The Company You Keep: Qualitative Uncertainty in the Provision of Club Goods.[1]

Bipasa Datta
Department of Economics and Related Studies, University of York, Y01 5DD
Tel: (44) (0)1904 433 780; Fax:: (44) (0)1904 433 759
E-mail: bd11@york.ac.uk

and

Clive D. Fraser[2]
Department of Economics, University of Leicester, LE1 7RL
Tel: (44) (0) 116 252 5365; Fax: (44) (0) 116 252 5351
E-mail: cdf2@le.ac.uk

Version: 14 February 2006

Clubs are typically experience goods. Potential members cannot ascertain precisely beforehand their quality (dependent endogenously on the club's facility investment and number of users, itself dependent on the club's pricing policy). Members with unsatisfactory initial experiences discontinue visits. We show that a monopoly profit maximiser never offers a free trial period for such goods but, for a quality function homogeneous of any feasible degree, a welfare maximiser always does. When the quality function is homogeneous of degree zero, the monopolist provides a socially excessive level of quality to repeat buyers. In other possible regimes, the monopolist permits too little club usage.

Keywords: Clubs, Qualitative Uncertainty, Monopoly, Welfarist.

JEL Classification: D42, D80, D60, H4.

1. INTRODUCTION

This article studies the optimum provision and pricing rules for a club good. The quality of the club good is increasing in the supplier's investment in the club facility and decreasing in the amount of usage of it. We compare the provision and pricing

[1] A very preliminary version was presented at PSERC, University of Leicester, and an IFS/ESRC/Warwick Mini-Conference on the Environment and Externalities at Warwick University. Thanks to Todd Sandler for comments at the latter. We also thank Peter Simmons for comments on this version.

[2] Corresponding author.

by a monopolist with that by a welfare maximiser to show that the monopolist is likely to over-provide quality and allow too little use of the club relative to the welfare optimum. The particular feature of club goods that we emphasize is the qualitative uncertainty that consumers face as club goods are essentially experience goods. Ex ante, a potential club user is uncertain how agreeable she will find the experience of membership. E.g., the water in the swimming pool might turn out to be consistently too tepid or too enervating, she could find that she cannot get her head around the queues at the bar, the food is more than she can stomach, cigarette smoke gets in her eye or passive smoking just gets up her nose. Such a potential customer typically has to try the good before she really knows what she buys. Yet, this qualitative uncertainty aspect of club goods has largely been ignored in the club literature. This literature has dealt mainly with other important issues, such as multijurisdictionality in large economies with many competing clubs, congestion externalities or tiered pricing (see, for example, Wooders (1978, 1999), Cornes and Sandler (1996), Scotchmer (1985) and Glazer, Niskanen and Scotchmer (1997)).

We are not aware of another paper that deals with the qualitative uncertainty aspect of club goods from our perspective. Sandler, Sterbenz and Tschirhart (1985) do analyze consumers' uncertainty when individuals are certain about their own membership but uncertain about the congestion they will experience on any particular visit. But, they focus on the relationship between risk aversion and capacity provision and not, as we do, on the endogenous determination of club membership through the provider's pricing strategy and investment in facilities. Moreover, they study neither the market provision of the club good nor members' self-selection.

Since we model the club good as an experience good that generates the frequency of future visits to the club by its potential members, this paper is related to the literature on the economics of experience goods and repeat buying. Authors such as Cremer (1984), Liebeskind and Rumelt (1989), Hoerger (1993), Krähmer (2002), Villas-Boas (2004) and Bergemann and Välimäki (2005), analyze the effects of qualitative uncertainty associated with experience goods on buyers' learning and the intertemporal pricing strategy of an imperfectly competitive firm. However, none compare the behaviour of a monopoly supplier of the experience good with that of a benchmark supplier, such as a welfare maximizer. This comparison turns out to be important because, unless the welfarist and the monopolist's behaviour coincide (which logically cannot occur in our model), the fact that their regimes differ can be used to delimit the possible configuration of choice variables within each. Further, none of these authors analyse explicitly the case of club goods with specific features such as those mentioned in the paragraph above (although Cremer (1984) briefly mentions a club as an example of an experience good).

Cremer (1984) and Bergemann and Välimäki (2005) are among the above authors who, like us, look specifically at the behavior of a monopoly provider. Cremer (1984) shows that a monopolist will not offer an 'introductory' price to first time buyers but will charge a lower price to repeat buyers. Bergemann and Välimäki (2005) show that, in experience goods markets, the monopolist actually faces two different types of markets: a mass market (where buyers are willing to buy at the full information monopoly price) and a niche market (with uninformed buyers who are not) where pricing strategies differ. In the mass market, prices decline over time whereas, in the latter, lower prices are followed by higher ones. We show that, consistent with Cremer (1984) and with Bergemann and Välimäki's (2005) mass market result, the monopoly club provider will not make an "introductory offer" that allows consumers to "try before they buy," but the welfare maximizer might (*Proposition 1 and Observation 2*). More specifically, we consider the class of club quality functions that are homogeneous in the facility investment and the usage of the club. We show that (*Proposition 3*), in this class, the welfare maximiser will offer a free trial period for all degrees of homogeneity that lead to feasible outcomes (which necessitates homogeneity greater than or equal to minus unity). This is a very strong result. Furthermore, we show that, under plausible assumptions, when the degree of homogeneity exceeds minus unity, the monopolist always invests in a greater level of facility provision per use of the club than does the welfarist. In the much discussed case where the quality function is homogeneous of degree zero, this translates to the monopolist over-investing in the quality provided to repeat buyers compared to the welfare maximizer (*Proposition 4*).

The other papers mentioned above, by Liebeskind and Rumelt (1989), Hoerger (1993), Krähmer (2002) and Villas-Boas (2004), have a slightly different angle from ours. Liebeskind and Rumelt (1989) and Hoerger (1993) study the effects of product quality uncertainty in the presence of adverse selection on the producers' side, while Krähmer (2002) and Villas-Boas (2004) analyze how consumers' learning in the presence of quality uncertainty impacts on the pricing strategies of oligopolists.

In section 2, after presenting the basic framework of our two-period model, we analyze the second period club membership decision of first time visitors. We show how second period membership gets determined endogenously, depending on the provider's strategies on prices and quality provision. We also carry out some comparative static analysis of the sensitivity of club membership to prices and quality. In subsection 2.2, we examine the monopoly provider's pricing and investment decisions. In subsection 2.3, we do the same for the social welfare maximizer. In subsection 2.4, we compare and contrast the monopolist's equilibrium pricing and investment decisions with that of the social welfare maximizer under various possible regimes and derive results as described two paragraphs above. Section 3 presents our conclusions. In the appendix, we prove our more technical results and derive one of the key equations that drive some of our main results.

2. THE MODEL

We consider a two-period model of club membership in an economy with a single private good and a single club good ("a club" for short) with a sole supplier. The private good is essential, but the club is not. There are n consumers, n being large, who are identical ex ante and face uncertainty regarding the quality of the club. Individuals must join the club and experience the good in order to learn their valuation of it, which then becomes their private information. Thus, ex-ante homogeneous consumers become heterogenous ex-post with respect to their valuation of the club once they have become members. In order to ascertain its quality, an individual must make a fixed number of visits to the club in the first period, irrespective of the nature of the supplier. Based on his experience, the individual decides whether to remain as a member in the second period or to quit, and how many visits to make if he stays. Thus, part of the focus of our model is on members' exit decisions.

We assume that a typical member has a strictly concave time-separable utility function with per period utility given by $U((x_i, v_i, C(\varepsilon, y, V_i))$, where x_i is his period i's consumption of the private good, $i = 1, 2$, v_i is the number of visits she makes in period i, y is the quantity of the club good (equivalently, its facility size) *which, once provided, does not depreciate in value*, V_i is the total number of visits made by all members in period i, ε is a random-valued parameter capturing the 'qualitative uncertainty' and $C(\varepsilon, y, V)$ is the *quality* or *congestion* function. The following assumption is about the specification of the utility function:

A1. The function $U((x_i, v_i, C(\varepsilon, y, V_i))$ is quasi-linear of the form $U(.) = u(x_i) + \varepsilon v C(y, V_i)$, with $u(x_i)$ being strictly concave.

Assumption A2 is about the distribution of ε.

A2. The parameter ε is distributed over the interval $[\underline{\varepsilon}\ ,\ \overline{\varepsilon}]$ with density function $f(\varepsilon)$ and CDF $F(\varepsilon)$.

The following assumption says that the club quality increases with the level of provision ("the facility size") but decreases when it gets more crowded:

A3. $C_y(y, V) > 0; C_V(y, V) < 0$.

The club good is supplied by a profit-maximizing monopolist that acts as a Stackelberg leader in choosing the level of provision y and prices p_i for the periods i = 1, 2 at the start of period 1. For simplicity, we consider only linear pricing, although a firm can usually do better with nonlinear pricing[3]. Further, we assume that the unit cost of providing the club good is constant at unity.

[3] Although we only consider linear pricing, as first period visits are fixed, a consumer effectively has to pay a lump sum to join the club and try the club good. So, pricing has the flavour of intertemporal two-part pricing.

Without loss of generality, normalise the fixed number of visits an individual must make to the club to ascertain its quality at unity. Let $\overline{V}$ denote the aggregate number of visits made in the first period - i.e., $\bar{V} = n(= V_1)$. Budget constraints of a member in periods 1 and 2 can then be written respectively as:

$$M_1 - p_1 = x_1$$

and

$$M_2 - p_2 v_2 = x_2$$

where M_i is the period i income of a member (recall, members are homogeneous with respect to income). The sequence of events is as follows:

- *Period 1.* The leader sets the level of provision y and prices $p_1$and p_2. Members then decide to join (or not) the club and make a visit. After experiencing the good, members become heterogeneously informed about its quality, based on which they decide whether to stay in the club or to exit.
- *Period 2.* If a member remains with the club, he then decides how many visits to make in the second period, given his private valuation of it.

2.1. The members' problem in period 2.

2.1.1. The exit decision and club membership

For convenience, we shall denote v_2 by v and V_2 by V from now on. We assume that each member treats V (which gets determined endogenously later) as parametric and chooses v to maximize the second period utility subject to the budget constraint. For a given p_2 and y, we assume that both a supplier and consumers can infer the V that will occur in an equilibrium. Additionally, given the large number of consumers, V is taken to equal its expected value (or decision makers take it to be so when they make their decisions). A typical member then solves the following problem in the second period:

$$\max_{v} \; u(M_2 - p_2 v) + \varepsilon v C(y, V)$$

The first order condition (FOC) yields:

$$-p_2 u_{x_2}(M_2 - p_2 v) + \varepsilon C(y, V) \leqq 0 \ for \ v \geqq 0 \tag{1}$$

with $-p_2 u_{x_2}(M_2) + \varepsilon C(y, V) \leqq 0$ if $v = 0$. Now, with quality taken as parametric, $-p_2 u_{x_2}(M_2) + \varepsilon C(y, V)$ is increasing in ε. Then, given a plausible assumption on C[4] and a sufficiently wide support for $f(\varepsilon)$, there exists an ε^* such that

$$-p_2 u_{x_2}(M_2) + \varepsilon C(y, V) \gtreqqless 0 \ \text{ according as } \varepsilon \gtreqqless \varepsilon^*.$$

Call $\varepsilon^* \in [\underline{\varepsilon}, \overline{\varepsilon}]$ the *marginal quality valuation* - i.e., ε^* solves

$$-p_2 u_{x_2}(M_2) + \varepsilon^* C(y, V) = 0. \tag{2}$$

So, ε^* just leaves the consumer indifferent between choosing some club consumption and not. Clearly, ε^* is a function of p_2 and y (as well as other parameter values, e.g., M_2). Note that the number of visits at the marginal quality valuation is zero: $v(\varepsilon^*) = 0$. The following Lemma shows how period 2 club membership gets determined depending upon the realization of ε.

LEMMA 1. *(Single Crossing) Members having $\varepsilon \geqq \varepsilon^*$ remain in the club while those having $\varepsilon < \varepsilon^*$ exit the club.*

Proof. Given p_2 and C, $-p_2 u_{x_2}(M_2) + \varepsilon C(y, V)$ is increasing in ε, and $-p_2 u_{x_2}(M_2) + \varepsilon^* C(y, V) = 0$ at $\varepsilon = \varepsilon^*$. Hence, for $\varepsilon > \varepsilon^*$, $-p_2 u_{x_2}(M_2 - p_2 v) + \varepsilon C(y, V) = 0$ can be satisfied for some $v > 0$. But this implies that members having $\varepsilon \geqq \varepsilon^*$ remain in the club and make positive visits (the marginal member "remains" in the club but makes zero visit). Obviously, for $\varepsilon < \varepsilon^*$, members make zero visits and exit the club as $-p_2 u_{x_2}(M_2) + \varepsilon C(y, V) < 0$. ∎

[4] We assume $C(0, V) > 0$ and that $\overline{\varepsilon}$ is sufficiently large so that $\overline{\varepsilon} > p_2 u_x(M_2)/C(y, V)$ is satisfied for all possible values of p_2 and C. Then ε^* is strictly between its lower and upper bounds.

The (positive) number of visits v made by an individual who stays in the club is obtained as a function of ε, p_2, V and y, $v(\varepsilon, p_2, y, V)$, from

$$-p_2 u_{x_2}(M_2 - p_2 v(\varepsilon, p_2, y, V)) + \varepsilon C(y, V) = 0 \tag{3}$$

Thus, ex ante (when seen from period 1), for a given p_2 and y, the expected number of visits by a member is given by

$$\int_{\varepsilon^*}^{\overline{\varepsilon}} v(\varepsilon, p_2, y, V) dF(\varepsilon). \tag{4}$$

Denoting $v(\varepsilon, p_2, y, V)$ by only $v(\varepsilon)$ from now on, unless otherwise necessary, the expected number of visits in *aggregate* is therefore given by

$$V = n \int_{\varepsilon^*}^{\overline{\varepsilon}} v(\varepsilon) dF(\varepsilon) \tag{5}$$

The only technical task remaining in this subsection is to prove the existence and uniqueness of an equilibrium in expected visits for a given p_2 and y. The following Lemma is proven in the Appendix:

LEMMA 2. *For a given y and p_2, a unique equilibrium in expected second period visits exists.*

2.1.2. Some comparative statics

We use the following comparative static results (see the Appendix) for consumers' responses to magnitudes that they take as parametric in solving the leader's problem:

LEMMA 3. *(i) $\partial V/\partial y > 0$; (ii) $\partial V/\partial p_2 < 0$; (iii) $\partial \varepsilon^*/\partial y < 0$; (iv) sign $(\partial \varepsilon^*/\partial p_2) =$ sign $(C + VC_V)$.*

Thus: (i) as the level of facility provision increases, aggregate (and individual) visits increase; (ii) an increase in second period price reduces aggregate (and individual) visits; (iii) more people stay with the club if the level of provision increases;

(iv) a change in the second period price has an ambiguous effect on how many use the club. The last is the result of independent interest. We show below that the second period price can be set at a level where a further increase would produce either a rise or no change in club membership, depending on the club provider's objective.

2.2. The monopolist's problem

The monopolist acts as a Stackelberg leader, choosing (y, p_1, p_2) to maximize its profit knowing that members behave as described above. She maximizes subject to the constraint that agents join the club in the first period. We confine attention to a pure strategy equilibrium[5]. The maximization problem is:

$$\max_{p_1,p_2,y} n\left\{p_1 + \delta\int_{\varepsilon^*}^{\overline{\varepsilon}} p_2 v(\varepsilon)dF(\varepsilon)\right\} - y$$

subject to

$$u(M_1 - p_1 v) + C(y,\overline{V})E(\varepsilon) + \delta\int_{\varepsilon^*}^{\overline{\varepsilon}} \left[u(M_2 - p_2 v(\varepsilon)) + \varepsilon v(\varepsilon)C(y,V)\right]dF(\varepsilon) \geqq$$

$$u(M_1) + \delta u(M_2)(1 - F(\varepsilon^*)) \tag{6}$$

Equation (6) is an agent's participation constraint (after simplification), where $E(\varepsilon) = \int_{\underline{\varepsilon}}^{\overline{\varepsilon}} \varepsilon f(\varepsilon)d\varepsilon$ and δ is the discount factor. Letting superscript "m" show magnitudes for the monopolist, denote the Lagrangian equation by $\mathcal{L}^m$ and the

[5] Our model is a full commitment one wherein the monopolist commits deterministically to its pricing and quality strategies in period 1. We thereby rule out any "ratcheting effects" à la Laffont-Tirole - which normally give rise to mixed strategy equilibria in non-commitment games.

corresponding multiplier by λ^m.

$$\mathcal{L}^m = n\{p_1 + \delta \int_{\varepsilon^*}^{\overline{\varepsilon}} p_2 v(\varepsilon) dF(\varepsilon)\} - y + \lambda^m [u(M_1 - p_1) + C(y, \overline{V}) E(\varepsilon)$$
$$+\delta \int_{\varepsilon^*}^{\overline{\varepsilon}} [u(M_2 - p_2 v(\varepsilon)) + \varepsilon v(\varepsilon) C(y, V)] \, dF(\varepsilon)$$
$$-u(M_1) - \delta \int_{\varepsilon^*}^{\overline{\varepsilon}} u(M_2) dF(\varepsilon^*)$$

The FOC's to the above maximization problem, after simplification, are as follow:

$$\frac{\partial \mathcal{L}^m}{\partial p_1^m} = n - \lambda^m u_{x_1} \leqq 0 \quad for \quad p_1^m \geqq 0 \tag{7}$$

$$\frac{\partial \mathcal{L}^m}{\partial p_2^m} = n\delta [\int_{\varepsilon^*}^{\overline{\varepsilon}} \left\{ v(\varepsilon) + p_2^m \frac{\partial v(\varepsilon)}{\partial p_2} \right\} dF(\varepsilon)] +$$
$$\lambda^m \delta \int_{\varepsilon^*}^{\overline{\varepsilon}} \{\varepsilon v(\varepsilon) C_V \frac{\partial V}{\partial p_2} - v(\varepsilon) u_{x_2}\} dF(\varepsilon) \geqq 0 \text{ for } p_2^m \geqq 0 \tag{8}$$

$$\frac{\partial \mathcal{L}^m}{\partial y^m} = n[C_y(y, \overline{V}) E(\varepsilon) + \delta \int_{\varepsilon^*}^{\overline{\varepsilon}} \varepsilon v(\varepsilon) \{C_y + C_V \frac{\partial V}{\partial y}\} dF(\varepsilon)] +$$
$$n\delta u_{x_1} \int_{\varepsilon^*}^{\overline{\varepsilon}} p_2 \frac{\partial v(\varepsilon)}{\partial y} dF(\varepsilon) \leqq u_{x_1} \quad for \quad y^m \geqq 0. \tag{9}$$

These FOC's suggest a number of observations[6]. First, from equation (7), it follows that $\lambda^m \geqq n/u_{x_1} > 0$. This implies that the participation constraint of the agent binds, in turn implying that agents get no rent in equilibrium (as is expected) - regardless of whether p_1^m is positive or not. Second, p_2^m cannot be zero. Otherwise, the demand for second period visits to the club would be infinite and no solution to the maximization problem would exist. But can p_1^m be zero? I.e., could the monopolist make an introductory offer on the club good (where "introductory

[6] We take second order conditions (SOC's) to be satisfied, without further formal investigation. The monopolist's strategy space is closed and bounded and its objective and constraint functions are continuous, so equilibria will exist. It is easy to see that, if the SOC's are satisfied, these FOCs will identify an equilibrium in pure strategies. Consumers cannot deviate from it and improve welfare by not joining the club: their utility outside the club is just the reservation expected utility they get from membership. Given consumers do not deviate, the monopolist maximises profits if satisfying the SOC's and these FOC's.

offer" is interpreted as "a free trial period")?[7] The answer is no, as shown by the following proposition.

Proposition 1. *The monopolist does not make an introductory offer on the club good - i.e., $p_1^m > 0$.*

Proof. Since the participation constraint binds in equilibrium (regardless of whether p_1^m is zero or positive), rewrite (6) as

$$\begin{aligned} & \delta \int_{\varepsilon^*}^{\overline{\varepsilon}} \left[u(M_2 - p_2 v(\varepsilon)) + \varepsilon v(\varepsilon) C(y, V) - u(M_2) \right] dF(\varepsilon) \\ = \; & u(M_1) - U(M_1 - p_1) - C(y, \overline{V}) E(\varepsilon) \end{aligned}$$

If $p_1^m = 0$, then the RHS will be strictly negative while the LHS will be strictly positive, given that for $\varepsilon > \varepsilon^*$ the person get more utility in the club than out. This would violate the participation constraint. Hence $p_1^m > 0$. ∎

Now, $p_2^m > 0$ implies (8) holds as an equality. Substituting $\lambda^m = n/u_{x_1}$, using equation (23) in the Appendix and simplifying, we obtain (see the Appendix)

$$\left[u_{x_1} \int_{\varepsilon^*}^{\overline{\varepsilon}} \frac{u_{x_2}}{p_2 u_{xx}} dF(\varepsilon) - \int_{\varepsilon^*}^{\overline{\varepsilon}} u_{x_2} v(\varepsilon) dF(\varepsilon) \right] \left[\frac{V}{C} C_V + 1 \right] = 0 \qquad (10)$$

Define the visit elasticity of quality by $\eta_v = \frac{V}{C}\frac{\partial C}{\partial V}$ (< 0, since $C_V < 0$). As $[u_{x_1} \int_{\varepsilon^*}^{\overline{\varepsilon}} \frac{u_{x_2}}{p_2 u_{xx}} dF(\varepsilon) - \int_{\varepsilon^*}^{\overline{\varepsilon}} u_{x_2} v(\varepsilon) dF(\varepsilon)] < 0$, we can note the following about the second period pricing rule of the monopolist:

Observation 1. *The monopolist sets $p_2^m (> 0)$ such that $\mid \eta_v \mid = 1$.*

The condition $\left| \frac{V}{C} \frac{\partial C}{\partial V} \right| = 1$ is analogous to ones found in other contexts -e.g., the efficiency wage hypothesis. Its interpretation here is that of a marginal revenue = 0 condition. Having chosen y and p_1, the monopolist picks a p_2 that maximizes VC, the quality-adjusted aggregate expected visits, thereby maximizing consumers' willingness to pay for the club good.

[7] Note that $p_1^m = 0$ is conceivable, given $v_1 = 1$ is fixed.

Lastly, we cannot rule out at this stage the possibility that y^m can be zero.

2.3. A benchmark: social welfare maximisation (under an identical informational constraint).

As a benchmark, we consider the case where the club good is provided by a benevolent social welfare maximizer. Like the monopolist, the welfare maximizer also knows members' behaviour as described in sections 2.1.1 and 2.1.2, but is unable to observe agents' ex-post valuation of the good. (Hence, she cannot engage in discriminatory pricing ex post.) She incorporates this information while solving the following social welfare maximization problem:

$$\max_{p_1,p_2,y} \quad n[u(M_1-p_1)+C(y,\overline{V})E(\varepsilon)+ \delta\int_{\varepsilon^*}^{\overline{\varepsilon}}\{u(M_2-p_2v(\varepsilon))+\varepsilon v(\varepsilon)C(y,V)\}dF(\varepsilon)+\delta\int_{\varepsilon^*}^{\overline{\varepsilon}}u(M_2)dF(\varepsilon)]$$

subject to

$$np_1+n\delta\int_{\varepsilon^*}^{\overline{\varepsilon}}p_2v(\varepsilon)dF(\varepsilon)\geqslant y \tag{11}$$

where (11) is the revenue constraint that the (expected value) of the revenue raised from the club good must be large enough to cover its provision cost. We can reasonably ignore the participation constraint (6) here on the following ground. If it binds with a profit maximizer making positive profits, it will certainly be slack with a welfarist that just breaks even and leaves some surplus with consumers. The optimal values of the choice variables ε^*, p_2, etc., here will therefore generally differ from the corresponding values in the monopolist's problem. Let superscript "s" denote magnitudes in the welfarist's regime.

The Lagrangian for the welfarist's optimization is

$$\begin{aligned}\mathcal{L}^s &= n[u(M_1 - p_1^s) + C(y^s, \overline{V})E(\varepsilon) \\ &+\delta \int_{\varepsilon^*}^{\overline{\varepsilon}} \{u(M_2 - p_2^s v(\varepsilon)) + \varepsilon v(\varepsilon) C(y^s, V)\} dF(\varepsilon) + \delta \int_{\varepsilon^*}^{\overline{\varepsilon}} u(M_2) dF(\varepsilon)] \\ &+\lambda^s \{n p_1^s + n\delta \int_{\varepsilon^*}^{\overline{\varepsilon}} p_2^s v(\varepsilon) dF(\varepsilon) - y^s\}\end{aligned}$$

The FOC's are:

$$\frac{\partial \mathcal{L}^s}{\partial p_1^s} = -\, u_{x_1}^s + \lambda^s \leqq 0 \quad \text{for } p_1^s \geqq 0 \tag{12}$$

$$\begin{aligned}\frac{\partial \mathcal{L}^s}{\partial p_2^s} &= n\delta[\int_{\varepsilon^*}^{\overline{\varepsilon}} \{-v(\varepsilon)u_{x_2} + \varepsilon v(\varepsilon) C_V \frac{\partial V}{\partial p_2^s}\} dF(\varepsilon) + \\ &\lambda^s \int_{\varepsilon^*}^{\overline{\varepsilon}} \{v(\varepsilon) + p_2^s \frac{\partial v(\varepsilon)}{\partial p_2^s}\} dF(\varepsilon)] \leqq 0 \text{ for } p_2^s \geqq 0\end{aligned} \tag{13}$$

$$\begin{aligned}\frac{\partial \mathcal{L}^s}{\partial y^s} &= n[C_y E(\varepsilon) + \delta \int_{\varepsilon^*}^{\overline{\varepsilon}} \varepsilon v(\varepsilon)\{C_y + C_V \frac{\partial V}{\partial y^s}\} dF(\varepsilon)] + \\ &\lambda^s n\delta \int_{\varepsilon^*}^{\overline{\varepsilon}} p_2^s \frac{\partial v(\varepsilon)}{\partial y^s} dF(\varepsilon) \leqq \lambda^s \text{ for } y^s \geqq 0\end{aligned} \tag{14}$$

We observe the following about the FOC's. First, $p_2^s > 0$ in equilibrium (by the argument that otherwise, as in the monopoly case, second period demand will be infinite). So, (13) holds with equality. Second (after substituting for $\frac{\partial V}{\partial y}$), since

$$\{C_y + C_V \frac{\partial V}{\partial y}\} = \frac{C_y}{1 + C_V n \int_{\varepsilon^*}^{\overline{\varepsilon}} \frac{\varepsilon}{p_2^2 u_{xx}} dF(\varepsilon)} > 0 \tag{15}$$

(14) indicates $\lambda^s > 0$ - i.e., the revenue constraint binds in equilibrium[8]. This,

[8] To show that (14) indicates $\lambda^s > 0$, suppose not, thus $\lambda^s = 0$ by Kuhn-Tucker theory. Then (14) collapses to $n[\overline{v} C_y E(\varepsilon) + \delta \int_{\varepsilon^*}^{\overline{\varepsilon}} \varepsilon v(\varepsilon)\{C_y + C_V \frac{\partial V}{\partial y^s}\} dF(\varepsilon)] \leqq 0$. This cannot be, given $C_y + C_V \frac{\partial V}{\partial y^s} > 0$ and $C_y > 0$.

along with the fact that $p_2^s > 0$, then implies that $y^s > 0$ at the welfarist's optimum. Therefore equation (14) holds with strict equality. Note that members receive some rents at the welfarist's optimum (as opposed to in the monopolist's case) in any meaningful solution. However, as we show formally below with homogeneous $C(.)$, even with $\lambda^s > 0$, (12) can have a corner solution. Hence we have

Observation 2. *It is possible for the welfarist to make an introductory offer on the club good (i.e., $p_1^s = 0$).*

Observation 2 helps us show the following (proven in the Appendix):

PROPOSITION 2. *If the welfarist makes an introductory offer, then she also sets $p_2^s (> 0)$ so that $| \eta_v | > 1$ holds. Further, there is 'overprovision' of the good in the Samuelson rule sense that willingness to pay for the marginal investment in the club facility is less than its cost.*

It is also worth noting that (13) holding with equality can be rearranged to

$$n\delta[-\{\frac{V}{C}C_V + 1\} \int\limits_{\varepsilon^*}^{\bar{\varepsilon}} \varepsilon v(\varepsilon) dF(\varepsilon) + \lambda^s \int\limits_{\varepsilon^*}^{\bar{\varepsilon}} \{v(\varepsilon) + p_2^s \frac{\partial v(\varepsilon)}{\partial p_2^s}\} dF(\varepsilon)] = 0 \qquad (16)$$

So, as $VC_V + C < 0$ at the welfarist's optimum when it makes an introductory offer, it then follows that $\int_{\varepsilon^*}^{\bar{\varepsilon}} \{v(\varepsilon) + p_2^s \frac{\partial v(\varepsilon)}{\partial p_2^s}\} dF(\varepsilon) < 0$. I.e., other things equal, the welfarist could increase its expected revenue by lowering price. The rationale for this is simple: if $VC_V + C < 0 \Longleftrightarrow | \eta_v | > 1$, quality is very sensitive to visits at the welfarist's optimum and it will wish to discourage visits, other things equal. This it can do by raising p_2^s to above the level that is profit-maximizing, given its choice of y^s and p_1^s.

2.4. Monopolist's versus welfarist's equilibrium

Recall that at the monopoly equilibrium $p_1^m > 0$ and $p_2^m > 0$, although $y^m = 0$ is possible, and that at the social optimum $p_2^s > 0$ and $y^s > 0$, while p_1^s =0 is possible. Also, observe that not all the choice variables can *simultaneously* be positive for both the monopolist and the welfarist.[9] Thus, there are only three possible ways in which the monopolist's equilibrium can differ from the social optimum:

1.*Regime (a).* $p_1^m > 0$, $p_2^m > 0, y^m > 0$; $p_1^s = 0$, $p_2^s > 0, y^s > 0$;

2. *Regime (b).* $p_1^m > 0$, $p_2^m > 0, y^m = 0$; $p_1^s = 0, p_2^s > 0, y^s > 0$; and

3. *Regime (c).* $p_1^m > 0$, $p_2^m > 0, y^m = 0$; $p_1^s > 0$, $p_2^s > 0, y^s > 0$.

2.5. The characterisation of different regimes

This section explores which one(s) of the above regimes is (are) likely to occur and their characteristics. We first study cases when the quality function, $C(y, V)$, is homogeneous[10]. Our general result (proven in the Appendix) is the following:

PROPOSITION 3. *Suppose that the quality function $C(y, V)$ is homogeneous of degree k. Then: (i) regime (c) cannot occur for any k; (ii) regime (b) occurs if and only if $k = -1$; (iii) only regime (a) can occur for all k satisfying $k + 1 > 0$..*

Although $C(y, V)$ might not be homogeneous, homogeneity provides a convenient simplification that lets us visualise the consequences of different extents of qualitative returns to scale. One implication of Proposition 3 is that, if there are sufficiently large qualitative scale diseconomies, then the monopolist will not find it optimal to make any investment in the club facility. When $k = -1$, doubling y and V would keep the facility provision per use of the club constant but result in a halving of the quality level as perceived by its customers, simply because crowding or non-exclusiveness per se causes them such detriment. In that case, the monopolist would find it more profitable to not spend on the facility and keep visits low if it wishes to maintain quality. But, an even more striking and important implication of Proposition 3 is the following: the welfarist will *always* offer a free trial period for all degrees of homogeneity of the quality function that lead to a feasible solution (which the Appendix shows requires $k + 1 \geq 0$). This behaviour contrasts starkly with the monopolist's, which we know from Proposition 1 *never* offers a free trial

[9] Since that makes their first order conditions exactly identical, which cannot be possible given that the monopolist maximises profit while the welfarist breaks even!

[10] Some of the implications of homogeneous quality or congestion functions for club theory have been studied by Barro and Romer (1987), Fraser (2000) and Kolm (1974), among others.

period whether or not $C(.)$ is homogeneous.

In the arbitrary homogeneous case, the quality function satisfies $C(y.V) = V^k c(y/V)$ for some function $c(.)$, where k is the degree of homogeneity. It is easy to show that the monopolist will then always wish to offer a higher level of facility provision per visit than does the welfarist if the facility provision elasticity of quality, $(y/V)\, c^{/}(y/V)\,/c(y/V)$, is monotonic in the facility provision per visit, $z \equiv y/V$. First, we show in the next Lemma (proven in the Appendix) that $zc^{/}(z)\,/c(z)$ is decreasing in z at both the monopolist's and welfarist's equilibrium. Hence, if it is monotonic, it must be decreasing everywhere.

Lemma 4. *If there are diminishing returns to an investment in the facility provision (i.e., $c^{//} < 0$) and the facility provision elasticity of quality, $(y/V)\, c^{/}(y/V)\,/$ $c(y/V)$, is monotonic in $z \equiv y/V$, then it is decreasing everywhere.*

It immediately follows that, if the conditions of Lemma 4 are satisfied, then the monopolist will always invest in a greater level of facility provision per visit than the welfarist. To see this, note that as the monopolist's and welfarist's equilibria satisfy $z^s c^{/}(z^s)/c(z^s) > k+1 = z^m c^{/}(z^m)/c(z^m)$, then we must have $z^m > z^s$.

A much discussed case of homogeneity is when the quality function is homogeneous of degree zero ("h.o.d.0"). This case is one where quality just depends on the facility investment per use of the club. For example, swimmers might perceive the quality of a swimming pool to be determined by the average area each swimmer has to herself, and the construction cost per square metre of pool is constant[11]. In the h.o.d.0. case, $C(y,V) = c(y/V)$, for some function $c(.)$,with $c^{/}(y/V) > 0$. The following proposition is an immediate implication of Lemma 4 and the fact that $z^m > z^s$ if the conditions of this Lemma are satisfied.

Proposition 4. *If $C(.)$ is h.o.d.0. and the elasticity of quality w.r.t. the facility provision is monotonic, then $C(y^m, V^m) = c(y^m/V^m) > c(y^s/V^s) = C(y^s, V^s)$: the monopolist invests in a socially excessive level of quality provision for the second period.*

[11] Some of the implications of an h.o.d.0. quality function for club theory have been explored elsewhere - e.g., by Fraser (2000) and Kolm (1974).

The rationale for this result is that the monopolist both wishes to extract rent from those who might try in the first period but not buy in the second (hence it sets $p_1^m > 0$) and to provide an incentive for many first period tryers to remain second period buyers. It can do this by ensuring a high second period quality, which relaxes the participation constraint. The welfarist, conversely, is concerned about equity as well as efficiency. It is interested in equalizing the actual utility of stayers and leavers as nearly as possible. It prefers, therefore, to not charge in the first period, though this means relatively less funds might be available for facility provision to enhance second period quality. Perhaps surprisingly, this scenario is reminiscent of the one found in the literature on monopoly pricing under asymmetric information where prices signal products quality. It is well established in that field that high prices signal high-quality products in markets with unknown product quality (e.g., cf. Milgrom and Roberts 1986, Bagwell and Riordan 1991, Judd and Riordan 1994). In order to signal the product quality, the monopolist may charge a price well above the full information profit maximizing price level. Our model is not a signalling model, yet it can have an observationally equivalent implication. When the quality of the club good is yet to be learnt by the visitors, the monopolist credibly provides a higher quality club good than the welfarist does if it charges a higher first period price than does the latter (i.e., $p_1^m > p_1^s = 0$).

An Example. Suppose $C(y,V) = [(y/V)+\gamma]^{\vartheta}$, for some scalars $\gamma < 0$ and $\vartheta \in (0,1)$. Then, it is easy to show[12] that $y^m/V^m = \frac{\gamma}{\vartheta-1} > y^s/V^s$, hence $C(y^m,V^m) = [(y^m/V^m)+\gamma]^{\vartheta} > C(y^s,V^s) = [(y^s/V^s)+\gamma]^{\vartheta}$.

From Proposition 3, we know that homogeneity of $C(.)$ severely restricts the possibility for regimes other than (a) to occur. We will therefore now suppose that $C(y,V)$ is not homogeneous and that regimes (b) and (c) are possible. We can then ask what the characteristics of these regimes might be. In what follows, we make the following reasonable assumption:

[12] From (10), $-(y^m/V^m)[\gamma+(y^m/V^m)]^{\vartheta-1}\vartheta + [\gamma+(y^m/V^m)]^{\vartheta} = 0 \iff -(y^m/V^m)\vartheta + [\gamma+(y^m/V^m)] = 0 \iff (y^m/V^m)(1-\vartheta)+\gamma = 0 \iff y^m/V^m = \frac{\gamma}{\vartheta-1}$. Likewise, from Proposition 2, $-(y^s/V^s)[\gamma+(y^s/V^s)]^{\vartheta-1}\vartheta + [\gamma+(y^s/V^s)]^{\vartheta} < 0$, which simplifies to $y^s/V^s < \frac{\gamma}{\vartheta-1}$.

A4. $C_{VV} \leqq 0$ (increasing marginal disutility of congestion); $C_{Vy} \geqq 0$ (increased club facility provision ameliorates the negative impact of increased club usage).[13]

Proposition 5. *Under regimes (b) and (c) and A4, the aggregate second period visits to the club under monopoly are less than the socially optimal level:* $V^m < V^s$.

Proof. Recall that, under regime (b), the inequality $|\eta_v^s| > |\eta_v^m| = 1$ holds as $|\eta_v^m| = 1$ by observation 1 and $|\eta_v^s| > 1$ by Proposition 2. Conversely, under regime (c), $|\eta_v^s| = 1$ (combining equations (12) and (13) when $p_1^s > 0$) - i.e., under regime (c), $|\eta_v^s| = |\eta_v^m| = 1$ holds. We can use this, together with the properties of the visit elasticity of quality when C is non-homogeneous, to show that monopoly will plausibly result in less second period use of the club than is socially optimal in regimes (b) and (c). To see how the elasticity $\eta_v \equiv VC_V(y,V)/C(y,V)$ behaves in response to changes in y and V, we can totally differentiate and rearrange to obtain

$$d\eta_v = C^{-2}\left[\left(CC_V + CVC_{VV} - VC_V^2\right)dV + \left(CVC_{Vy} - VC_VC_y\right)dy\right] \tag{17}$$

Given our assumptions, $\left(CC_V + CVC_{VV} - VC_V^2\right) < 0$ and $\left(CVC_{Vy} - VC_VC_y\right) > 0$. Thus, other things equal, an increase in V will decrease the elasticity η_v (make it more negative), while an increase in y will increase it (make it less negative). In both these regimes $y^m = 0$. Therefore, to compare the monopolist and the welfarist's behavior in them, as $y^m = 0$, we can let $dy = y^s > 0 = y^m$. Then, to satisfy $V^mC_V(0,V^m)/C(0,V^m) = -1 \geq V^sC_V(y^s,V^s)/C(y^s,V^s)$ and (17), we must have $V^s > V^m$. ∎

Note that the assumption $C_{Vy} \geqq 0$ can actually be violated and yet we get $V^s > V^m$ in regimes (b) and (c). For example, if $C(y,V) = h(y)/g(V)$ for some positive increasing functions h and g, then $C_{Vy} = -h'(y)g'(V)/g(V)^2 < 0$. However, direct calculation shows that $CVC_{Vy} - VC_VC_y = 0$ in this case and, so,

[13] We also assume $C(0,V) > 0$. Otherwise, regimes (b) and (c) could not occur as the monopolist would not get any second-period customers and the participation constraint could not be satisfied if $y^m = 0$.

we must have $V^s > V^m$ as before.

We are unable to compare second period quality levels in regimes (b) and (c). This is because, although $y^s > 0 = y^m$, the lower level of second period visits under monopoly might still mean $C(0, V^m) > C(y^s, V^s)$ occurs. However, as the total number of first period visits ($\overline{V}$) are the same under monopoly and welfarism, it follows that $C\left(y^s, \overline{V}\right) > C\left(0, \overline{V}\right)$: the welfarist offers a higher first period quality than the monopolist in these regimes. This is consistent with the suggestion that, compared with the welfarist, the monopolist is more focused on treating retained customers well, even if at the expense of disappointed first period customers. These arguments suggest that it might be possible for the monopolist to offer a higher quality to repeat customers, yet a lower quality to first-time and once-only customers, than does the welfarist. So, unlike in a single-period model, we cannot say unambiguously that the monopolist will over- or under-supply quality.

3. CONCLUSIONS

We have examined the pricing and investment strategies of a club good provider when potential club users are uncertain about the quality of the shared facility. Essentially, club goods are experience goods: " you have to try before you know you want to buy." Yet this aspect of clubs has not been studied in the literature. Incorporating this feature is important as it can give rise to very contrasting strategies for a monopoly profit maximizer and a welfare maximizer, as we have shown.

In our model, potential club members are unsure of the quality of a club's facilities beforehand. They must make a fixed number of visits in the first period in order to ascertain their evaluation of the quality, which they then learn perfectly. Based on this learning experience, they then decide whether to continue their membership and the number of visits to make, or to leave the club for good. Pricing strategies announced in the first period and the investment undertaken to maintain the shared facilities by the provider are therefore crucial in determining the ultimate membership of the club.

In this scenario, one might expect a provider to offer an introductory discount to the consumers who have no prior knowledge about the quality of the good they are about to experience. However, we show that is not necessarily so - it depends upon who the provider is. If it is a social welfare maximizer, she might indeed give consumers an "introductory offer" of a free trial period in which to decide whether it is agreeable to them - and definitely does this if the quality function is homogeneous. She does this in order to reduce the disparity in welfare between those who try the product and find it unsatisfactory and therefore leave the club, and those who find it satisfactory and wish to continue as consumers. In the extreme, only the latter pay for providing the club good facility. Conversely, if the provider is a monopolist, then her focus is on extracting as much rent as possible from consumers. As a result, the monopolist never makes an introductory offer. Thus all consumers, whether stayers or leavers, contribute to any cost of facility provision and to profits. This enables the monopolist to increase (in some cases) the size of the club facility (thereby its quality), therefore increasing the incentive for consumers to remain with it.

The latter results about the monopoly provider are consistent with those from models of monopoly pricing with experience goods and repeat purchases (such as Cremer (1984) and Bergemann and Välimäki (2005)), and also with ones that establish the signalling role about the future quality of the product played by today's price in a dynamic setting (such as Bagwell and Riordan (1991), Milgrom and Roberts (1986) and Judd and Riordan (1994)). However, none of these other papers have considered explicitly, as we have done, the implications of the peculiar features of clubs - such as the congestion externality and the endogenous determination of quality arising both from the utilization choice of members and the entrepreneurial club provider's pricing strategy and level of facility provision.

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APPENDIX

Proof of Lemma 2. For a given p_2, y and V, the club usage choice of someone with experience ε is a continuous and differentiable mapping $v(\varepsilon, p_2, y, V)$: $[0, M_2/p_2] \rightarrow [0, M_2/p_2]$ satisfying (3).The ex ante expected visits for this consumer satisfy (4) and those for all consumers must satisfy $V = n\int_{\varepsilon^*}^{\overline{\varepsilon}} v(\varepsilon, p_2, y, V)dF(\varepsilon)$ uniquely if a unique equilibrium exists. Define the aggregate expected visit mapping $V(p_2, y, V)$ by $V(p_2, y, V) = n\int_{\varepsilon^*}^{\overline{\varepsilon}} v(\varepsilon, p_2, y, V)dF(\varepsilon) : [0, nM_2/p_2] \rightarrow [0, nM_2/p_2]$. This mapping is also continuous and differentiable. By differentiating,

$n\partial\left[\int_{\varepsilon^*}^{\overline{\varepsilon}} v(\varepsilon, p_2, y, V)dF(\varepsilon)\right]/\partial V = -n\left(C_V(y, V)/C(y, V)\right)\int_{\varepsilon^*}^{\overline{\varepsilon}} p_2 \frac{u_{x_2}}{u_{xx}} dF(\varepsilon) < 0$, using (3) and Leibnitz's rule. So, $V(p_2, y, V)$ is monotonically decreasing in V and takes it maximum value at $V(p_2, y, 0)$, where $nM_2/p_2 > V(p_2, y, 0) > 0$, with the first inequality following from the fact that the private good is essential. As $nM_2/p_2 > V(p_2, y, 0) > V(p_2, y, nM_2/p_2)$, the graph of $V(p_2, y, V)$ against V

must cross the 45^0 line uniquely from above at a point where $V(p_2, y, V) = V$. Thus, a unique equilibrium in expected visits exists for a given p_2 and y. ■

Proof of Lemma 3. (i) Differentiation of (5) with respect to y, (using Leibnitz's rule) yields

$$\frac{\partial V}{\partial y} = n[\int_{\varepsilon^*}^{\overline{\varepsilon}} \frac{\partial v(\varepsilon)}{\partial y} dF(\varepsilon) - v(\varepsilon^*)\frac{\partial \varepsilon^*}{\partial y}] = n\int_{\varepsilon^*}^{\overline{\varepsilon}} \frac{\partial v(\varepsilon)}{\partial y} dF(\varepsilon) \tag{18}$$

since $v(\varepsilon^*) = 0$. Differentiating (3) with respect to v and y and integrating over ε, we obtain

$$\int_{\varepsilon^*}^{\overline{\varepsilon}} \frac{\partial v(\varepsilon)}{\partial y} dF(\varepsilon) = -[C_y + C_v \frac{\partial V}{\partial y}] \int_{\varepsilon^*}^{\overline{\varepsilon}} \frac{\varepsilon}{p_2^2 u_{xx}} dF(\varepsilon) \tag{19}$$

where u_{xx} is the second derivative with respect to x_2. Hence

$$\frac{\partial V}{\partial y} = -n\left\{C_y \int_{\varepsilon^*}^{\overline{\varepsilon}} \frac{\varepsilon}{p_2^2 u_{xx}} dF(\varepsilon)\right\} / \left\{1 + nC_V \int_{\varepsilon^*}^{\overline{\varepsilon}} \frac{\varepsilon}{p_2^2 u_{xx}} dF(\varepsilon)\right\} > 0. \tag{20}$$

So, as the level of provision increases, both individual and aggregate visits increase.

(ii) Differentiating (5) with respect to p_2, we find

$$\frac{\partial V}{\partial p_2} = n[\int_{\varepsilon^*}^{\overline{\varepsilon}} \frac{\partial v(\varepsilon)}{\partial p_2} dF(\varepsilon) - v(\varepsilon^*)\frac{\partial \varepsilon^*}{\partial p_2}] = n\int_{\varepsilon^*}^{\overline{\varepsilon}} \frac{\partial v(\varepsilon)}{\partial p_2} dF(\varepsilon) \tag{21}$$

Differentiation of (3) with respect to v and p_2 yields

$$\frac{\partial v(\varepsilon)}{\partial p_2} = \frac{\left\{-\varepsilon C_V \frac{\partial V}{\partial p_2} - u_{xx} p_2 v(\varepsilon) + u_x\right\}}{p_2^2 u_{xx}} \tag{22}$$

Thus, integrating and rearranging to isolate $\partial V/\partial p_2$,

$$\frac{\partial V}{\partial p_2} = -n\int_{\varepsilon^*}^{\overline{\varepsilon}} \left[\frac{v(\varepsilon)}{p_2} - \frac{u_x}{p_2^2 u_{xx}}\right] dF(\varepsilon) / \left[1 + nC_V \int_{\varepsilon^*}^{\overline{\varepsilon}} \frac{\varepsilon}{p_2^2 u_{xx}} dF(\varepsilon)\right] < 0 \tag{23}$$

(iii) Differentiating (2) w.r.t. y, using (20) and rearranging yields

$$\frac{\partial \varepsilon^*}{\partial y} = -C_y \varepsilon^*/C \left[1 + nC_V \int_{\varepsilon^*}^{\overline{\varepsilon}} \frac{\varepsilon}{p_2^2 u_{xx}} dF(\varepsilon) \right] < 0. \tag{24}$$

(iv) From the condition defining the marginal quality valuation,

$$\begin{aligned}
\frac{\partial \varepsilon^*}{\partial p_2} &= \left\{ u_x(M_2) - \varepsilon^* C_V \frac{\partial V}{\partial p_2} \right\} \frac{1}{C} \\
&= \left\{ C + C_V n \int_{\varepsilon^*}^{\overline{\varepsilon}} v(\varepsilon) dF(\varepsilon) \right\} \left[1 + nC_V \int_{\varepsilon^*}^{\overline{\varepsilon}} \frac{\varepsilon}{p_2^2 u_{xx}} dF(\varepsilon) \right]^{-1} \frac{\varepsilon^*}{p_2} \\
\therefore \quad & sign\left(\frac{\partial \varepsilon^*}{\partial p_2} \right) = sign\left\{ C + C_V V \right\} \lesseqgtr 0
\end{aligned} \tag{25}$$

after substituting from (23).

Derivation of equation (10). From (8) in the text, substituting $\lambda = n/u_{x_1}$,

$$n\delta u_{x_1} \left[\int_{\varepsilon^*}^{\overline{\varepsilon}} \{ v(\varepsilon) + p_2^m \frac{\partial v(\varepsilon)}{\partial p_2^m} \} dF(\varepsilon) \right] + n\delta \left[\int_{\varepsilon^*}^{\overline{\varepsilon}} \{ -v(\varepsilon) u_{x_2} + \varepsilon v(\varepsilon) C_V \frac{\partial V}{\partial p_2^m} \} dF(\varepsilon) \right] = 0$$

Rearranging,

$$n\delta \int_{\varepsilon^*}^{\overline{\varepsilon}} (u_{x_1} - u_{x_2})\, v(\varepsilon)\, dF(\varepsilon) + n\delta \int_{\varepsilon^*}^{\overline{\varepsilon}} \left\{ u_{x_1} p_2^m \frac{\partial v(\varepsilon)}{\partial p_2^m} + \varepsilon v(\varepsilon)\, C_V \frac{\partial V}{\partial p_2^m} \right\} dF(\varepsilon) = 0$$

Cancelling $n\delta > 0$, this becomes

$$\int_{\varepsilon^*}^{\overline{\varepsilon}} (u_{x_1} - u_{x_2})\, v(\varepsilon)\, dF(\varepsilon) + u_{x_1} p_2^m \int_{\varepsilon^*}^{\overline{\varepsilon}} \frac{\partial v(\varepsilon)}{\partial p_2^m} dF(\varepsilon) + C_V \frac{\partial V}{\partial p_2^m} \int_{\varepsilon^*}^{\overline{\varepsilon}} \varepsilon v(\varepsilon)\, dF(\varepsilon) = 0.$$

As $\frac{\partial V}{\partial p_2} = \frac{-n \int_{\varepsilon^*}^{\overline{\varepsilon}} \left[\frac{v(\varepsilon)}{p_2} - \frac{u_x}{p_2^2 u_{xx}} \right] dF(\varepsilon)}{\left[1 + nC_V \int_{\varepsilon^*}^{\overline{\varepsilon}} \frac{\varepsilon}{p_2^2 u_{xx}} dF(\varepsilon) \right]} \equiv -n \int_{\varepsilon^*}^{\overline{\varepsilon}} \left[\frac{v(\varepsilon)}{p_2} - \frac{u_x}{p_2^2 u_{xx}} \right] dF(\varepsilon)/D$ from (23), the last equation can be written as

$$\int_{\varepsilon^*}^{\overline{\varepsilon}} (u_{x_1} - u_{x_2})\, v(\varepsilon)\, dF(\varepsilon) -$$

$$\int_{\varepsilon^*}^{\overline{\varepsilon}} \left[\frac{v(\varepsilon)}{p_2} - \frac{u_x}{p_2^2 u_{xx}}\right] dF(\varepsilon) \left\{ u_{x_1} p_2^m + nC_V \int_{\varepsilon^*}^{\overline{\varepsilon}} \varepsilon v(\varepsilon)\, dF(\varepsilon) \right\} /D = 0$$

Or

$$\int_{\varepsilon^*}^{\overline{\varepsilon}} (u_{x_1} - u_{x_2})\, v(\varepsilon)\, dF(\varepsilon) \left[1 + nC_V \int_{\varepsilon^*}^{\overline{\varepsilon}} \frac{\varepsilon}{p_2^{m2} u_{xx}} dF(\varepsilon)\right] -$$

$$\int_{\varepsilon^*}^{\overline{\varepsilon}} \left[\frac{v(\varepsilon)}{p_2^m} - \frac{u_x}{p_2^{m2} u_{xx}}\right] dF(\varepsilon) \left\{ u_{x_1} p_2^m + nC_V \int_{\varepsilon^*}^{\overline{\varepsilon}} \varepsilon v(\varepsilon)\, dF(\varepsilon) \right\} = 0$$

Now, from the first and second terms,

$$\begin{aligned} & \int_{\varepsilon^*}^{\overline{\varepsilon}} (u_{x_1} - u_{x_2})\, v(\varepsilon)\, dF(\varepsilon) - u_{x_1} p_2^m \int_{\varepsilon^*}^{\overline{\varepsilon}} \left[\frac{v(\varepsilon)}{p_2^m} - \frac{u_x}{p_2^{m2} u_{xx}}\right] dF(\varepsilon) \\ = & \int_{\varepsilon^*}^{\overline{\varepsilon}} \left[u_{x_1} \frac{u_{x_2}}{p_2^m u_{xx}} - u_{x_2} v(\varepsilon)\right] dF(\varepsilon) \equiv (A). \end{aligned}$$

From the terms in C_V, we have

$$\begin{aligned} & -\int_{\varepsilon^*}^{\overline{\varepsilon}} \left(\frac{\varepsilon}{p_2^{m2} u_{xx}}\right) dF(\varepsilon) \int_{\varepsilon^*}^{\overline{\varepsilon}} u_{x_2} v(\varepsilon)\, dF(\varepsilon) + \int_{\varepsilon^*}^{\overline{\varepsilon}} \frac{u_{x_2}}{p_2^m u_{xx}} dF(\varepsilon) \int_{\varepsilon^*}^{\overline{\varepsilon}} \varepsilon v(\varepsilon)\, dF(\varepsilon) \\ = & -\int_{\varepsilon^*}^{\overline{\varepsilon}} \left(\frac{\varepsilon}{p_2^{m2} u_{xx}}\right) dF(\varepsilon) \int_{\varepsilon^*}^{\overline{\varepsilon}} u_{x_2} v(\varepsilon)\, dF(\varepsilon) + \int_{\varepsilon^*}^{\overline{\varepsilon}} \frac{u_{x_2}}{p_2^m u_{xx}} dF(\varepsilon) \int_{\varepsilon^*}^{\overline{\varepsilon}} u_{x_2} v(\varepsilon) \frac{p_2^m}{C} dF(\varepsilon) \end{aligned}$$

= (using $\varepsilon = p_2^m u_{x_2}/C$ from the FOC for an agent with experience ε in the first period)

$$\begin{aligned}
& -\int_{\varepsilon^*}^{\overline{\varepsilon}} \frac{p_2^m}{C}\left(\frac{u_{x_2}}{p_2^{m2}u_{xx}}\right) dF(\varepsilon) \int_{\varepsilon^*}^{\overline{\varepsilon}} u_{x_2} v\left(\varepsilon\right) dF(\varepsilon) \\
& + \int_{\varepsilon^*}^{\overline{\varepsilon}} \frac{u_{x_2}}{p_2^m u_{xx}} dF(\varepsilon) \int_{\varepsilon^*}^{\overline{\varepsilon}} u_{x_2} v\left(\varepsilon\right) \frac{p_2^m}{C} dF(\varepsilon) \\
= \; & 0
\end{aligned}$$

The residual terms in C_V equal

$$nC_V \left[\int_{\varepsilon^*}^{\overline{\varepsilon}} \left(\frac{\varepsilon}{p_2^{m2}u_{xx}}\right) dF(\varepsilon) \int_{\varepsilon^*}^{\overline{\varepsilon}} u_{x_1} v\left(\varepsilon\right) dF(\varepsilon) - \int_{\varepsilon^*}^{\overline{\varepsilon}} \frac{v(\varepsilon)}{p_2^m} dF\left(\varepsilon\right) \int_{\varepsilon^*}^{\overline{\varepsilon}} \varepsilon v\left(\varepsilon\right) dF(\varepsilon) \right] \equiv (B).$$

Amalgamating (A) and (B),

$$\begin{aligned}
& u_{x_1} \int_{\varepsilon^*}^{\overline{\varepsilon}} \frac{u_{x_2}}{p_2^m u_{xx}} dF\left(\varepsilon\right) + u_{x_1} nC_V \int_{\varepsilon^*}^{\overline{\varepsilon}} \left(\frac{\varepsilon}{p_2^{m2}u_{xx}}\right) dF(\varepsilon) \int_{\varepsilon^*}^{\overline{\varepsilon}} v\left(\varepsilon\right) dF(\varepsilon) \\
= \; & u_{x_1} \left[\int_{\varepsilon^*}^{\overline{\varepsilon}} \frac{u_{x_2}}{p_2^m u_{xx}} dF\left(\varepsilon\right) + C_V \int_{\varepsilon^*}^{\overline{\varepsilon}} \left(\frac{\varepsilon}{p_2^{m2}u_{xx}}\right) dF(\varepsilon) V \right] \\
= \; & \left[u_{x_1} \int_{\varepsilon^*}^{\overline{\varepsilon}} \frac{u_{x_2}}{p_2^m u_{xx}} dF\left(\varepsilon\right) \right] \left[1 + \frac{C_V V}{C} \right] \quad \text{(using } \varepsilon = p_2^m u_{x_2}/C \text{ again)}
\end{aligned}$$

Also,

$$\begin{aligned}
& -\int_{\varepsilon^*}^{\overline{\varepsilon}} u_{x_2} v\left(\varepsilon\right) dF\left(\varepsilon\right) - nC_V \int_{\varepsilon^*}^{\overline{\varepsilon}} \frac{v(\varepsilon)}{p_2^m} dF\left(\varepsilon\right) \int_{\varepsilon^*}^{\overline{\varepsilon}} \varepsilon v\left(\varepsilon\right) dF(\varepsilon) \\
= \; & -\int_{\varepsilon^*}^{\overline{\varepsilon}} u_{x_2} v\left(\varepsilon\right) dF\left(\varepsilon\right) \left[1 + \frac{C_V V}{C} \right] \text{ (again using } \varepsilon = p_2^m u_{x_2}/C\text{)}
\end{aligned}$$

So, resubstituting, the FOC (8) becomes

$$\left[u_{x_1} \int_{\varepsilon^*}^{\overline{\varepsilon}} \frac{u_{x_2}}{p_2^m u_{xx}} dF\left(\varepsilon\right) - \int_{\varepsilon^*}^{\overline{\varepsilon}} u_{x_2} v\left(\varepsilon\right) dF\left(\varepsilon\right) \right] \left[1 + \frac{C_V V}{C} \right] = 0$$

which is (10) in the text.

Proof of Proposition 2. (i) First, we show that if $p_1^s = 0$ then $u^s_{x_1} > \lambda^s$. Suppose otherwise, so $p_1^s = 0$ yet $u^s_{x_1} = \lambda^s$. Suppose the welfarist were then to increase p_1^s to $p_1^s = \varepsilon > 0$, for some very small ε. To first-order, the loss of welfare in first period utility is exactly counter-balanced by the value of extra funds, λ^s. Thus the welfarist could equally well set $p_1^s = \varepsilon > 0$, contradicting the unique optimality of $p_1^s = 0$. Next, note from (13), since $u^s_{x_1} > \lambda^s > 0$,

$$n\delta[\int_{\varepsilon^*}^{\overline{\varepsilon}} \{-v(\varepsilon)u_{x_2} + \varepsilon v(\varepsilon)C_V \frac{\partial V}{\partial p_{2^s}}\}dF(\varepsilon) + u^s_{x_1}\int_{\varepsilon^*}^{\overline{\varepsilon}} \{v(\varepsilon) + p_2^s \frac{\partial v(\varepsilon)}{\partial p_2^s}\}dF(\varepsilon)] > 0$$

which, after simplification (similar to the derivation of (10) above), yields

$$\left[u_{x_1}\int_{\varepsilon^*}^{\overline{\varepsilon}} \frac{u_{x_2}}{p_2 u_{xx}} dF(\varepsilon) - \int_{\varepsilon^*}^{\overline{\varepsilon}} u_{x_2} v(\varepsilon) dF(\varepsilon)\right]\left[\frac{V}{C}C_V + 1\right] > 0$$

Since $\left[u_{x_1}\int_{\varepsilon^*}^{\overline{\varepsilon}} \frac{u_{x_2}}{p_2 u_{xx}} dF(\varepsilon) - \int_{\varepsilon^*}^{\overline{\varepsilon}} u_{x_2} v(\varepsilon) dF(\varepsilon)\right] < 0$, hence $\left[\frac{V}{C}C_V + 1\right] < 0 \Rightarrow \mid \eta_v \mid > 1$ as $C_V < 0$.

(ii) Using the fact that $u^s_{x_1} > \lambda^s$ for $p_1^s = 0$, equation (14) - which holds with equality as $y^s > 0$ - can be rewritten as

$$\begin{aligned} & n\left[C_y E(\varepsilon) + \delta\int_{\varepsilon^*}^{\overline{\varepsilon}} \varepsilon v(\varepsilon)\{C_y + C_V \frac{\partial V}{\partial y^s}\}dF(\varepsilon)\right] + u^s_{x_1} n\delta \int_{\varepsilon^*}^{\overline{\varepsilon}} p_2^s \frac{\partial v(\varepsilon)}{\partial y^s} dF(\varepsilon) \\ < \quad & u^s_{x_1} \end{aligned}$$

The left hand side represents the (expected) marginal 'valuation' obtained from increased facility provision. This is a sum of two terms: the first $n\,[.]$ term represents the expected benefit attained from increased facility size (taking into account any direct and indirect impact on quality, the latter from any induced change in congestion), at an unchanged level of usage of the club. The second term, $u^s_{x_1} n\delta \int_{\varepsilon^*}^{\overline{\varepsilon}} p_2^s \frac{\partial v(\varepsilon)}{\partial y^s} dF(\varepsilon)$, represents the valuation of the expenditure on extra visits induced by the increased facility provision. The right hand side is the utility value of the cost incurred to increase the facility size. There is overprovision of the club good in the Samuelson rule sense since the valuation of the good induced by an increase in facility size falls short of the cost of providing that increase in the facility. ■

Proof of Proposition 3 The strategy of the proof is to show, first, that if $C(y,V)$ is homogeneous, then the monopolist's behaviour under regimes (b) and (c) (i.e., $y^m = 0$) occurs *iff* $C(y,V)$ is homogeneous of degree -1 (abbreviated "h.o.d.-1"). We then show that $C(y,V)$ being h.o.d.-1 is inconsistent with the welfarist's behaviour under regime (c). So, if $C(y,V)$ is h.o.d.-1, then only regime (b) holds. For all other degrees of homogeneity, only regime (a) is possible. However, the welfarist's behaviour under regime (a) is only consistent with $C(y,V)$ being h.o.d.k , where $k+1>0$.

Suppose that $C(y,V)$ is h.o.d.k. i.e.

$$C(ty,tV) = t^k C(y,V) \text{ for all } t > 0 \tag{26}$$

Then, by Euler's theorem,

$$yC_y + VC_v = kC(y,V) \tag{27}$$

At the monopoly equilibrium: $|\eta_v^m| = 1 \Rightarrow V^m C_v^m = -C(y^m, V^m)$. Substituting in (27) then yields:

$$y^m C_y^m = (k+1)C(y^m, V^m) \tag{28}$$

Proof of part (i): *regime (c) cannot occur for any k.*

In regime (c), $p_1^m > 0$, $p_2^m > 0, y^m = 0$; $p_1^s > 0$, $p_2^s > 0, y^s > 0$.With $y^m = 0$ for the monopolist and $C(0,V) > 0$ (see footnote 4), (28) then implies the only possible value of k, for this regime to occur is $k=-1$.However, as $p_1^s > 0$,we must have $[\frac{V^s}{C^s}C_v^s + 1] = 0 \Rightarrow V^s C_v^s = -C^s$ for the welfarist which then yields, similar to the monopoly case, the following form of (27): $y^s C_y^s = (k+1)C(y^s, V^s) \Rightarrow y^s = 0$ if $k=-1$ thereby contradicting the fact that $y^s > 0$ in this regime.

Proof of part (ii): *regime (b) occurs if and only if* $k = -1$.

In regime (b), $p_1^m > 0$, $p_2^m > 0, y^m = 0$; $p_1^s = 0$, $p_2^s > 0, y^s > 0$.The '⇒'part: If $k = -1$, then (28) implies $y^m C_y^m = 0$. As $C_y(0, V) > 0$ by (A3), we must have $y^m = 0$.This means, from the monopolist's point of view, both regimes (b) and (c) are possible. However, as just shown above, with $k = -1$, for the welfarist, regime (c) is not possible. Therefore, the only candidate for a plausible regime, when $k = -1$ is regime (b). We need to verify that $y^s > 0$ is consistent with regime (b). We do that as follows: By part (i) of the proof of proposition 2, at the welfarist equilibrium in regime (b) we have

$$[\frac{V^s}{C^s}C_v^s + 1] < 0 \Rightarrow V^s C_v^s + C^s < 0 \tag{29}$$

Now, from (27), $y^s C_y^s + V^s C_v^s = kC(y^s, V^s)$. Rewrite this by adding $C(.)$ on both sides,

$$y^s C_y^s + V^s C_v^s + C(y^s, V^s) = (k+1)C(y^s, V^s) \tag{30}$$

$$i.e.,\ V^s C_v^s + C(y^s, V^s) = (k+1)C(y^s, V^s) - y^s C_y^s \tag{31}$$

Then, using (29),

$$(k+1)C(y^s, V^s) - y^s C_y^s < 0 \tag{32}$$

$$i.e.,\ (k+1)C(y^s, V^s) < y^s C_y^s \tag{33}$$

When $k = -1$, (33) ⇒

$$y^s C_y^s > 0 \Rightarrow y^s > 0 \text{ as } C_y^s > 0 \tag{34}$$

Thus, if C is h.o.d.-1, then only regime (b) holds.

Proof of part (iii): *Only regime (a) can occur for all* k *satisfying* $k + 1 > 0$.

We know from parts (i)-(ii) that we can rule out regimes (b) and (c) *iff* $k+1 \neq 0$. So, if $k + 1 \neq 0$, only regime (a) can occur and we must have $p_1^s = 0$, $y^m > 0$, and

$y^s > 0$. Now, for the monopolist, $y^m C_y^m = (k+1)C^m$ (equation (28)) implies $y^m > 0 \Leftrightarrow k+1 > 0$, by (A.3). ■

Proof of Lemma 4 By definition, if $C(.)$ is homogeneous of arbitrary degree k, then $C(y,V) = V^k c(y/V)$ for some function $c(.)$. As $V^m C_V^m + C^m = 0$ at the monopoly equilibrium and $C_V = kV^{k-1}c(y/V) - yV^{k-2}c(y/V)$ then, using $z^m \equiv y^m/V^m$, $z^m c^/(z^m)/c(z^m) = k+1$. Likewise, as $V^s C_V^s + C^s < 0$ at the welfarist equilibrium, we can show $z^s c^/(z^s)/c(z^s) > k+1$. Now, by differentiation, $d\left[zC^/(z)/C(z)\right]/dz = \left[C^/(z)\right]^{-2}\left[zC(z)C^{//}(z) + C^/(z)\left\{C(z) - zC^/(z)\right\}\right]$. As $C(z^m) - zC^/(z^m) = 0$, then $d\left[z^m C^/(z^m)/C(z^m)\right]/dz < 0$ must hold. Likewise, $C(z^s) - zC^/(z^s) < 0$, so $d\left[z^s C^/(z^s)/C(z^s)\right]/dz < 0$ also. Therefore, if $zC^/(z)/C(z)$ is monotonic, it must be decreasing everywhere. ■