Problem set 5: Scattering from a single obstacle

As we showed in the lecture, the scattered field φ_s from a single object with refractive index $n^2(\boldsymbol{x}) = 1 + \chi(\boldsymbol{x})$ is to leading order

$$\varphi_s(\boldsymbol{x}) = -k_0^2 \int d^3\boldsymbol{x}' G(\boldsymbol{x} - \boldsymbol{x}')\chi(\boldsymbol{x}')\mathrm{e}^{\mathrm{i}\boldsymbol{k}\cdot\boldsymbol{x}}$$

where $\boldsymbol{k}$ is the wave vector of the incident wave.

(i) Derive the second order expression for the scattered field in terms of an integral over a product of two Green functions.

(ii) Derive a spatial dependence of the scatterer $\chi(\boldsymbol{x})$ such that the first order scattering is zero at a particular angle.

(iii) Using the Jupyter notebook provided with this lecture, numerically verify the scattering behavior predicted in part (ii).

(iv) Using the tensor Green function for electromagnetic waves given in the notes at the end of lecture 3, derive the vector version of the Rayleigh scattering formula given in the notes (within the Born approximation). What does this tell you about the polarization of light scattered from a small sphere.