Problem set 6: Scattering from an infinite number of obstacles (Part 1)

1) Consider a lattice made up of an infinite number of *pairs* of non–identical delta function scatterers of strengths α_1 and α_2 (the medium has period a, and the two scatterers are spaced by d within the unit cell)

$$\frac{d^2\phi}{dx^2} + k_0^2\left[1 + \sum_{n=-\infty}^{\infty}\left[\alpha_1\delta(x - na + d/2) + \alpha_2\delta(x - na - d/2)\right]\right] \tag{1}$$

(a) Find the dispersion relation connecting the Bloch vector K to the frequency k_0.

(b) Calculate the group velocity $d\omega/dK$ (either analytically or numerically). Plot the group velocity within the first Brillouin zone. What happens to the group velocity at the zone boundary?

(c) Under what conditions (i.e. values of α_1, α_2 and d) can you eliminate reflection from a single pair of scatterers? What happens when this condition coincides with the Brillouin zone boundary?

2 a) Write your own program that will—using the transfer matrix $\boldsymbol{T}$—calculate K as a function of k_0 for any one dimensional periodic refractive index profile.

b) Suppose I am limited to materials that have a refractive index ranging from 1 to 2. Using whatever method you like find a periodic profile of fixed period a that has the largest peak value of $\mathrm{Im}[K]$ within the first band gap. [Such a periodic medium ought to be a very good reflector for even a small number of layers]