Components of the neural signal underlying congenital nystagmus

Ozgur E. Akman · David S. Broomhead · Richard V. Abadi · Richard A. Clement

Abstract Congenital nystagmus is an involuntary bilateral horizontal oscillation of the eyes that develops soon after birth. In this study, the time constants of each of the components of the neural signal underlying congenital nystagmus were obtained by time series analysis and interpreted by comparison with those of the normal oculomotor system. In the neighbourhood of the fixation position, the system generating the neural signal is approximately linear with 3 degrees of freedom. The shortest time constant was in the range of 7–9 ms and corresponds to a normal saccadic burst signal. The other stable time constant was in the range of 22–70 ms and corresponds to the slide signal. The final time constant characterises the unidentified neural mechanism underlying the unstable drift component of the oscillation cycle and ranges between 31 and 32 ms across waveforms. The characterisation of this unstable time constant poses a challenge for the modelling of both the normal and abnormal oculomotor control system. We tentatively identify the unstable component with the eye position signal supplied to the superior colliculus in the normal eye movement system and explore some of the implications of this hypothesis.

Keywords Nystagmus · Muscle plant · Time constant · Time series analysis · Nonlinear dynamics

Introduction

From a systems perspective, the oculomotor system can be considered to consist of: (1) the vestibulo-ocular and optokinetic reflexes, which are responsible for maintaining gaze direction, and (2) the saccadic, pursuit and vergence mechanisms, which are responsible for gaze shifting (Carpenter 1988; Leigh and Zee 2006). These subsystems produce eye velocity commands which are all fed into a final common pathway that comprises a velocity to position converter, referred to as the neural integrator, together with the muscle plant. In general, when combining nonlinear systems, a sufficient condition to ensure that the overall system remains stable is that the response of each component to temporary disturbances decays exponentially fast.
(Slotine and Lohmiller 2001). In the context of the oculomotor system, the brainstem neural integrator can be considered stable because in the dark the eyes return to the straight ahead position with a long time constant around 25 s (Becker and Klein 1972). Although a leaky integrator is considered to be a clinical disorder, by this definition the system is in fact more stable as it returns faster to the equilibrium position of looking straight ahead following perturbation (the time constant is shorter than normal).

The advantage of defining overall stability in terms of stable and unstable time constants is that it allows us to quantitatively identify the class of oculomotor disorders that arise when not all perturbations decay exponentially fast. For example, the oculomotor disorder of congenital nystagmus is characterised by involuntary conjugate bilateral oscillations of the eyes, usually in a horizontal direction. Although the congenital nystagmus oscillation typically comprises different combinations of sawtooth and pendular waveforms, historically its defining characteristic has been identified as an exponentially increasing drift away from the point of fixation (Dell’Osso and Daroff 1972; Abadi and Dickinson 1986). The question that follows immediately from this characterisation is: what is the time constant of the exponential drift in nystagmus?

This question is challenging to answer because congenital nystagmus waveforms are usually very variable. Consequently, it is not obvious: (1) how many equilibrium states there are; (2) how many individual subsystems are contributing to the oscillations; and (3) how to distinguish the time constants of the various contributions. However, previous studies have shown that these issues can be addressed by applying geometric techniques of time series analysis that treat successive neural states as a trajectory in a state space and equilibrium states as fixed points of the trajectory (Abadi et al. 1997; Akman et al. 2006; Shelhamer 2007). These equilibria represent states of the oculomotor control system at which the activity of all neural populations is constant. Close to an equilibrium the eye velocity is low, corresponding physiologically to a foveation period (Akman et al. 2006). State space analysis of congenital nystagmus data based on the eye position signal has shown that even during steady fixation there can be more than one equilibrium state (Theodorou and Clement 2007). Where there are multiple equilibria these represent alternative potential foveation directions, one of which is aligned with the target and thus corresponds to fixation.

At these equilibria, three different components of the movement can be identified, each with an associated time constant (Abadi et al. 1997; Akman et al. 2006). In order to clarify the sources of these components, we have applied geometric analysis techniques to the neural command to the muscle plant, rather than the eye position signal, and then compared the time constants of this signal with those associated with a stable oculomotor system.

**Materials and methods**

**Experimental**

**Subjects**

Subject 1 was a 26-year-old female with binocular visual acuity of 6/6. Subject 2 was a 37-year-old male with a binocular visual acuity of 6/9 and a left head turn. Ethical approval was obtained from the Institute of Child Health/ Great Ormond Street Hospital Research Ethics Committee before the study commenced.

**Eye movement recording**

The stimulus consisted of a red LED that subtended an angle of 0.35° at the eye. The head of the subject was supported by a chin rest. Horizontal eye movements were recorded using an IRIS 6500 infrared limbal tracker (Skalar Medical, Delft). A micro 1401 interface (CED, Cambridge, UK) was used to digitise the analogue signal to 12 bit resolution at a rate of 1,000 samples per second. The system was linear over the range ±20°, with a resolution of 0.03°. The data were calibrated by recording saccades made to targets located 15° to the left and right and then manually adjusting the scale of the traces so that the foveating part of the waveform was located at ±15°.

**Computational**

**Data transformation**

The neural control signal was reconstructed by using Fourier techniques to remove the effects of the linear muscle plant (Van Opstal et al. 1985; Harris 1998). The muscle plant was described by two Voigt elements in series, with time constants of 0.012 and 0.260 s (Goldstein 1987). This model was used as it correctly predicts the torque found in normal muscles (Goldstein 1987) and the eye movements generated by neural signals (Krauzlis 2000).

A state space description of the behaviour of the oculomotor system was obtained from the neural control signal by applying the method of delays (Abadi et al. 1997; Akman et al. 2006; Shelhamer 2007). Given a time series of \( n \) measurements from the system, the method of delays involves treating \( p \) consecutive measurements each separated by a fixed number of sampling intervals as the
coordinates of a vector in a \( p \)-dimensional delay space. This yields a geometric reconstruction, or delay embedding, of the dynamical system underlying the observed oculomotor behaviour. Crucially, the embedding preserves key properties of the ambient dynamics, particularly the time constants of equilibria (Ott et al. 1994). Here, we used a value of 5 for \( p \). This was chosen to be larger than the dimensionality of the system, which previous studies had shown to be no more than 4 (Akman et al. 2006). The accuracy of the estimated time constants will reflect a balance between contamination by noise over short sampling intervals and nonlinearities over longer intervals. Previously we recorded data at a rate of 200 samples per second and used a separation of 1 sampling interval (Abadi et al. 1997). More recent data were recorded at 1,000 samples per second and for this data we have used a separation of 2 sampling intervals in the calculation.

**Local linear analysis**

Local linear analysis techniques exploit the constraint that close to an oculomotor equilibrium (or fixed point), the behaviour of a dynamical system is approximately linear. In practice, this means that any point \( x(n) \) in the state space of a discrete system \( x(n + 1) = F(x(n)) \) that is close to a fixed point \( x^* \) will be transformed into the point \( x(n + 1) \) according to the system of linear equations:

\[
x(n + 1) - x^* = M(x(n) - x^*).
\] (1)

The constant \( M_{ij} \) in the \( i \)th row and \( j \)th column of the \( p \times p \) matrix \( M \) specifies the rate at which the \( i \)th element of the difference vector \( x(n + 1) - x^* \) varies with changes in the \( j \)th element of the difference vector \( x(n) - x^* \). Any linear transformation defined by such an array of numbers will have \( p \) eigenvalues and eigenvectors associated with it. An eigenvector does not change its direction under the linear transformation but does change its length by a factor determined by the associated eigenvalue. The aim of the local linear analysis is to determine the eigenvalues and eigenvectors associated with \( x^* \), so that the behaviour of the neural network from which the signal was recorded can be interpreted as a weighted sum of the activities of different populations of neurons corresponding to each of the eigenvectors (Clement 1993).

The first step in applying the local linear analysis is to identify the fixed points, if any, underlying the generation of the measured neural signal. The terms of Eq. (1) can be rearranged to give a set of linear equations which can be solved for \( x^* \) (So et al. 1997). Each point in the delay space can give a different estimate of \( x^* \). Some of these estimates will be clearly spurious because they do not lie close to the diagonal of the delay space and can thus be discarded. A histogram of the frequency of the remaining estimates will peak at the actual locations of any fixed points (So et al. 1997; Theodorou and Clement 2007).

When there are isolated peaks in the histogram, one can then determine the number of eigenvalue/eigenvector pairs required to characterise the behaviour of the neural system close to a given fixed point \( x^* \) (this number, \( m \), is referred to as the dimensionality of \( x^* \)). Any point on a trajectory close to \( x^* \) can then be expressed as a sum of multiples of the \( m \) eigenvectors, with the eigenvalues describing the rate and direction of motion along each eigenvector. The \( m \) eigenvalue/eigenvector pairs can be identified using principal components analysis of the coordinates of the delay vectors because the \( m \) components of variation associated with the fixed point dynamics will generally be larger than the other \( p-m \) components. A more reliable method involves calculating the relative sizes of the components of variation for a range of spherical neighbourhoods of the fixed point. Only \( m \) components of variation will typically scale linearly with neighbourhood radius. These will be separated in size from the remaining \( p-m \) components, which tend to scale nonlinearly with sphere radius (Broomhead and Jones 1989).

The simplest way of testing for linear scaling is by applying singular value decomposition (SVD) to the trajectory matrix \( X \) formed by gathering successive delay vectors into rows (Broomhead and Jones 1989; Abadi et al. 1997; Akman et al. 2006). For a set of delay vectors obtained from zero-mean data, the sizes of the components of variation correspond to the eigenvalues of the covariance matrix \( XX^T \). SVD can be used to express \( X \) as a product of three matrices. The first matrix is made up of the eigenvectors of \( X^TX \) and the third comprises the eigenvectors of \( XX^T \). The second matrix has diagonal elements equal to the singular values of \( X \) and is zero everywhere else. The singular values are equal to the square roots of the required components of variation (Broomhead and Jones 1989).

The final stage of the local linear analysis involves projecting the delay vectors onto the first \( m \) eigenvectors of the covariance matrix and then calculating the eigenvalues and eigenvectors of the linear map governing the projected dynamics. This map was estimated by using a least squares techniques to fit a linear model to points in the neighbourhood of the fixed point (Healey et al. 1991; Abadi et al. 1997; Akman et al. 2006). Along an eigenvector of a sampled system with a fixed point at the origin, it holds that

\[
x(n + 1) = \mu x(n),
\] (2)

where \( \mu \) is the corresponding eigenvalue. For such a discrete system, the eigenvector is stable if \( \mu \) is less than one and unstable if \( \mu \) is greater than one. In the interval \( \tau \) between samples, the state of the underlying continuous system will have changed so that
\[ x((n+1)\tau) = \exp(\lambda \tau)x(n\tau), \]

where \( \lambda \) is the corresponding eigenvalue. It follows that the eigenvalues of the sampled and continuous systems are related by \( \lambda = \log(\mu)/\tau \). The eigenvector is stable in the continuous system if \( \lambda \) is negative and unstable if \( \lambda \) is positive. The corresponding time constant is equal to the reciprocal of the eigenvalue of the continuous system, \( 1/\lambda \).

To summarise, first the fixed points were identified, then the dimensionality of the fixed points was estimated, and finally, the eigenvalues and eigenvectors of the equilibrium were calculated from a least squares estimate of the linear map. Full details of our methods of local linear analysis can be found in previous publications (Abadi et al. 1997; Akman et al. 2006; Theodorou and Clement 2007).

**Results**

Identification of fixed points

Examples of the eye movements recorded from the subjects are illustrated in Figs. 1a and 2a. The movements of subject 1 are examples of jerk nystagmus, in which a steadily increasing drift away from the point of fixation alternates with a saccade in the opposite direction that returns the fovea to the point of fixation. Subject 2 shows pendular nystagmus with foveating saccades in which the saccadic phase brakes an extended quasi-sinusoidal slow phase (Dell’Osso and Daroff 1975). These two waveforms are representative of the oscillations commonly found amongst congenital nystagmus subjects (Abadi and Bjerre 2002). The corresponding neural control signals are plotted beneath the position traces and have been normalised to have a maximum value of unity.

The histograms of the neural data used to identify the location of the fixed points are shown in Figs. 1b and 2b. These plots reveal a single fixed point for subject 1 and a pair of fixed points for subject 2, indicating a single foveation position and a pair of alternative foveation positions, respectively. The height of a peak in the histogram reflects the amount of evidence for a fixed point, and so for subject 2 we concentrated on the fixed point associated with the largest peak to obtain the most numerically reliable results. The position of the fixed point estimate in each case was assessed by calculating the standard deviations of the peak in the corresponding histogram using a bootstrap method (Akman et al. 2006). This gave standard deviations of 0.02 (subject 1) and 0.03 (subject 2), which were less than the bin width of the histogram (0.05°). For both data sets, an offset was added to the neural signal so that the fixed point was located at the origin of the delay space, which was assumed to correspond to the position of the fixation target.

Number of neural components

The dimensionality of the neural system generating the behaviour at the fixed points was estimated by constructing a set of spherical neighbourhoods centred on the origin of the delay space. The singular values of the trajectory matrix within each sphere were computed and for each singular value its dependence on sphere radius was fitted with a straight line. Means and standard deviations of the slopes were estimated using the bootstrap method employed previously for analysing the fixed point histograms. The criterion used to decide that a singular value was scaling directly with neighbourhood radius was a mean slope between 0.5 and 1.5. Both subjects had three such singular values with mean slopes in the range 0.635–1.143. The remaining singular values had mean slopes in the range 0.019–0.368, indicating nonlinear scaling. The standard deviations of all the slopes were within the range 0.010–0.053.

Estimation of time constants

For both subjects, the trajectories in delay space close to the fixed point were found to have three invariant directions (eigenvectors), as can be seen in Figs. 1c and 2c. Along each of these, the point representing the state of the system either approaches or moves away from the fixed point depending on whether the corresponding rate of motion (eigenvalue) is less than or greater than 1, respectively. Accordingly, eigenvalues of the delay space dynamics that are less than 1 correspond to negative eigenvalues of the underlying continuous system from which the data have been sampled, and eigenvalues of the sampled system that are greater than 1 correspond to positive eigenvalues of the continuous system (cf. Eqs. (2) and (3) of the Methods section). The associated time constants are the absolute values of the reciprocals of these continuous eigenvalues and thus specify the rate of exponential motion close to the fixed point (Abadi et al. 1997). To facilitate interpretation, we do not take absolute values so that a negative time constant indicates convergent (stable) motion and similarly a positive time constant denotes divergent (unstable) motion. The three time constants calculated in this manner are given in Table 1. For both subjects, two of the time constants (negative) are associated with stable drift towards the fixed point and the third time constant (positive) is associated with unstable drift away from the fixed point.

**Discussion**

In this study, we set out to characterise the components of the neural signal underlying congenital nystagmus in terms of its linearised behaviour close to an oculomotor
equilibrium. Our analysis procedure has three parts: first the equilibrium states are identified, then the dimensionality of the system at the equilibrium states is determined and finally the time constants that govern the rates at which the state evolves along each dimension are estimated. We begin by assessing the validity of the estimated time constants and then interpret their functional significance in terms of neural populations.

Validity of estimated time constants

Previous applications of local linear analysis to nystagmus used the position signal rather than the reconstructed neural signal (Abadi et al. 1997; Akman et al. 2006; Theodorou and Clement 2007). The advantage of using the neural signal is that it facilitates the interpretation of the time constants of the local linear model by removing the neural dynamics associated with the muscle plant (Van Opstal et al. 1985). For both our subjects, the time constants found by analysis of the position signal were different to those calculated from the neural signal. To distinguish between the two sets of eigenvalues, numerical values for the position signal are given in italics. For subject 1, the calculated time constants were $-0.005$, $-0.027$ and $0.048$ s; for subject 2, the time constants were $-0.007$, $-0.037$ and $0.027$ s. The previous analyses of eye position of two subjects with jerk nystagmus gave time constants of $-0.003$, $-0.769$ and $0.049$ for the first subject and $-0.004$, $-0.769$ and $0.049$ for the second subject (Akman et al. 2006). Whilst the time constants associated with the unstable and most stable components are comparable in magnitude to those found in this study, those associated with the second stable decaying exponential are longer. This is due to the numerical instability involved in calculating the inverse of a near-zero eigenvalue (Akman et al. 2006). The inversion problem is reduced with the neural signal, which we had not previously used. The original data were not recorded at sufficiently high temporal resolution to be able to reconstruct the neural signal, but subject B of

Fig. 1 Local linear analysis of subject 1. a 5 s recording of horizontal eye position (upper panel) and the corresponding reconstructed neural signal (lower panel). The thickness and colour of the time series are proportional to eye velocity so that saccadic movements appear in red. b The first step in the analysis is to identify any fixed points (oculomotor equilibria) of the control system. The signals were embedded in a 5-dimensional delay space and a transformation applied that concentrated the data at the fixed points. Peaks in the histograms of the transformed data are evidence of fixed points: here there is a single peak indicating the existence of a unique equilibrium. c The next step in the analysis is to determine the dimensionality of the system in the neighbourhood of the fixed point. The singular values of the trajectory matrix for 5 neighbourhood sizes are plotted as points and straight lines have been fitted to these points. The 3 singular values that scale linearly with neighbourhood size have been plotted in red and the remaining 2 singular values have been plotted in grey. The dashed line shows the slope that would be expected with linear scaling. The linear scaling of 3 of the singular values implies that locally the system is 3-dimensional in this case.
that study was re-recorded by us subsequently. For this re-recorded data, the time constants computed from the neural signal were $-0.009$, $-0.041$ and $0.031$, giving a similar spectrum of values to those obtained from subjects 1 and 2 of this study (see Table 1 for the eigenvalue estimates which are presented as subject 3).

It should be noted that the precise eigenvalues computed from the neural signal will have some dependence on the plant model used. Current research into the orbital plant is directed towards providing a mechanically correct description that is accurate for the entire range of eye movements (Angelaki 2011). It has been found that a linear model with 4 time constants is required to accommodate the behaviour of the plant for both transient and extended periods of constant innervation (Sklavos et al. 2005) and that linear models do not capture the behaviour of eye muscles during passive movements (Quaia et al. 2009a, b, 2010). Here, we have analysed a restricted range of eye movements and this approach has the advantage that the effects of many of the nonlinearities of the orbital plant are minimised. Consequently, within this narrow range of movements a linear model of the plant is appropriate.

To test the dependency of our recovered time constants on the choice of plant model, we perturbed the viscosities of each of the Voigt elements by $\pm 10\%$, as well as using the original values. This gave nine different versions of the plant model with short time constants in the range $11–13$ ms and long time constants in the range $233–286$ ms, consistent with the ranges reported in the experimental literature (Sklavos et al. 2005). The average values of the recovered neural signal time constants with these plants were almost identical to the unperturbed values for the shorter stable time constant and differed by less than $3$ ms for the unstable time constant. For subjects 2 and 3, the longer stable time constant exhibited a deviation of around $2$ ms, whilst that of subject 1 showed the greatest deviation (9.3 ms), attributable to the numerical errors associated with inverting small eigenvalues mentioned previously. The details of these calculations are given in Tables S1–S5 of the Supplementary Material. We conclude that our findings are relatively independent of the choice of plant parameters.

### Interpretation of estimated time constants

Historically, one of the most important advances in quantifying the mechanisms involved in generating eye movements was the determination of a 25 s time constant for the neural integrator (Becker and Klein 1972). The neural integrator concept had previously been introduced by Robinson (1968) as necessary to compensate for the plant dynamics. Subsequently, the plant was more accurately modelled as a pair of Voigt elements in series that has three
time constants with values 0.012, 0.259 and 0.072 s (Goldstein 1987). The first two time constants describe the passive behaviour of the muscle plant. The third represents the rate at which active tension is transferred to the plant and must be compensated by an additional neural signal referred to as the slide. Optican and Miles (1985) argued that the neural signal controlling saccades must have a pulse, step and a slide component, each derived from a single velocity command (their step component was generated by passing the burst signal through the neural integrator). They showed that the sum of these 3 signals derived from the velocity command was equivalent to the neural signal controlling saccades.

Ideally, local linear analysis should recover the time constants of all three neural components of the saccadic signal, together with any additional time constants associated with the pulse generator mechanism (e.g. pause cells). However, the delay embedding process can result in some of the eigenvectors lying very close to each other so that the corresponding eigenvalues cannot be easily distinguished, especially in the presence of noise (Akman et al. 2006). In particular, the final common pathway structure of the oculomotor system means that the step and slide signals are likely to be combined by the embedding.

Our experimental findings with the nystagmus waveforms recover three components in all, two of which are exponentially decreasing and one of which is exponentially growing. The stable time constants recovered from the data are associated with the saccadic portions of the waveforms, as identified by their main sequence properties (Abadi et al. 1997). These are therefore expected to match the pulse and combined slide-step components. The smallest stable time constant of 7 ms can be attributed to the pulse, as this is the most rapid component of the neural signal. The longer stable time constant of 70 ms in pure jerk nystagmus is close to the value of 72 ms characteristic of the slide component in the normal saccadic system. We therefore interpret this time constant as reflecting the combined slide-step signal. This second time constant was found to be much shorter in the pendular with foveating saccades waveform (see Table 1). One possible explanation for this finding rests on the established role of the slide component, which is to maintain the eye in position following a saccadic movement by compensating for slowly relaxing viscoelasticities in the eye muscles. The effects of the slide signal will persist for over 100 ms, which is two-fifths of a cycle in 4 beats per second nystagmus. If the pendular waveform is produced by a greater involvement of the pursuit system than in jerk nystagmus, then shortening the slide time constant would reduce the opposition to the pursuit movement following the saccade. Since the slide component is known to originate in the adaptive mechanism of the cerebellum (Optican et al. 1986), it may be the case that the reduced slide time constant in pendular nystagmus is part of a strategy to improve the effectiveness of pursuit.

Table 1  Eigenvalues and time constants estimated from experimental data

<table>
<thead>
<tr>
<th>Subject</th>
<th>Eigenvalue</th>
<th>Time constant</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>1</td>
<td>0.752 ± 0.060</td>
<td>0.972 ± 0.047</td>
</tr>
<tr>
<td>2</td>
<td>0.752 ± 0.038</td>
<td>0.912 ± 0.037</td>
</tr>
<tr>
<td>3</td>
<td>0.817 ± 0.044</td>
<td>0.952 ± 0.032</td>
</tr>
</tbody>
</table>

The derivation of the estimates for subjects 1 and 2 are described in the “Results” section. The estimates for subject 3 are included for comparison with previous work; their derivation is described in the “Discussion”. In each case, the eigenvalues shown are for the sampled (discrete) system. The corresponding eigenvalues for the underlying continuous system are obtained by dividing the logarithm of the discrete eigenvalue by the sampling interval. Inverting the continuous eigenvalues yields the time constants, specified in seconds. Errors represent one standard deviation, calculated from bootstrap samples (the standard deviation estimates are only valid for the discrete eigenvalues, owing to the nonlinear transformations involved in calculating the time constants).

Possible implications of the unstable time constant for understanding the normal eye movement control system

Various models have been proposed to explain how congenital nystagmus might arise; the sawtooth nystagmus waveform can be generated by an abnormality of the saccadic system (Akman et al. 2005), pursuit system (Wang and Dell’Osso 2011) or neural integrator (Optican and Zee 1984; Barreiro et al. 2009), whilst the pendular waveform can be generated by an abnormality of the optokinetic (Huang et al. 2006) or pursuit systems (Harris 1995; Jacobs and Dell’Osso 2004). Although such models can produce plausible nystagmus waveforms, this does not imply that the local linear dynamics of the models matches that found experimentally (Akman et al. 2006). But a problem of greater importance than simply matching the local linear dynamics in individual cases is the need to explain why the instability in nystagmus has a time constant of 30 ms across diverse waveforms.
Congenital nystagmus typically involves successive alternations between an unstable fixation and a small saccade. So it is natural to consider the role of the superior colliculus in nystagmus, as this structure is directly involved in controlling the balance between fixation and redirection of gaze. Ideas about the mechanism of the superior colliculus have been developing rapidly over recent years (see Gandhi and Katnani 2011 for a review). In particular, it is now thought that steady fixation depends on a balance between the activity in the left and right rostral poles of the colliculus (Hafed et al. 2008) and that the spatio-temporal firing pattern in the superior colliculus specifies the trajectory of a saccadic eye movement (Goossens and Van Opstal 2006). Furthermore, the output of the superior colliculus is modulated by an eye position signal although its exact role is not yet clear (Groh 2011).

To interpret our results within this neurobiological framework, we make two assumptions. The first is that each half of the colliculus starts out with a sensory map containing a portion of its ipsilateral hemifield and that during development these representations diminish so that the two most rostral portions end up representing the same direction, except in subjects who go on to develop nystagmus where the representations are distinct. It follows from this assumption that there will be two equilibrium positions, one for the left and one for the right rostral pole directions. The second assumption is that the collicular motor map is updated after every saccade so that the trajectory specifications are correct for future movements. This would require the colliculus to be supplied with an internally generated eye position signal, which we associate with the 31 ms time constant. In subjects with nystagmus, a saccade will end on one of the directions associated with the rostral poles but not on the other. This imbalance will generate an offset in the eye position signal (Hafed et al. 2008) that leads to an exponentially increasing drift of the eye away from the target due to the subsequent discrepancy between the sensory and motor maps. This explanation of nystagmus is obviously speculative, but if the assumptions hold then one would expect that when the difference between the directions associated with the left and right rostral colliculi is small the deviation of the eye after a saccade should take longer to develop, generating the jerk with extended foveation waveform. We are in the process of testing this hypothesis in a larger group of subjects by comparing the type of waveform observed to the angular separation of the equilibrium positions.

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