Problem set 1: Complex analysis

(a) Which of the following functions satisfies the Cauchy–Riemann conditions:

- $f(z) = z + z^\star$
- $f(z) = az^3 + bz^2 + cz$
- $f(z) = (z^\star)^8$

(b) Plot the vector fields $\boldsymbol{V}$ and $\boldsymbol{W}$ equivalent to the three functions given above. Using these plots explain why each does or doesn't satisfy the Cauchy–Riemann conditions.

(c) Use Cauchy's integral formula to evaluate the following integrals

- $$I = \int_{-\infty}^{\infty} \frac{\mathrm{e}^{\mathrm{i}kx}}{(k - k_1 - 0.01\mathrm{i})} dk$$
- $$I = \int_{-\infty}^{\infty} \frac{1}{(x - \mathrm{i})(x - 2\mathrm{i})} dx$$
- $$I = \int_{-\infty}^{\infty} \frac{\sin^2(kx)}{x^2} dx$$

(d) Adapt the Jupyter notebook "Numerical demonstration of Cauchy's theorem" (or write your own code in the open source language of your choice) to numerically verify the results of the above three integrals, in whatever way you like.