (high output, $y = n$).

The indirect utility function V_i in (3) is clearly not single-peaked (quasi-concave). This is fairly typical for models like this. Even with perfect divisibility and in regular cases, like with Cobb-Douglas preferences, the induced indirect utility function over a firm's output will be convex for low shareholdings and (at least) triple-peaked for higher shareholdings. This is because an agent, who holds almost no shares cares exclusively about the externality, while for an investor, who holds more shares, the profit motive becomes more important. The failure of single-peakedness forbids the application of the median voter theorem, which would otherwise be a feasible theory of control for publicly held corporations.[2] This is why we will develop a somewhat more involved theory about shareholder voting (along the lines of Ritzberger(2005)) below.

[2] Some authors apply the median voter theorem in analoguous contexts nevertheless, e.g. Renström and Yalçin (2003).

2.2 Bliss Points

A shareholder with low valuation $v_i = \underline{v}$ prefers the high price $p = \overline{v}$ (low output, $y = m$) to the low price $p = \underline{v}$ (high output, $y = n$) if and only if

$$\theta_i \left[\overline{v} - c\right] m \geq \theta_i \left[\underline{v} - c\right] n \Leftrightarrow \overline{\pi} \geq \underline{\pi}$$

(the maximum profit at the high price is at least as large as the maximum profit at the low price). Such a shareholder prefers the high price ($y = m$) to a zero price (excessive output, $y = n + 1$) if and only if

$$\theta_i \left[\overline{v} - c\right] m \geq \underline{v} - \theta_i c \left(n + 1\right) \Leftrightarrow \theta_i \geq \frac{\underline{v}}{\overline{\pi} + c\left(n + 1\right)} = \frac{\underline{v}}{\overline{v}m + c\left(n - m + 1\right)} \equiv \alpha$$

that is, if and only if her share is larger than a threshold α. Finally, the low price $p = \underline{v}$ (high output, $y = n$) is preferred by such a shareholder to a zero price (excessive output, $y = n + 1$) if and only if

$$\theta_i \left[\underline{v} - c\right] n \geq \underline{v} - \theta_i c \left(n + 1\right) \Leftrightarrow \theta_i \geq \frac{\underline{v}}{\underline{\pi} + c\left(n + 1\right)} = \frac{\underline{v}}{\underline{v}n + c} \equiv \beta$$

Note that $\beta < 1/n$ and $\beta > \alpha$ if and only if $\overline{\pi} > \underline{\pi}$.

That is, low-valuation types with shares below $\min\{\alpha, \beta\}$ favor a zero price (excessive output, $y = n+1$); with shares above this threshold $\min\{\alpha, \beta\}$ they favor the high (resp. low) price if $\overline{\pi} > \underline{\pi}$ (resp. $\overline{\pi} \leq \underline{\pi}$). In other words, low-valuation types with sufficiently many shares ($\theta_i \geq \min\{\alpha, \beta\}$) favor the alternative that yields higher profits.

A shareholders with high valuation $v_i = \overline{v}$ prefers a low price $p = \underline{v}$ (high output, $y = n$) to a high price $p = \overline{v}$ (low output, $y = m$) if and only if

$$\theta_i \left[\overline{v} - c\right] m \leq \theta_i \left[\underline{v} - c\right] n + \overline{v} - \underline{v} \Leftrightarrow \theta_i \leq \frac{\overline{v} - \underline{v}}{\overline{\pi} - \underline{\pi}} \equiv \gamma$$

provided $\overline{\pi} > \underline{\pi}$. (Under this condition the threshold γ satisfies $\gamma \geq 1/m$, because it is strictly decreasing in c and $c \leq \underline{v}$.) If $\overline{\pi} \leq \underline{\pi}$, then she *always* prefers the low price $p = \underline{v}$ (high output, $y = n$) to the high price $p = \overline{v}$ (low output, $y = m$). She prefers a low price $p = \underline{v}$ (high output, $y = n$) to a zero price (excessive output, $y = n + 1$) if and only if

$$\theta_i \left[\underline{v} - c\right] n + \overline{v} - \underline{v} \geq \overline{v} - \theta_i cn \Leftrightarrow \theta_i \geq \frac{\underline{v}}{\underline{\pi} + c\left(n + 1\right)} = \beta$$

that is, if her share is above the threshold β. Finally, she prefers a high price $p = \overline{v}$ (low output, $y = m$) to a zero price (excessive output, $y = n + 1$) if and only if

$$\theta_i \left[\overline{v} - c\right] m \geq \overline{v} - \theta_i c n \Leftrightarrow \theta_i \geq \frac{\overline{v}}{\overline{\pi} + c\,(n+1)} \equiv \eta$$

i.e., if her share is at least as large as a threshold η. Note that $n > m$ and $\overline{v} > c$ imply $\beta < \eta$. Moreover, because $\overline{v} > c$, the inequality $\eta < \gamma$ holds if and only if $\overline{\pi} > \underline{\pi}$.

That is, high-valuation types with shares below β favor a zero price, with shares between β and η they favor a low price. If $\overline{\pi} > \underline{\pi}$, then $\eta < \gamma$, and high-valuation types with shares between η and γ also favor a low price. Otherwise, if $\overline{\pi} \leq \underline{\pi}$, the threshold γ is negative, $\gamma < 0 < \eta$, and high-valuation types with shares above η continue to favor the low price. With shares above γ (resp. η) they favor a high (resp. low) price if $\overline{\pi} > \underline{\pi}$ (resp. $\overline{\pi} \leq \underline{\pi}$).

The thresholds are ordered as follows: $1/m > \eta > \max\{\alpha, \beta\}$, and $\overline{\pi} > \underline{\pi}$ if and only if $\alpha < \beta$ and $\eta < \gamma$. The following table summarizes the bliss points of the two types in terms of the desired production plans as functions of their shareholdings.

$\overline{\pi} > \underline{\pi}$	$0 \leq \theta_i < \alpha$	$\alpha \leq \theta_i < \beta$	$\beta \leq \theta_i \leq \gamma$	$\gamma < \theta_i$
$v_i = \underline{v}$	$y = n+1$	$y = m$	$y = m$	$y = m$
$v_i = \overline{v}$	$y = n+1$	$y = n+1$	$y = n$	$y = m$

$\overline{\pi} \leq \underline{\pi}$	$0 \leq \theta_i < \beta$	$\beta \leq \theta_i$
$v_i = \underline{v}$	$y = n+1$	$y = n$
$v_i = \overline{v}$	$y = n+1$	$y = n$

Therefore, for the parameter constellation $\overline{\pi} \leq \underline{\pi}$ the bliss points of shareholders are independent of their types. All shareholders with shares below β ($< 1/n$) will favor a zero price (excessive output, $y = n + 1$), and all shareholders with shares above β will favor the low price $p = \underline{v}$ (high output, $y = n$). No shareholder will ever want a high price.

A conflict between shareholders with equal shares, but different types, arises only when $\overline{\pi} > \underline{\pi}$ and shares are at least α ($< 1/m$). With shares between α and β low-valuation types favor a high price, but high-valuation types favor a zero price. With shares between β and γ low-valuation types continue favoring a high price, but high-valuation types favor a low price. Only with shares above γ the two types again agree on the high price.

The interesting parameter range, therefore, is $\overline{\pi} > \underline{\pi}$, where a conflict between shareholders with equal shares, but different types, can arise. Since γ can be large (as compared to α), when $\overline{\pi} - \underline{\pi} > 0$ becomes small (compared to $\overline{v} - \underline{v}$), the region of potential conflict can be wide. For this reason we henceforth concentrate on the case $\overline{\pi} > \underline{\pi}$. (This will hold, for instance, if $m/n \geq \underline{v}/\overline{v}$, because $\overline{\pi} > \underline{\pi}$ is equivalent to $c\,(1 - m/n)\,/\overline{v} > \underline{v}/\overline{v} - m/n$.)

Moreover, it will be *assumed* that the alternatives of the firm are restricted to $y = m$ (low output, high price) and $y = n$ (high output, low price). This is for two reasons. First, a zero price entails negative profits. If this were an equilibrium outcome, rational investors, foreseeing that the firm will lose money, would not invest into shares in the first place—so, the firm would not come into existence in equilibrium. Second, only investors with very small shares (below $\beta < 1/n$) will favor a zero price. Most likely, a shareholder assembly, where excessive output is proposed, can never be won.

Among the two possible output choices of the firm, $y = m$ and $y = n$, the efficient one is $y = n$. For, by quasi-linearity of utility there exists a (welfare) representative consumer with indirect utility function

$$V\,(y) = \sum_{i=1}^{n} V_i\,(\theta_i, y) = n + [p\,(y) - c]\,y + m \max\{0, \overline{v} - p\,(y)\} \tag{4}$$

because $\sum_{i \in I} \omega_i = n - q \sum_{i \in I} \left(\theta_i - \bar{\theta}_i\right) = n$ by stock market clearing, where $\bar{\theta}_i$ is i's initial share and q the stock market price of shares. Since $V\,(n) - V\,(m) = (n - m)\,(\underline{v} - c) \geq 0$ with strict inequality if $c < \underline{v}$, it follows that $y = n$ is welfare maximizing. Henceforth we refer to $y = n$ as the *efficient* production plan and to $y = m$ as the *monopolistic* production plan.

Since for $c = \underline{v}$ the two production plans are both efficient, the focus will be on the case $c < \underline{v}$ in the sequel. Thus, in summary, the assumptions on parameters are henceforth $\overline{\pi} > \underline{\pi}$, $y \in \{m, n\}$, and $c < \underline{v}$.

2.3 Budget Constraints

In principle there are two possibilities to account for budget constraints. The first possibility consists of a sequence of two budget constraints for each agent, one for the stage of share trading, where a third commodity is traded against shares, and one for the period of production and consumption. This would mean that agents cannot take credit on account of their future income.

In particular, takeover attempts could not be debt financed. Yet, debt financed bootstrap acquisitions have been important in the US takeover wave of the 1980ties (see Müller and Panunzi (2004)). Moreover, a sequence of two budget constraints for each agent would take a specification of the utility derived from the third commodity against which shares are traded at the initial stage. The advantage of this modelling alternative would be that the ultimate budget constraint can be made contingent on the firm's production plan.

The other possibility is a single budget constraint that extends over all three stages of the model. For each agent $i \in I$ this is of the form

$$1 + \theta_i \pi + q\bar{\theta}_i \geq px_1 + q\theta_i$$

where $x_1 \in \{0, 1\}$ is the purchasing decision of the indivisible commodity $l = 1$, q is the stock market price, $\bar{\theta}_i$ is i's initial endowment with shares, and both π and p depend on the firm's output y (and $\omega_i = 1 - q\left(\theta_i - \bar{\theta}_i\right)$). Since first-date transactions (at the stock market) enter this budget constraint, it has to hold for all production plans of the firm. On the other hand, a single budget constraint fits better with general equilibrium and allows for debt financed share acquisitions.

The single budget constraint implies for agent $i \in I$ that

$$q\left(\theta_i - \bar{\theta}_i\right) \leq \begin{cases} 1 + \theta_i \underline{\pi} - \underline{v} & \text{if} \quad y = n \\ 1 + \theta_i \overline{\pi} - \overline{v} & \text{if} \quad y = m \text{ and } v_i = \overline{v} \\ 1 + \theta_i \overline{\pi} & \text{if} \quad y = m \text{ and } v_i = \underline{v} \end{cases} \tag{5}$$

Since $\underline{v} < \overline{v} \leq 1$ was assumed, the right hand side of this budget constraint is always strictly larger than $\theta_i \underline{\pi}$, provided $\theta_i > 0$. Therefore, any share purchase that does not involve paying a control premium (that is, a price above the after-trade value of the firm) is affordable. Since there is always some slack, also share purchases that do involve a moderate premium are affordable under a single budget constraint per agent. As a consequence, verifying solvency boils down to showing that, as a buyer, i either pays no control premium or, if he does, the premium is not too high.

3 Shareholder Voting

Shareholders' bliss points determine their voting behavior on the firm's production plan in a shareholder assembly. By the assumption that $y \in \{m, n\}$

the vote among shareholders is a binary decision. As a consequence, a variant of the shareholder voting model by Ritzberger (2005) can be applied.

For a given share distribution $\theta = (\theta_1, ..., \theta_n) \in \Theta = \left\{\theta \in \mathbf{R}^n_+ \,|\sum_{i=1}^n \theta_i = 1\right\}$ denote the set of shareholders supporting the efficient production plan by $H(\theta) = \{i \in H \,|\theta_i < \gamma\}$ and the set of supporters of monopolistic output by $L(\theta) = L \cup \{i \in H \,|\theta_i > \gamma\}$. High-valuation shareholders $i \in H$ with $\theta_i = \gamma$ are indifferent with respect to the firm's production plan.

The assumptions on the shareholder assembly are as follows. Every shareholder ($i \in I$ with $\theta_i > 0$) has to decide whether or not to participate in the assembly and, if he does, how to cast his vote. Participation in the assembly carries a small privately born cost—a "lexicographic" preference to get one's preferred alternative without participation. The participation costs of agent $i \in I$ are random, observed only privately by i, and are distributed according to a continuous (cumulative) distribution function $F_i : \mathbf{R}_+ \to [0, 1]$. The decision in the assembly is with simple majority of represented votes, where one share counts for one vote.

Every equilibrium comes with a status quo. This is the production plan supported by (one of) the largest shareholder(s). This "dominant" shareholder is committed to participate in the assembly. (His participation costs are zero or negative with certainty.) Therefore, ties in the vote are broken in favor of the status quo, that is, the dominant shareholder breaks ties. The assumption of small participation costs implies that, with the exception of the dominant shareholder, an agent participates in the assembly if and only if he is pivotal. And, once he participates, he will cast his vote in favor of his preferred alternative.

For this situation Dorofeenko et al. ((2005), Proposition 1) prove a characterization of pure strategy Nash equilibria. There exists a pure strategy Nash equilibrium of the voting game if and only if one of the following two conditions holds: (a) Either the dominant shareholder ι holds at least as many shares as any one of the opponents of his preferred production plan; in this case the status quo survives with certainty. (b) Or there is a subgroup $B \subseteq I \setminus A$ of the set of opponents of the status quo such that

$$\sum_{i \in B} \theta_i > \theta_\iota + \max_{j \in A \setminus \{\iota\}} \theta_j \text{ and } \theta_\iota \geq \sum_{i \in B \setminus \{k\}} \theta_i \text{ for all } k \in B$$

where A denotes the set of supporters of the dominant shareholder $\iota \in A$; in this case the status quo is dismissed by the assembly with certainty.

Without the assumption of a committed dominant shareholder the first type of pure equilibrium disappears and only the second type with $B = \{i\}$ for some $i \in I \setminus A$ survives (see Ritzberger (2005)). The advantage of the variant with a committed "president," therefore, is that the status quo is supported by a pure strategy equilibrium. Nevertheless, the characterization result implies that there are many share distributions $\theta \in \Theta$ for which no pure strategy equilibrium exists. For such share distributions only mixed strategy equilibria exist. Those are described in what follows.

3.1 Voting Equilibria

Suppose that at $\theta \in \Theta$ the status quo is monopolistic output $y = m$, but a shareholder assembly is called. Denote by $\sigma_i \in [0, 1]$ shareholder i's participation probability and by $\Lambda^* (\sigma)$ the success probability of monopolistic output, $\sigma = (\sigma_1, ..., \sigma_n) \in [0, 1]^n$. Define the set of coalitions for which the monopolistic production plan wins as

$$W = \left\{ C \in 2^I \left| \sum_{i \in C \cap L(\theta)} \theta_i \geq \sum_{i \in C \cap H(\theta)} \theta_i \right. \right\}$$

where 2^I denotes the power set of I. (The efficient production plan $y = n$ wins in all coalitions $C \in 2^I \setminus W$.) When the status quo is $y = m$, then $\emptyset \in W$. For each $i \in L(\theta)$ let $P_i = \{C \in W \,|C \setminus \{i\} \notin W\}$ be the set of coalitions for which monopolistic output is adopted and i is pivotal. For $i \in H(\theta)$ let $P_i = \left\{C \in 2^I \setminus W \,|C \setminus \{i\} \in W\right\}$ be the set of coalitions for which efficient output wins and i is pivotal. Of course, if $C \in P_i$ then $i \in C$ is also pivotal in $C \setminus \{i\}$; but $C \setminus \{i\}$ is not a coalition for which i's preferred alternative wins.

For every coalition $C \in 2^I$ and strategy combination $\sigma \in [0, 1]^n$ denote by $\rho_\sigma (C) = \prod_{i \in C} \sigma_i \prod_{i \in I \setminus C} (1 - \sigma_i)$ the realization probability of C under σ. The success probability of monopolistic output $\Lambda^* (\sigma)$ is a function $\Lambda^* : [0, 1]^n \to [0, 1]$ defined by $\Lambda^* (\sigma) = \sum_{C \in W} \rho_\sigma (C)$. The probability that i is pivotal, given $\sigma \in [0, 1]^n$, is

$$p_i (\sigma) = \sum_{C \in P_i} [\rho_\sigma (C) + \rho_\sigma (C \setminus \{i\})] = \sum_{C \in P_i} \prod_{j \in C \setminus \{i\}} \sigma_j \prod_{j \in I \setminus C} (1 - \sigma_j)$$

If $C \in W \setminus P_i$ for some $i \in L (\theta)$, then $C \setminus \{i\} \in W \setminus P_i$, because if $C \setminus \{i\} \notin W$ were true, then $C \in P_i$ would obtain (and $C \setminus \{i\}$ cannot belong to P_i by definition). Conversely, if $C \backslash \{i\} \in W \backslash P_i$ for some $i \in L(\theta)$, then $C \in W \backslash P_i$, because adding i's votes cannot make i's preferred alternative lose. Thus, $C \in W \setminus P_i$ if and only if $C \setminus \{i\} \in W \setminus P_i$ for any $i \in L(\theta)$. It follows that for any $i \in L(\theta)$

$$\begin{aligned}\Lambda^*(\sigma) &= \sum_{i \in C \in W \backslash P_i} [\rho_\sigma(C) + \rho_\sigma(C \setminus \{i\})] + \sum_{C \in P_i} \rho_\sigma(C) \\ &= \sum_{i \in C \in W \backslash P_i} [\rho_\sigma(C) + \rho_\sigma(C \setminus \{i\})] + \sigma_i p_i(\sigma)\end{aligned}$$

so that $\partial \Lambda^*(\sigma) / \partial \sigma_i = p_i(\sigma)$, which is Russo's formula (Russo (1982)). Likewise, for any $i \in H(\theta)$ a similar argument implies that $p_i(\sigma) = -\partial \Lambda^*(\sigma) / \partial \sigma_i$. Defining

$$\Lambda_i^*(\sigma) = \sum_{i \in C \in W \backslash P_i} [\rho_\sigma(C) + \rho_\sigma(C \setminus \{i\})]$$

one can write $\Lambda^*(\sigma) = \Lambda_i^*(\sigma) + \sigma_i p_i(\sigma)$ for any $i \in L(\theta)$.

If shareholder $i \in L$ participates in the assembly with probability σ_i and her vote is pivotal with probability $p_i(\sigma)$, he obtains

$$\begin{aligned}&\Lambda^*(\sigma)[V_i(\theta_i, m) - V_i(\theta_i, n)] + V_i(\theta_i, n) - \sigma_i \varepsilon_i \\ =\ &\Lambda_i^*(\sigma)[V_i(\theta_i, m) - V_i(\theta_i, n)] + V_i(\theta_i, n) \\ &+\sigma_i p_i(\sigma)[V_i(\theta_i, m) - V_i(\theta_i, n)] - \sigma_i \varepsilon_i\end{aligned}$$

where ε_i is the participation cost. Therefore, participation is at least as good as not to participate if $p_i(\sigma)[V_i(\theta_i, m) - V_i(\theta_i, n)] \geq \varepsilon_i$. If the cost ε_i is distributed according to the distribution function F_i, shareholder $i \in L$ will participate with probability

$$\sigma_i = F_i(p_i(\sigma)[V_i(\theta_i, m) - V_i(\theta_i, n)]) = F_i(p_i(\sigma)\theta_i \Delta) \tag{6}$$

where $\Delta = \overline{\pi} - \underline{\pi} > 0$. An analogous computation for high-valuation types $i \in H \cap L(\theta)$, who prefer the monopolistic output $(\theta_i > \gamma)$, yields

$$\sigma_i = F_i(p_i(\sigma)[\theta_i - \gamma]\Delta) \tag{7}$$

because for such shareholders $V_i(\theta_i, m) - V_i(\theta_i, n) = \theta_i \Delta - \overline{v} + \underline{v}$, where $\gamma = (\overline{v} - \underline{v}) / \Delta$. For high-valuation types $i \in H(\theta)$ with shares $\theta_i \leq \gamma$ a similar argument implies that

$$1 - \Lambda^*(\sigma) = \sum_{i \in C \in (2^I \backslash W) \backslash P_i} [\rho_\sigma(C) + \rho_\sigma(C \setminus \{i\})] + \sigma_i p_i(\sigma)$$

$$
\begin{aligned}
&= 1 - \Lambda_i^* (\sigma) + \sigma_i p_i (\sigma) , \text{ where} \\
\Lambda_i^* (\sigma) &= 1 - \sum_{i \in C \in (2^I \setminus W) \setminus P_i} [\rho_\sigma (C) + \rho_\sigma (C \setminus \{i\})]
\end{aligned}
$$

for any $i \in H(\theta)$. Therefore, a high-valuation type $i \in H(\theta)$, who participates in the assembly with probability σ_i and whose vote is pivotal with probability $p_i(\sigma)$, obtains

$$
\begin{aligned}
& (1 - \Lambda^* (\sigma)) [V_i (\theta_i, n) - V_i (\theta_i, m)] + V_i (\theta_i, m) - \sigma_i \varepsilon_i \\
= \; & (1 - \Lambda_i^* (\sigma)) [V_i (\theta_i, n) - V_i (\theta_i, m)] + V_i (\theta_i, m) \\
& + \sigma_i p_i (\sigma) [V_i (\theta_i, n) - V_i (\theta_i, m)] - \sigma_i \varepsilon_i
\end{aligned}
$$

where again ε_i denotes participation costs. Hence, it is preferable to participate if $p_i(\sigma)[V_i(\theta_i, n) - V_i(\theta_i, m)] \geq \varepsilon_i$. For high-valuation types $V_i(\theta_i, n) - V_i(\theta_i, m) = \overline{v} - \underline{v} - \theta_i \Delta$, so that for stochastic participation costs distributed according to F_i a shareholder $i \in H(\theta)$ will participate with probability

$$
\sigma_i = F_i (p_i (\sigma) [\gamma - \theta_i] \Delta) \tag{8}
$$

Note that for a high-valuation shareholder $i \in H$ with share $\theta_i = \gamma$ the participation probability is $\sigma_i = F_i(p_i(\sigma) \Delta [\gamma - \theta_i]) = F_i(0) = 0$, that is, such a shareholder will never participates, because she is indifferent with respect to the firm's decision.

3.2 Selection

Denote by $\lambda \in [0, 1]$ the probability that the firm will adopt the monopolistic production plan and let $\pi(\lambda) = \lambda \overline{\pi} + (1 - \lambda) \underline{\pi} = \underline{\pi} + \lambda \Delta$, where $\Delta = \overline{\pi} - \underline{\pi} > 0$. As shown above, the probability λ depends on the share distribution $\theta = (\theta_1, ..., \theta_n) \in \Theta$, as $\sigma \in [0, 1]^n$ is the solution to the equation system (6), (7), and (8), that is, $\sigma = \sigma(\theta)$. Since the solutions to this equation system are given by a correspondence, a selection from this correspondence is required.[3]

Fix a selection that satisfies (a) whenever (6), (7), and (8) admit a solution in pure strategies, $\sigma \in \{0, 1\}^n$, this is selected, and (b) if there is a subset $H' \subseteq H(\theta)$ (resp. a subset $L' \subseteq L(\theta)$) such that $\sum_{k \in H'} \theta_k >$

[3] A *selection* from a correspondence $F : \Theta \to [0, 1]^n$ is any function $f : \Theta \to [0, 1]^n$ such that $f(\theta) \in F(\theta)$ for all $\theta \in \Theta$.

$\max_{i,j \in L(\theta), i \neq j} (\theta_i + \theta_i) \geq \sum_{k \in H' \setminus \{i\}} \theta_k$ for all $i \in H'$ (resp. such that $\sum_{k \in L'} \theta_k > \max_{i,j \in H(\theta), i \neq j} (\theta_i + \theta_i) \geq \sum_{k \in L' \setminus \{i\}} \theta_k$ for all $i \in L'$), but also $\max_{i \in L(\theta)} \theta_i \geq \theta_j$ for all $j \in H(\theta)$ (resp. $\max_{i \in H(\theta)} \theta_i \geq \theta_j$ for all $j \in L(\theta)$), then the selection assigns the solution, where the monopolistic production plan is adopted (resp. where the efficient production plan is adopted).

That is, the selection is chosen such that, whenever shareholder voting has a pure strategy equilibrium, the selection picks it. Moreover, if there are multiple pure equilibria, the selection picks the one, where less coordination of voters' expectations is required. Denote by $\Lambda : \Theta \to [0,1]$ the composition of the selection with the function Λ^* that maps $\sigma \in [0,1]^n$ into the success probability λ of the monopolistic production plan.

The function Λ captures the outcome of shareholder voting as far as production plans are concerned. At the same time the selection pins down what is required to alter the production plan by changes in the share distribution. In particular, if $\Lambda(\theta) = 1$ (resp. $\Lambda(\theta) = 0$), then $\Lambda(\theta') < 1$ (resp. $\Lambda(\theta') > 0$) requires that $\max_{i \in L(\theta)} \theta_i \geq \theta_j$ for all $j \in H(\theta)$ (resp. $\max_{i \in H(\theta)} \theta_i \geq \theta_j$ for all $j \in L(\theta)$), but $\max_{i \in H(\theta')} \theta'_i > \theta'_\iota$ for some $\iota \in \arg\max_{i \in L(\theta)} \theta_i$ (resp. $\max_{i \in L(\theta')} \theta'_i > \theta'_\iota$ for some $\iota \in \arg\max_{i \in H(\theta)} \theta_i$). In other words, a change (in the probabilities) of the production plan(s) by trading from θ to θ' requires that after trade (at θ') a different type than before trade (at θ) is the largest shareholder. Otherwise the dominant shareholder would still be of the same type as before trade, which would imply no change in the production plan.

4 The Stock Market

The shareholdings that are relevant for the vote on the production plan are those that result from trade at the stock market prior to the shareholder assembly. Denote by $\bar{\theta} \in \Theta$ the original share distribution that constitutes part of the agents' initial endowments.

Modelling the stock market is a nontrivial exercise, because share trades imply possibly different outcomes (λ's) in shareholder voting than with $\bar{\theta}$. Trading at the stock market is, therefore, genuinely strategic. In particular, unless no change of λ is implied by stock market activity, how a given net trade is evaluated by an agent depends on what other agents trade. More precisely, it depends on whether or not an agent's net trade is pivotal or not for a target change in the probability λ of the monopolistic production plan.

At a given share distribution $\theta \in \Theta$ and an implied probability $\lambda = \Lambda(\theta)$ of monopolistic output shareholder i's utility is

$$u_i(\theta_i, \lambda) \equiv 1 + \theta_i \pi(\lambda) + (1-\lambda)\max\{0, v_i - \underline{v}\}$$

Accordingly, the utility difference resulting from a trade from θ_i to θ_i' with an implied change in the probability of monopolistic output from $\lambda = \Lambda(\theta)$ to $\lambda' = \Lambda(\theta')$ is given by

$$\begin{aligned} u_i(\theta_i', \lambda') - u_i(\theta_i, \lambda) - q(\theta_i' - \theta_i) &= \\ (\lambda' - \lambda)[\theta_i'\Delta - \max\{0, v_i - \underline{v}\}] - (\theta_i' - \theta_i)[q - \pi(\lambda)] &= \\ (\lambda' - \lambda)[\theta_i\Delta - \max\{0, v_i - \underline{v}\}] - (\theta_i' - \theta_i)[q - \pi(\lambda')] & \end{aligned} \tag{9}$$

Consequently, $i \in I$ prefers shares θ_i' purchased (sold) at price q combined with the probability λ' of monopolistic output to the status quo of shares θ_i and λ if and only if

$$\begin{aligned} u_i(\theta_i', \lambda') - u_i(\theta_i, \lambda_i) &\geq q(\theta_i' - \theta_i) \Leftrightarrow \\ (\lambda' - \lambda_i)\Delta\left[\theta_i - \frac{\max\{0, v_i - \underline{v}\}}{\Delta}\right] &\geq (\theta_i' - \theta_i)[q - \pi(\lambda')] \Leftrightarrow \\ (\lambda' - \lambda_i)\Delta\left[\theta_i' - \frac{\max\{0, v_i - \underline{v}\}}{\Delta}\right] &\geq (\theta_i' - \theta_i)[q - \pi(\lambda_i)] \end{aligned} \tag{10}$$

where $\lambda_i = \Lambda\left(\theta_{-i}', \theta_i\right)$ is the probability of monopolistic output when all others trade to θ_{-i}', but i refuses to trade—assuming that this is feasible. If i is pivotal, in the sense that the change from λ to λ' fails if he refuses to trade, then $\lambda_i = \lambda$. If it succeeds irrespective of whether or not i trades, then $\lambda_i = \lambda'$. Consequently, at $\lambda' = \lambda_i$ shareholder i will want to buy (sell) an arbitrary amount of shares whenever $q < \pi(\lambda) = \pi(\lambda')$ $(q > \pi(\lambda) = \pi(\lambda'))$, and at $q = \pi(\lambda) = \pi(\lambda')$ he is indifferent.

To model the full complexity of stock market trades as a non-cooperative game seems intractable. Therefore, we limit the present analysis to a criterion that identifies when an initial share distribution $\bar{\theta} = \theta$ will not be changed by stock market activity aimed at buying control over the firm. Methodologically this bears some resemblance to solution concepts for cooperative games, as it declares something (in this case some $\bar{\theta} = \theta$) a "solution" if it is immune against a certain class of "deviations" (in this case stock market trades).

Two such criteria will be studied. For both the object is an initial share distribution $\bar{\theta} = \theta$, and for both the class of allowed "deviations" will be stock market (net) trades at a *single* price. The two criteria differ with respect to how trades are initiated. The first criterion will consider tender offers that are unilaterally proposed by an agent. The second will allow for coordinated trades that are proposed by a benevolent "auctioneer."

4.1 Tender Offers

Tender offers are offers by an agent to buy a specified quantity of shares at a specified price with the goal to alter the chances of the firm's production plans in shareholder voting. For an initial share distribution $\bar{\theta} = \theta$ to qualify as an "equilibrium" distribution it is necessary that no agent can make a tender offer which is successful in the sense that other agents will voluntarily supply the shares required for the offer to succeed.

For the formal development, the following definition the notion of a rationing rule is needed. For $x > 0$ define $\Omega_x = \left\{\xi \in \mathbf{R}_-^{n-1} \left| x + \sum_{i=1}^{n-1} \xi_i \leq 0 \right.\right\}$ and $\partial\Omega_x = \left\{\xi \in \mathbf{R}_-^{n-1} \left| x + \sum_{i=1}^{n-1} \xi_i = 0 \right.\right\}$. A (supply side) *rationing rule* is a function $r_x : \Omega_x \to \partial\Omega_x$ such that $\xi \leq r_x(\xi)$ for all $\xi \in \Omega_x$.[4] Denote by $\mathcal{R}(x)$ the set of all such rationing rules r_x for given $x > 0$.

Definition 1 *A* ***tender offer*** *by $i \in I$ is a pair $(q, x_i) \in \mathbf{R}_{++}^2$ consisting of a bid price $q > 0$ and a quantity $x_i > 0$ of shares demanded by i. The* ***game induced*** *by a tender offer (q, x_i) by i at the share distribution $\bar{\theta} \in \Theta$ is the n-player normal form game $\Gamma_{\bar{\theta}}(q, x_i) = (S, \upsilon)$, where $S = \times_{j=1}^{n} S_j$ with $S_j = \left[-\bar{\theta}_j, 0\right]$ for all $j \neq i$ and $S_i = \mathcal{R}(x_i)$, and $\upsilon = (\upsilon_1, ..., \upsilon_n) : S \to \mathbf{R}^n$ is given by*

$$\begin{aligned} \upsilon_j(s) = 1 - q r_j(s_{-i}) + \left(\bar{\theta}_j + r_j(s_{-i})\right) \pi\left(\Lambda\left(\bar{\theta} + r(s_{-i})\right)\right) \\ + \left[1 - \Lambda\left(\bar{\theta} + r(s_{-i})\right)\right] \max\left\{0, \upsilon_j - \underline{\upsilon}\right\} \end{aligned} \tag{11}$$

for all $s \in S$ and all $j \in I$, where $r = (s_i, x_i) \in S_i \times \{x_i\} = \mathcal{R}(x_i) \times \{x_i\}$ if $x_i + \sum_{j \neq i} s_j \leq 0$ and $r(s_{-i}) = 0 \in \mathbf{R}^n$ if $x_i + \sum_{j \neq i} s_j > 0$, and $s_{-i} = (s_1, ..., s_{i-1}, s_{i+1}, ..., s_n)$.

[4] Notation for vector inequalities is standard: $x \geq y$ means $x_i \geq y_i$ for all i, $x > y$ means $x \geq y$ but not $x = y$, and $x \gg y$ means $x_i > y_i$ for all i.

That the agent, who made the offer, can choose a rationing rule in the game induced by the tender offer expresses his discretion to trade with whom he wishes. That strategy sets of suppliers are bounded (by $-\bar{\theta}_i$) from below excludes short sales. After all, negative shares would have no meaning in a shareholder assembly, nor would shares above 1. A tender offer (q, x_i) that is not met by sufficient supply, that is, $x_i + \sum_{j \neq i} s_j > 0$, fails and all trades are cancelled.

Definition 2 *A tender offer (q, x_i) by $i \in I$ is* ***successful at*** $\bar{\theta} \in \Theta$ *if the game $\Gamma_{\bar{\theta}}(q, x_i)$ induced by it has a pure strategy Nash equilibrium $s^{\bar{\theta}} \in S$ such that $x_i + \sum_{j \neq i} s_j^{\bar{\theta}} \leq 0$ and at least one agent $j \in I$ is strictly better off with than without trade, i.e. $\upsilon_j\left(s^{\bar{\theta}}\right) > \upsilon_j\left(s_{-j}^{\bar{\theta}}, 0\right)$.*

For a tender offer to be successful it takes an equilibrium in the game induced by it at which the demands are fulfilled. But the equilibrium needs to be nontrivial in the sense that at least one player has a strict incentive to play the equilibrium. This condition excludes trades at indifference. It also excludes offers that amount to a reshuffling of shares, say, between agents of the same type, without altering (the probabilities of) the production plan(s).

Since Grossman and Hart (1980) it has become customary to assume that traders, who are not pivotal to a trade, will not tender unless they are offered more than the after-trade value of the firm: the "atomistic shareholder model." This is what the definition of a successful tender offer captures.[5] If no trader is pivotal, then sellers will demand a price of at least the after-trade value of the firm. But the raider, who wishes to buy, will optimally only make an offer with a price not exceeding the after-trade value of the firm. Therefore, when no agent is pivotal, by (9) no trader has a strict incentive to trade—thus excluding this as a successful tender offer.

Likewise, a trade between agents of the same type that implies no change of the probability $\Lambda(\theta)$ does not qualify as a successful tender offer. In fact, a successful tender offer by i at $\bar{\theta}$ implies that $\Lambda\left(\bar{\theta}\right) \neq \Lambda\left(\bar{\theta} + \left(s_i^{\bar{\theta}}\left(s_{-i}^{\bar{\theta}}\right), x_i\right)\right)$. For, suppose that equality would hold. Then by the argument above trade can only take place at $q = \pi\left(\Lambda\left(\bar{\theta}\right)\right)$. But, under the assumption that $\Lambda\left(\bar{\theta}\right) = \Lambda\left(\bar{\theta} + r\left(s_{-i}^{\bar{\theta}}\right)\right)$, (9) implies that all players are indifferent among all their strategies in $\Gamma_{\bar{\theta}}\left(\pi\left(\Lambda\left(\bar{\theta}\right)\right), x_i\right)$.

[5] Note, however, that Bagnoli and Lipman (1988) have shown that the argument by Grossman and Hart (1980) does not survive in a fully specified game model.

The following is the first criterion to solve for a share distribution that is not vulnerable to successful tender offers.

Definition 3 *A share distribution* $\bar{\theta} \in \Theta$ *is* ***uncontestable*** *if there exists no successful tender offer at* $\bar{\theta}$*. It* ***supports*** *the efficient (resp. monopolistic) production plan if* $\Lambda\left(\bar{\theta}\right) = 0$ *(resp.* $\Lambda\left(\bar{\theta}\right) = 1$*).*

Uncontestability requires that the shares will not change due to a tender offer by a raider, who attempts to take over. In this sense an uncontestable share distribution is stable and constitutes an equilibrium notion. A share distribution that is not uncontestable is called *contestable.*

4.1.1 Existence

In order to prove logical consistency of uncontestability a few contestable share distributions are identified first. The results to follow delimit the "basin of attraction" of an uncontestable share distribution that supports the efficient production plan. The first result states that any distribution that supports efficiency must be such that all low-valuation types together own no more than the dominant high-valuation shareholder.

Lemma 1 *Any share distribution* $\theta \in \Theta$ *with* $\Lambda(\theta) = 0$ *and* $\sum_{i \in L(\theta)} \theta_i > \max_{i \in H(\theta)} \theta_i$ *is contestable.*

Proof. If $\Lambda(\theta) = 0$, then there is some $i \in H(\theta)$ such that $\theta_i \geq \theta_j$ for all $j \in L(\theta)$ by the selection. Fix $\iota \in \arg\max_{i \in L(\theta)} \theta_i$ and two small but positive numbers ε and δ with $0 < \delta < \min\{\theta_i \,|i \in I, \, \theta_i > 0\}$. Consider a tender offer by $\iota \in L(\theta)$ to buy $x_\iota = \max_{i \in H(\theta)} \theta_i - \theta_\iota + \delta$ at a price $q = \underline{\pi} + \varepsilon$. If this offer succeeds, then $\iota \in L(\theta)$ will hold more shares than any high-valuation type, so that some $\lambda' > 0$ will obtain after trade, with $\pi(\lambda') > q$. The latter implies that the offer is affordable for i according to (5), because there is no control premium.

To see that the offer succeeds, consider first $\iota \in L(\theta)$. By (9)

$$
\begin{aligned}
\lambda' \Delta(\theta_\iota + x_\iota) - x_\iota(q - \underline{\pi}) &= \lambda' \Delta(\theta_\iota + x_\iota) - x_\iota \varepsilon > 0 \text{ if } \iota \in L \text{ and} \\
\lambda' \Delta(\theta_\iota + x_\iota - \gamma) - x_\iota(q - \underline{\pi}) &= \lambda' \Delta(\theta_\iota + x_\iota - \gamma) - x_\iota \varepsilon > 0 \text{ if } \iota \in H
\end{aligned}
$$

(the latter, because $x_\iota > 0$ and $\theta_\iota \geq \gamma$ imply $\theta_\iota + x_\iota > \gamma$ for $\iota \in H \cap L(\theta)$) for $\varepsilon > 0$ small enough.

Since $q < \pi(\lambda')$, no shareholder i, who is not pivotal, will want to sell, as $\theta'_i < \theta_i$ implies by (9) that $(\theta_i - \theta'_i)(q - \pi(\lambda')) < 0$. A shareholder $i \in H(\theta)$ will also not sell, even if he were pivotal, as for $\theta'_i < \theta_i < \gamma$ by (9) $\lambda' \Delta (\theta'_i - \gamma) - (\theta'_i - \theta_i)\varepsilon < 0$ for small enough $\varepsilon > 0$.

On the other hand, all low-valuation types $i \in L \setminus \{\iota\}$ other than ι have an incentive to sell if and only if they are pivotal, because by (9)

$$\lambda' \Delta (\theta_i + x_i) - x_i (q - \underline{\pi}) = \lambda' \Delta (\theta_i + x_i) - x_i \varepsilon > 0$$

for small enough $\varepsilon > 0$, provided $\theta_i + x_i > 0$. We claim that it is an equilibrium for each $i \in L(\theta) \setminus \{\iota\}$ to sell precisely

$$x_i = \frac{-x_\iota \theta_i}{\sum_{j \in L(\theta) \setminus \{\iota\}} \theta_j}$$

For, given that all other $j \in L(\theta) \setminus \{\iota\}$ sell these quantities, a given $i \in L(\theta) \setminus \{\iota\}$ is pivotal and (9) yields

$$\lambda' \Delta \left(1 - \frac{x_\iota}{\sum_{j \in L(\theta) \setminus \{\iota\}} \theta_j}\right) \theta_i + \frac{x_\iota \theta_i}{\sum_{j \in L(\theta) \setminus \{\iota\}} \theta_j} > 0$$

since $\sum_{j \in L(\theta) \setminus \{\iota\}} \theta_j > x_\iota = \max_{j \in H(\theta)} \theta_j - \theta_\iota + \delta$ by the hypothesis. Tendering less than $-x_i$ cannot be optimal, because then the offer fails and $\underline{\pi} < q < \pi(\lambda')$. Tendering more also cannot be optimal, because it will not change λ' by rationing and selling one extra share earns q, but costs $\pi(\lambda')$, which is more than q. Therefore, each $i \in L(\theta) \setminus \{\iota\}$ sells precisely $-x_i$. It follows that $x_\iota + \sum_{i \in L(\theta) \setminus \{\iota\}} x_i = 0$ and the offer succeeds. ■

Intuitively, when all low-valuation types together own more than the dominant high-valuation type, they will "pool" their shares to increase the chances of monopolistic output in shareholder voting. This "pooling" does not even involve a control premium, that is, the bid price is below the after-trade value of the firm. This works, because low-valuation types only tender an amount just enough to take over and benefit through their retained shares from the increased after-trade value of the firm.

For the case, where the dominant high-valuation shareholder holds at least as much as all low-valuation types together, the following characterizes contestable distributions.

Proposition 1 *A share distribution $\theta \in \Theta$ with $\Lambda(\theta) = 0$ and $\sum_{i \in L(\theta)} \theta_i \leq \max_{i \in H(\theta)} \theta_i$ is contestable if and only if $\max_{i \in L} \theta_i + \max_{i \in H(\theta)} \theta_i > \gamma$.*

Proof. "if:" That $\lambda = \Lambda(\theta) = 0$ implies by the selection that $\gamma > \max_{i\in H(\theta)} \theta_i \geq \theta_j$ for all $j \in L(\theta)$. Let $\iota \in \arg\max_{i\in H(\theta)} \theta_i$, $j \in \arg\max_{i\in L} \theta_i$, and $\lambda' = \Lambda(\theta')$, where $\theta' \in \Theta$ satisfies $\theta'_j = \theta_j + \theta_\iota$, $\theta_\iota = 0$, and $\theta'_i = \theta_i$ for all $i \in (H(\theta) \cup L(\theta)) \setminus \{j, \iota\}$. Since $\theta_\iota \geq \theta_i$ for all $i \in H(\theta)$ and $\theta_j > 0$ by the hypothesis, the selection implies, by $\theta'_j > \theta'_i$ for all $i \in H(\theta)$, that $\lambda' > 0$. Consider an offer by $j \in L$ to buy $x_j = \theta_\iota$ at a price

$$q = \underline{\pi} + \lambda'\Delta\frac{\gamma}{\theta_\iota} > \pi(\lambda')$$

(the latter, because $\iota \in H(\theta)$ implies $\theta_\iota < \gamma$). This is profitable for $j \in L$, because by (9) and the hypothesis

$$\lambda'\Delta(\theta_j + x_j) - x_j(q - \underline{\pi}) = \lambda'\Delta[\theta_j + \theta_\iota - \gamma] > 0$$

The offer is affordable for $j \in L$, because $qx_j = \theta_\iota\underline{\pi} + \lambda'(\overline{v} - \underline{v})$, so that from (5) in case $y = n$

$$\begin{aligned} qx_j = \theta_\iota\underline{\pi} + \lambda'(\overline{v} - \underline{v}) &\leq 1 + (\theta_j + \theta_\iota)\underline{\pi} - \underline{v} \Leftrightarrow \\ \lambda'\overline{v} + (1 - \lambda')\underline{v} &\leq 1 + \theta_j\underline{\pi} \end{aligned}$$

follows from $\underline{v} < \overline{v} \leq 1$, and from (5) in case $y = m$

$$\begin{aligned} qx_j = \theta_\iota\underline{\pi} + \lambda'(\overline{v} - \underline{v}) &\leq 1 + (\theta_j + \theta_\iota)\overline{\pi} \Leftrightarrow \\ \lambda'(\overline{v} - \underline{v}) &\leq 1 + \theta_j\overline{\pi} + \theta_\iota\Delta \end{aligned}$$

again by $\underline{v} < \overline{v} \leq 1$. It is also not costly to sell for $\iota \in H(\theta)$, because by (9)

$$-\lambda'\Delta\gamma - (\theta'_\iota - \theta_\iota)(q - \underline{\pi}) = \lambda'\Delta[\gamma - \gamma] = 0$$

No $i \in H(\theta)$ with $0 < \theta_i < \theta_\iota$ is willing to sell, though, because by (9) and $x_i \geq -\theta_i$

$$\begin{aligned} \lambda'\Delta(\theta_i - \gamma) - x_i(q - \pi(\lambda')) &= \lambda'\Delta\left(\theta_i - \gamma - x_i\frac{\gamma - \theta_\iota}{\theta_\iota}\right) \leq \\ \lambda'\Delta\left(\theta_i - \gamma + \theta_i\frac{\gamma - \theta_\iota}{\theta_\iota}\right) &< \lambda'\Delta(\theta_i - \theta_\iota) < 0 \end{aligned}$$

High-valuation types $i \in H(\theta)$ with $\theta_i = \theta_\iota$ are indifferent to trade, so it is optimal for them not to tender. All $i \in L(\theta) \setminus \{j\}$ are willing to sell, because

$q > \pi(\lambda')$, irrespective of whether or not they are pivotal. But even if they all sell, they cannot fulfill j's demand, because from $\theta_j > 0$ it follows that $\theta_\iota \geq \sum_{i \in L(\theta)} \theta_i > \sum_{i \in L(\theta) \setminus \{j\}} \theta_i$, so their combined supply falls short of θ_ι. Therefore, they must be rationed in equilibrium: If not, $\iota \in H(\theta)$ would still be pivotal, but could only sell less than θ_ι; selling less than θ_ι at q would not be profitable for $\iota \in H(\theta)$, though.

"only if:" If at $\lambda = \Lambda(\theta) = 0$ there is a successful tender offer at price q by, say, $j \in L$, then for some $\lambda' > 0$ and $x_j > 0$

$$\begin{aligned} \lambda' \Delta \theta_j - x_j (q - \pi(\lambda')) &= \lambda' \Delta (\theta_j + x_j) - x_j (q - \underline{\pi}) \geq 0 \\ &\Leftrightarrow \pi(\lambda') + \lambda' \Delta \frac{\theta_j}{x_j} \geq q \end{aligned}$$

By the hypothesis that $\sum_{i \in L(\theta)} \theta_i \leq \max_{i \in H(\theta)} \theta_i$ success ($\lambda' > 0$) implies that at least one $i \in H(\theta)$ must tender some of his shares, that is, there is $i \in H(\theta)$ with $x_i < 0$ such that

$$\begin{aligned} \lambda' \Delta (\theta_i - \gamma) - x_i (q - \pi(\lambda')) &= \lambda' \Delta (\theta_i + x_i - \gamma) - x_i (q - \underline{\pi}) \geq 0 \\ &\Leftrightarrow \pi(\lambda') + \lambda' \Delta \frac{\gamma - \theta_i}{-x_i} \leq q \end{aligned}$$

which implies $(\gamma - \theta_i) / (-x_i) \leq \theta_j / x_j$, as $\lambda' > 0$ and $\Delta > 0$. Since either for $i \in H(\theta)$ or for $j \in L$ strict inequality must hold at a successful offer, it follows from $x_j > 0$ and $x_i < 0$ that $x_j (\gamma - \theta_i) + x_i \theta_j < 0$. Since $x_j \geq -x_i$ must hold, $\theta_j \geq 0$ and $x_j > 0$ imply $\theta_i + \theta_j > \gamma$. Therefore, $\max_{i \in L} \theta_i + \max_{i \in H(\theta)} \theta_i > \gamma$ as required. ■

The intuition for this result is straightforward. When the characterizing condition is met, the largest low-valuation type can "bribe" the largest high-valuation type by offering a premium over the after-trade value of the firm that compensates the high-valuation seller for the lost utility from the externality. This premium is affordable for the low-valuation raider, because by $1/m \leq \gamma < 1$ (as implied by the characterizing condition) there remain enough high-valuation types that can be exploited by producing monopolistic output.

An important consequence of this proposition is an existence result for uncontestable share distributions. For, let $\bar{\theta} = \theta^* \in \Theta$ be given by $\theta_i^* = 1/m \leq \gamma$ for all $i \in H$ and $\theta_i^* = 0$ for all $i \in L$. Then $H(\theta^*) = H$, $L(\theta^*) = L$, and $\max_{i \in L} \theta_i^* + \max_{i \in H} \theta_i^* = 1/m \leq \gamma$. Therefore, the "only if"-part of Proposition 1 implies that θ^* is uncontestable.

Corollary 1 *There exists an uncontestable share distribution* $\bar{\theta} = \theta^* \in \Theta$ *and it supports the efficient production plan,* $\Lambda\left(\bar{\theta}\right) = 0$.

Intuitively, this result states that shares sufficiently dispersed among high-valuation types (and small enough stakes among low-valuation types) will guarantee uncontestability of the efficient production plan. In this sense competitive behavior goes along with dispersed ownership in the firm.

While Corollary 1 establishes existence, uniqueness remains open even with respect to the production plan supported by an uncontestable share distribution. This is to be studied next.

4.1.2 Multiplicity

Lemma 1 implies that if all low-valuation types together hold more than the dominant high-valuation shareholder, then the efficient production plan is contestable. The argument relies on the largest low-valuation shareholder buying all shares from other low-valuation types and taking over. In order to establish the possibility of multiple uncontestable share distributions, it is now shown that the distribution that results from the tender offer in the proof of Lemma 1 may be uncontestable.

Proposition 2 *If* $\gamma < 1/2$, *any distribution* $\bar{\theta} \in \Theta$ *with* $\bar{\theta}_\iota = \sum_{i \in L(\bar{\theta})} \bar{\theta}_i \geq 2\gamma$ *for some* $\iota \in L$ *is uncontestable and supports the monopolistic production plan,* $\Lambda\left(\bar{\theta}\right) = 1$.

Proof. If $\bar{\theta}_\iota \geq 2\gamma$, then $\bar{\theta}_i < \gamma$ for all $i \in H\left(\bar{\theta}\right)$ implies by the selection that $\Lambda\left(\bar{\theta}\right) = 1$. Suppose $i \in \arg\max_{j \in H(\bar{\theta})} \bar{\theta}_j$ can make a tender offer (q, x_i) such that after trade a probability $\lambda' < 1$ of monopolistic output will obtain.

If the offer is successful, then by (9)

$$(1 - \lambda')\left(\gamma - \bar{\theta}_i - x_i\right) - x_i\left(q - \overline{\pi}\right) \geq 0$$

must hold. Consider first the case, where $i \in H\left(\bar{\theta}\right)$ buys the quantity $\bar{\theta}_\iota - \theta'_\iota > 0$ from $\iota \in L\left(\bar{\theta}\right)$ in the equilibrium of the game induced by (q, x_i). In that case

$$(1 - \lambda')\Delta\theta'_\iota - \left(\theta'_\iota - \bar{\theta}_\iota\right)\left(q - \overline{\pi}\right) \geq 0$$

must obtain, which implies $q \geq \overline{\pi} = \pi(\lambda) > \pi(\lambda')$. Therefore, $\bar{\theta}_i + x_i \leq \gamma$ must hold from the previous inequality for i. That $\lambda' < 1 = \lambda$ implies that after trade $\bar{\theta}_i + x_i > \theta'_\iota$, so that $\bar{\theta}_\iota \geq 2\gamma > \gamma \geq \bar{\theta}_i + x_i > \theta'_\iota$ implies $\bar{\theta}_\iota - \theta'_\iota \geq 2\gamma - \theta'_\iota$ and $\bar{\theta}_i + x_i \geq \bar{\theta}_i + \bar{\theta}_\iota - \theta'_\iota \geq \bar{\theta}_i + 2\gamma - \theta'_\iota > \bar{\theta}_i + \gamma$. But the latter yields the contradiction $\bar{\theta}_i + x_i > \bar{\theta}_i + \gamma \geq \gamma$.

Therefore, the distribution $\bar{\theta}$ can only be contestable, if $i \in H\left(\bar{\theta}\right)$ manages to take over *without* buying from $\iota \in L\left(\bar{\theta}\right)$. But then after trade i would have to hold more than ι which implies that $\bar{\theta}_i + x_i > \bar{\theta}_\iota > \gamma$. In that case, though, i also prefers monopolistic output, $\lambda' = 1$. Hence, as $\lambda = \lambda' = 1$, trade can only take place at indifference. But this does not qualify as a successful tender offer. ■

The intuition for this result is straightforward. A high-valuation type, who attempts to buy control, must after trade own more shares than the dominant low-valuation shareholder. But when the latter controls at least 2γ, this implies that by purchasing such a big quantity the buyer changes his preferences so as to also prefer monopolistic output. The high share of the dominant low-valuation shareholder constitutes a "barrier" against tender offers by high-valuation types that operates through preference-reversal. Therefore, concentrated ownership in the firm correlates with monopolistic behavior.

The condition $\bar{\theta}_\iota \geq 2\gamma$, while sufficient, is not necessary for a distribution, that supports monopolistic output, to be uncontestable, though. This is illustrated in the following example.

Example 1 *Suppose that a low-valuation type* $\iota \in L\left(\bar{\theta}\right)$ *is in control, holding a share* $\bar{\theta}_\iota = 1 - (n-1)\delta$ *and all other shareholders* $i \neq \iota$ *hold precisely* $\bar{\theta}_i = \delta < (1-2\gamma)/(n-2) < 1/n$*, where it is assumed that* $\gamma < 1/2$*. Consider a tender offer* (q, x_i)*, where a high-valuation type* i *buys* $x_i > 0$ *such that the probability of monopolistic output decreases from* $\lambda = 1$ *to* $\lambda' < 1$*. By the selection it takes at least relative majority to change* λ*, so that* $\lambda' < 1 = \lambda$ *implies that* $\delta + x_i \geq \bar{\theta}_\iota + x_\iota \geq \bar{\theta}_\iota - x_i$*, because in the best case* i *buys all his demands* $x_i \geq -x_\iota > 0$ *from the controlling shareholder* $\iota \in L\left(\bar{\theta}\right)$*. This implies* $\delta + x_i \geq 1 - (n-1)\delta - x_i \Leftrightarrow \delta + x_i \geq [1-(n-2)\delta]/2$ *which implies by the choice of* δ *that*

$$\delta + x_i > \frac{1}{2}\left[1 - \frac{(n-2)(1-2\gamma)}{n-2}\right] = \gamma$$

That is, if the offer is successful, $i \in H$ will have his incentives changed such that he also wants to produce monopolistic output—a contradiction to $\lambda' < 1$.

Intuitively, control by low-valuation types is robust against takeover attempts by high-valuation types, because the latter reverse their preferences when they hold high stakes in the company. This can only change when takeover attempts by high-valuation types are coordinated so that each of them buys only very little, but collectively they buy enough to unseat the low-valuation controlling shareholder.

4.2 Coordinated Action

The scope of the uncontestability criterion is limited to decentralized trading, where one agent takes the initiative and makes a tender offer. More coordination is conceivable, though. As an extreme case imagine an auctioneer or a market-maker, who suggests net trades to agents until all gains from trade are exhausted. This can potentially overcome the coordination problem of high-valuation types in trading towards a share distribution that supports the efficient production plan.

Definition 4 *An **offer at** $\bar{\theta} \in \Theta$ is a pair (q, x) consisting of a share price $q > 0$ and a net trade vector $x \in \mathbf{R}^n$ such that $\bar{\theta} + x \in \Theta$. The **game induced** by an offer (q, x) at $\bar{\theta}$ is the n-player normal form game $\hat{\Gamma}_{\bar{\theta}}(q, x) = \left(\hat{S}, \hat{v}\right)$, where $\hat{S} = \times_{i \in I} \hat{S}_i$ with $\hat{S}_i = \{\min\{0, x_i\}, \max\{0, x_i\}\}$ for all $i \in I$ and $\hat{v} = (\hat{v}_1, ..., \hat{v}_n) : \hat{S} \to \mathbf{R}^n$ with*

$$\hat{v}_i(s) = 1 - q\left(\vartheta_i(s) - \bar{\theta}_i\right) + \vartheta_i(s)\,\pi(\Lambda(\vartheta(s))) + [1 - \Lambda(\vartheta(s))]\max\{0, v_i - \underline{v}\} \tag{12}$$

for all $s \in S$ and all $i \in I$, where $\vartheta(x) = \bar{\theta} + x \in \Theta$ and $\vartheta(s) = \bar{\theta} \in \Theta$ for $s \neq x$.

The definition of an offer at $\bar{\theta}$ allows for the trivial case $x = 0 \in \mathbf{R}^n$ for generality. It will soon be shown that this does not do any harm. Short sales are excluded, though, as $\bar{\theta} + x \geq 0$.

The game induced by an offer captures several centralized trading mechanisms. That agents are quantity constrained by the x_i's resembles *limit orders*. If $x_i > 0$ for some i, but $x_j \leq 0$ for all $j \neq i$, this resembles a unilateral *tender offer*, where agent i offers to buy a quantity x_i at a price q.

If for a unilateral tender offer there is a unique $j \neq i$ such that $x_j < 0$, this constitutes a *bilateral* trade between i and j. The restriction incorporated in the definition of an offer is, however, that all investors trade at the same price q and their market operations are coordinated, that is, the structure of trades is not decided individually, but elicited by the auctioneer.

Intuitively, an offer is a trade proposal to the agents that each one, who is affected ($x_i \neq 0$), can either accept ($s_i = x_i$) or reject ($s_i = 0$). If all accept, it is implemented, otherwise there is no trade. Clearly, this is a stylized way to represent a stock market as it requires consensus among affected agents. But, because all possible offers will be considered and they cannot be restricted by the agents, it captures the best that a centralized market can achieve. In this sense such centralized offers constitute a benchmark case.

If trade were allowed when some, but not all affected agents accept, markets would not clear and quantity rationing would be required. But, if an offer is accepted by some affected agents, but not by others, and the trades among the accepting agents are consistent with market clearing, possibly after some rationing, then there is an alternative offer that is accepted by all affected agents. In this sense there is no loss of generality involved with this mechanism, save for the implicit coordination.

Definition 5 *A* ***control trade at*** $\bar{\theta} \in \Theta$ *is an offer* (q, x) *such that, in the game* $\hat{\Gamma}_{\bar{\theta}}(q, x)$ *induced by the offer* (q, x)*, the strategy profile* $s = x \in \hat{S}$ *constitutes a Nash equilibrium, where at least one affected agent* i *is strictly better off with trading than without, i.e.* $\hat{v}_i(x) > \hat{v}_i(x_{-i}, 0)$*.*

A control trade is an offer that is successful in equilibrium. But again it is so in a nontrivial way. The condition that at least one player has a strict incentive to play the equilibrium again excludes trades at indifference. In particular, it excludes trivial offers with $x = 0 \in \mathbf{R}^n$. The following is the stronger equilibrium criterion that is used to analyze coordinated market interaction.

Definition 6 *A share distribution* $\bar{\theta} \in \Theta$ *is* ***universally uncontestable*** *if there exists no control trade at* $\bar{\theta}$*.*

The same share distribution as in Corollary 1 establishes existence of universally uncontestable share distributions. But, because the criterion is stronger than uncontestability, the production plan supported by a universally uncontestable share distribution is now unique: the efficient production plan.

Proposition 3 *There exists a universally uncontestable share distribution $\bar{\theta} = \theta^* \in \Theta$, and every universally uncontestable share distribution supports the efficient production plan.*

Proof. (a) Let $\theta^* \in \Theta$ be given by $\theta_i^* = 1/m$ for all $i \in H$ and $\theta_i^* = 0$ for all $i \in L$. At θ^* the status-quo is the efficient production plan, $\lambda = \Lambda(\theta^*) = 0$, because by $c < \underline{v}$ it follows that $\gamma > \theta_i^* = 1/m > \theta_j^* = 0$ for all $i \in H$ and all $j \in L$. Consider an offer (q, x) such that $x_j > 0$ for some $j \in L$. If (q, x) is a control trade, then for $j \in L$ by (9)

$$u_j(x_j, \lambda') - u_j(0, 0) - qx_j = x_j(\pi(\lambda') - q) \geq 0$$

must hold, where $\lambda' = \Lambda(\theta^* + x) > 0$. This implies that $q \leq \pi(\lambda')$. Denote by $H' \subseteq H$ the set of high-valuation sellers, that is $i \in H' \subseteq H \Leftrightarrow x_i < 0$. The set H' must be nonempty, because $\sum_{i=1}^n x_i = 0$, $x_j > 0$ for $j \in L$, and $x_k \geq 0$ for all $k \in L$ by $\theta^* + x \geq 0$. But for any $i \in H'$ that $q \leq \pi(\lambda')$ implies from (9) that

$$\begin{aligned} u_i(\theta_i^* + x_i, \lambda') - u_i(\theta_i^*, 0) - qx_i &= \\ \lambda'\Delta\left(\frac{1}{m} - \gamma\right) - x_i(q - \pi(\lambda')) \leq \lambda'\Delta\left(\frac{1}{m} - \gamma\right) &< 0 \end{aligned}$$

that is, no $i \in H'$ is willing to accept the sales offer—a contradiction.

There is the possibility that no $i \in H'$ is pivotal, for instance, if $x_j \geq 2/m$ and all $i \in H'$ tender their shares. Then, given that all others sell, $i \in H'$ sells only if $q \geq \pi(\lambda')$. Therefore, if (q, x) is a control trade, $q = \pi(\lambda')$. But this and the hypothesis that no high-valuation type is pivotal implies that no trader can strictly profit from trade. Hence, this does not qualify as a control trade either.

There remains the possibility that a high-valuation type $j \in H$ buys $x_j > \gamma - 1/m > 0$, so that after trade he prefers to produce monopolistic output. If this is a control trade, then from (9)

$$\begin{aligned} u_j\left(\theta_j^* + x_j, \lambda'\right) - u_j\left(\theta_j^*, 0\right) - qx_j &= \\ \lambda'\Delta\left(\frac{1}{m} - \gamma\right) - x_j(q - \pi(\lambda')) &\geq 0 \end{aligned}$$

implies $q < \pi(\lambda')$. But at $q < \pi(\lambda')$ no high-valuation type $i \in H$ is willing to sell, because

$$u_i(\theta_i^* + x_i, \lambda') - u_i(\theta_i^*, 0) - qx_i \; =$$

$$\lambda' \Delta \left(\frac{1}{m} - \gamma \right) - x_i \left(q - \pi \left(\lambda' \right) \right) \quad < \quad 0$$

It follows that there can be neither a control trade with $x_i > 0$ for some $i \in L$, nor one with $x_i > 0$ for some $i \in H$. Since $x \leq 0$ implies $x = 0$ by the definition of an offer, it follows that θ^* is universally uncontestable.

(b) At any $\bar{\theta} \in \Theta$, where $\lambda = \Lambda \left(\bar{\theta} \right) > 0$, consider the offer (q, x) with $q = \pi (\lambda) + \varepsilon$ for some small ε with $0 < \varepsilon < \lambda \Delta (\gamma m - 1)$, $x_i = -\bar{\theta}_i$ for all $i \in L$, and $x_i = 1/m - \bar{\theta}_i$ for all $i \in H$. Then (9) implies for $i \in L$ with $\bar{\theta}_i > 0$ that

$$u_i \left(\bar{\theta}_i + x_i, 0 \right) - u_i \left(\bar{\theta}_i, \lambda \right) - q x_i = x_i \left(\pi \left(\lambda' \right) - q \right) = \bar{\theta}_i \varepsilon > 0$$

and for $i \in H$ that

$$u_i \left(\frac{1}{m}, 0 \right) - u_i \left(\bar{\theta}_i, \lambda \right) - q x_i = \lambda \Delta \left(\gamma - \frac{1}{m} \right) - \left(\frac{1}{m} - \bar{\theta}_i \right) \varepsilon > 0$$

for ε sufficiently small. ■

The intuition for this result is as follows. This is a transferable utility model. Therefore, appropriate transfers among the agents can always achieve efficiency. Since there are only two types, the market-maker can find such a transfer scheme that is equivalent to net trades at a single price. This is, in particular, the logic underlying uniqueness of the supported production plan (part (b) of the proof).

This argument reveals that, unlike Corollary 1, the conclusion from Proposition 3 may be peculiar to the present model. While existence of an uncontestable share distribution that supports efficient production seems to hold in general, that *only* efficient production plans are supported by (universally) uncontestable share distributions depends crucially on a transferable utility framework—and, of course, on the presence of a coordinating auctioneer. Nevertheless, the power of universal uncontestability in a transferable utility model can serve as a useful benchmark for more general models of corporate control and stock market trade.

5 Conclusions

The present paper studies the interaction between two core institutions of industrial democracy: shareholder voting and stock market trade. In the model, economic agents can trade voting stock before the firm's production plan is decided by a vote among shareholders (under the one-share-one-vote rule). Even though this leads to a highly strategic interaction at the stock market, we identify surprising support for competitive behavior and efficiency.

A share distribution is *uncontestable* if it cannot be changed by a tender offer—an equilibrium criterion about share distributions. It is shown that there always exists an uncontestable share distribution that supports the efficient production plan. If the equilibrium criterion is extended so as to exclude all possible control trades ("universally uncontestable"), then only efficiency can be supported. But this conclusion appears to depend on the transferable utility framework employed here.

In general the picture is a bit more ambiguous. Even though there is always an uncontestable share distribution that supports the efficient production plan, the latter appears fragile. This is because the types supporting efficiency are endangered by preference reversal when their stakes in the company become too high. Therefore, supporting efficiency takes dispersed shares. But dispersed distributions are vulnerable to "bribing" some supporters of efficient production on account of exploiting the rest.

This opens the possibility of uncontestable distributions that support monopolistic behavior, that is, multiplicity of uncontestable share distributions. If shares are sufficiently concentrated in the hands of a type, who supports monopolistic behavior, such a distribution may be uncontestable. This is because a takeover attempt by a supporter of efficient production may be thwarted by preference reversal, as he needs to accumulate too many shares.

These observations combine to testable implications of the theory. According to the present model, competitive behavior (the efficient production plan) should be associated with dispersed share ownership. Reciprocally, concentrated ownership tends to foster monopolistic behavior. An empirical investigation of these relations is a topic for further research.

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