1 Abstract

The need for increased reliability, low-cost design, and a shorter time to market, are all factors contributing to the increased use of Rapid Prototyping (RP). Many small to medium sized engineering companies own Computer Numerically Controlled (CNC) vertical machining centres, which, when equipped with a relatively low-cost hot-melt adhesive depositor unit, can be used to build simple RP models. Hot-melt adhesive can be used as a low-cost alternative to casting wax, and models are of sufficient quality to be used as cores for investment casting.

If model geometry has overhang features then support structures are normally required for this type of bottom-up horizontally sliced building approach. However, our work has focused on using the forth and fifth axes of the CNC machining centre to eliminate the need for support structures. Algorithms for locating suitable directions of build, model decomposition, and non-horizontal slicing are described. Model segmentation is demonstrated using novel surface analysis and geometric recognition techniques.

2 Keywords

Algorithms, Multi-Axis, Rapid Prototyping, Shape Decomposition.

3 Introduction

Rapid Prototyping (RP) is now an integral part of the design and testing phases of manufacture. Solid Freeform Fabrication from CAD-generated descriptions can be achieved with a variety of technologies such as selective laser sintering, stereo-lithography, 3-D printing, laminated object manufacturing and fused deposition modelling [1]. Although RP machine developers are constantly working on new techniques [2, 3] to improve the resolution of
models produced by their machines, most deposition systems work in a sim-
ilar manner - they employ a bottom-up building approach based on "slicing"
the artefact to be prototyped into a series of horizontal layers, which are then
built successively, deposited or attached, one upon another.

This paper examines the technique of multiple-direction layered man-
ufacturing of a model. The original motivation for the work comes from
a RP system under development at the University of Exeter which uses a
hot-melt adhesive depositor attached to a multi-axis Computer Numerically
Controlled (CNC) vertical machining centre. Models are built on the CNC
machining centre bed by moving it under the depositor mounted on the
vertical axis, whilst controlling the flow of a hot-melt adhesive from the de-
positor. Hot-melt adhesive [4, 5] is used as a low cost alternative to casting
wax for cores for investment casting. The adhesive has a short open-time,
good resistance to aging degradation, is non-soluble, and non-toxic.

Models are built in the CNC machining centre by depositing a series of
parallel layers, in a similar manner to conventional layered manufacturing
systems. The range of artefacts that can be built in this way is limited be-
cause models which exhibit overhang structures collapse before the deposited
adhesive has time to solidify. The overhang structure problem is also a limi-
tation of (for example) stereo-lithography systems, and is usually countered
by building supports, which must subsequently be removed during the fi-
nal hand finishing process. An alternative solution is offered by multi-axis
CNC machining centres because the bed on which the model is built may
be rotated about an additional axis (4-axis machines) or pair of axes (5-axis
machines). Utilising these additional degrees of freedom permits portions of
the model to be built from different directions, so that the model is always
self-supporting during construction. This eliminates, or at least reduces,
the need for supports. We dub this multi-axis additive rapid prototyping,
MAARP.

RP utilising CNC machining centres is attractive for a variety of reasons.
Many small and medium sized engineering businesses already own CNC ma-
chining centres and the cost of adding a hot-melt adhesive deposition unit
is small, far less than the cost of a full RP system. MAARP offers the pos-
sibility of building models that cannot be built by traditional RP methods,
in particular it is feasible to attach multiple depositors to a CNC machining
centre tool carousel permitting prototypes to be built from several materials.

In this paper we first discuss horizontal slicing in the context of hot-
melt adhesive deposition. This forms the basis for multi-axis construction of
many simple objects, which can be decomposed into a union of pieces, each
of which is bound by planar slices and is suitable for layered construction. To
facilitate this we describe a new method for locating planar slices in faceted
artefacts, such as those produced by 3D CAD packages. Finally we discuss the construction of more complicated objects. Our approach is illustrated with an example of a model which had been previously impossible to build using a single direction approach.

4 Horizontal Slicing

The majority of CAD systems produce a description of the modelled object in the STL format [6], which gives a triangulated boundary representation of the polyhedral model. Layered manufacturing systems slice the model into a series of layers of uniform thickness, which are successively deposited to build up the physical prototype. A straightforward slicing algorithm was proposed by Kirschman and Jara-Almonte [7], which is similar to our algorithm, and more recently McMains and Squin explored sweep plane slicing [8].

In our slicing algorithm, we assume that the machining centre co-ordinate system is used as reference, that each model stands on the machining centre bed with its lowest point at \( z = 0 \) and that all measurements are in mm. We also assume that all models are solids, not thin walled. Slicing is performed with a virtual plane positioned at \( nh \) where \( n \) is the current layer and \( h \) is the layer thickness. For each facet intersection with the virtual plane, a line segment is recorded. The resulting line segments are assembled into closed lists forming loops. An optimisation by removing co-linear segments is also applied to the loops.

The interior filling is performed using a variant of the scan-line fill algorithm [9] and the parity rule to determine the interior of the filled model. These methods are both fast and robust. The fill direction is alternated with each layer to produce a crosshatch effect giving the model additional physical strength. When building the model, filled slices are built one on top of another with the perimeter of each slice laid before the filled section. This yields a superior surface finish as illustrated in figure 1.

Horizontal building from a single direction gives good results for models which have vertically aligned perimeters or where the perimeter of each layer lies inside that of the previous layer. Models where each layer does not overhang the previous layer by more than 0.5 of a bead-width can also be built satisfactorily. The slumping problem is illustrated in figure 2 which shows a progressively increasing overhang encountered when building an arch. Close to the base, layers are almost vertically aligned and build quality is good, however higher layers overhang each other leading to a progressively worsening build quality.

We can see from these results that the perimeter is of key importance
Figure 1: The left hand photograph shows the surface finish of the perimeter-only build. The right hand photograph shows the same model built using the fill-only routines. By combining the two, a smooth surface is obtained for a solid model.

when evaluating the likelihood of successful building. If the perimeter can be built without falling, the interior can be filled, so we concentrate attention on building the perimeter line segments.

We can identify criteria which allow a model to be optimally built. Clearly, the model will be self-supporting if each line segment of adhesive is deposited vertically above an existing line segment of adhesive. It is also preferable if line segments are deposited horizontally.

Figure 2: The arch exhibits the classic problem of no underlying support for higher layers. The outer surface is smooth because each layer overlaps the previous. The inner surface has each layer slightly overhanging the previous layer and therefore the layer 'spills' over the edge. This is most evident at the inside top of the arch where the layers overhang by a large distance.
5 Multi-axis Building

The methods described for building from a single direction work well for shapes with no overhangs. Many overhangs can be removed by using the forth and fifth axes to rotate the model so that each individual overhanging line segment is vertically aligned above a previously built section of the model. Continuous rotation can only be achieved using machining centres with five axes of freedom. In many situations however, models do not require continuous rotation as whole sections of the model may be built from one direction without overhangs. Rotation of the model too quickly can cause weakness in the build, therefore it should only be used when necessary.

MAARP, using the forth and the fifth axes within the machining centre, provides the possibility for building from a range of directions for each part of the model, however, it is known that the best surface finish is obtained when the criteria derived for single direction building are met. These criteria can be stated as: The nozzle lies in the plane of the facet being constructed. The line segment being deposited is horizontal. Let be the downward unit normal representing the direction of the deposition nozzle, and let and be the position vectors of the line segment being deposited. If is the normal to the current facet, then these criteria may be expressed as:

The nozzle lies in the plane of the facet being constructed.
The line segment being deposited is horizontal

Let be the downward unit normal representing the direction of the deposition nozzle, and let and be the position vectors of the line segment being deposited. If is the normal to the current facet, then these criteria may be expressed as:

\[(p_1 - p_2) \times \hat{n} = \hat{f}\] (1)

The geometrical arrangement is illustrated in figure 3.

By solving equation 1, and by taking into account the limiting factors of the possible rotations of the forth and fifth axes, a single solution for the direction of build and position of the machining centre bed can be found. However, it is not necessary to constantly re-evaluate this equation to find the best solution for building as many consecutive layers may share a result due to their shared facets. The sections of the model that share a common direction for building are referred to as segments.

One of the aims of the MAARP algorithm is to build models by decomposing them into wedge segments that can be processed using the conventional horizontal slicing approach. There are however two fundamental issues that need to be considered when building models this way. Firstly, not all models
can be broken down into a series of simple wedges for slicing due to the geometry of their shape. Some models are too complex, or have no identifiable features that can be used to decompose the model into sections using the MAARP approach. Secondly, the MAARP algorithm identifies planar slices within the model which form the dividers for the wedge segments. Locating these slices using a simplistic method would be a computationally expensive process, MAARP therefore uses an alternative method.

6 Locating Planar Slices

The STL format only describes the triangularly faceted surface of the model and therefore if geometry or topology are required they must be recreated from the raw data. In order to decompose the model into segments for building using the MAARP approach, planar slices need to be located across the model. A typical STL file may contain between $10^4$ and $10^8$ facets [10]. A simplistic routine for locating planar slices across the model might test a series of vertices to determine if they lie in a plane. Any three vertices will of course form a plane, but once this plane has been defined, every other vertex from the model will need to be tested against the plane in order to locate the complete list of planar vertices. The operation must be repeated for every plane found from every trio of vertices. To check that each list is suitable to be used as a slice across the model, the edges which lie on the surface between
each of the vertices will also need to be identified. This list of edges will have to be assembled into a continuous loop in order to ensure that this was a suitable slice across the model. This algorithm, although comprehensive, would have a running time of at least $O(n^2)$, which is prohibitive for larger models.

As a result of the automated process in which CAD packages produce STL data, the facets created are often present in repeated symmetrical patterns. The lines of symmetry created by the edges of such facets are found in planes around the tessellated surface of the model as illustrated in figure 4. It is these planes that identify the breaks between one segment and another, and which are located by the MAARP algorithm.

![Figure 4: The base of a tessellated model showing lines of repetition and symmetry across the model.](image)

The MAARP algorithm uses a small triangle which we refer to as a micro-plane. A micro-plane is created from two surface edges with the third side either existing in free space between the two end vertices, or being another surface edge. To form micro-planes, each vertex on the model is identified and a list of surrounding edges are generated and stored with the vertex. Pairs of edges around the vertex are then used to create micro-planes. We are interested in any micro-planes created anywhere across the model, whose normal vectors are similar. This will indicate that they could be co-planar. A method for hashing vertices into a table for quick reference when reconstructing the geometry of an STL file was described by Vclav Skala and Martin
Kuchar [10] and we adapt this method for use with the normal vectors of the micro-planes.

By hashing on the normals of each of the micro-planes, we can determine that every micro-plane in a particular bin in the hash table must share a normal direction. By ordering the items in each bin by distance from the origin with respect to the direction of their normals, we produce clearly divided groups of micro-planes. These groups can be tested to see if all the items lie in a plane, and if the real edges extracted from the micro-planes form a closed loop around the surface. If both of these conditions are true then the group of micro-planes represent a slice that can be used to segment the model for multi-axis building.

The micro-plane and hashing method enables fast location of complete loops around the model surface without having to test all possible combinations of vertices. By ordering within each bin, the time taken to find a complete loop is reduced substantially to $O(n)$.

An additional increase in speed is gained by using the connected list of facets in conjunction with each loop discovered to locate new loops. By using each edge in a loop list and identifying the two facets which share the edge and which point away from the loop, a 'step-away' method is used to locate a series new vertices. Vertices which are planar are then used to calculate a normal which references another bin in the hash table. Figure 5 shows the results of the 'step-away' routine used on a pipe bend.

![Figure 5: The loop around the top of the first cylinder is located and the 'step-away' method is used to find the start of a new plane on a higher level. The extended facets are shown here in bold.](image)

By repeating the loop finding and 'step-away' operations a connected list
of loops is found.

By hashing micro-planes we generate a large amount of useful data about the surface and non-surface sections of the model. By examining the number of entries in a bin in the hash table we can determine possible candidates for good build directions. Figure 6 shows an example of the number of entries per bin for a typical hash table produced for a pipe bend.

![Figure 6: Example of results from hash table showing number of entries per bin for 10^4 bins. Peaks help identify possible directions for build.](image)

### 7 Complicated Regions

The hash table and micro-plane methods find planar loops around the model wherever there is a complete set of aligned edges in a plane. However, in areas with complex geometry, or where there are few lines of symmetry, the number of complete loops found will be less. In these situations the algorithm identifies the complete loops, finds as many closed segments as it can, and marks any non-connected loops that are not end-of-model loops as ’dangling’. Currently, any ’dangling’ loops that remain after the complete segment list is generated are simply sliced in the same way as identified segments during the slicing phase. The model is processed from the base upwards (as ordered by the connection of the segment list). Slicing is performed on each identified segment starting from the start loop in a direction normal to the loop, using regular slices. Slicing stops at the stop loop of the identified segment. In the case where the start loop of the next segment is ’dangling’ the slicing routine simply continues slicing in the direction of the normal to the start loop until it reaches another loop. Slicing of this segment is complete where there are no more free facets to slice between the first ’dangling’ start loop and any other loops.
Future development will concentrate on adding topology to the data collected so that multiple unknown segments may be identified and correctly positioned in the segment list.

8 Pipe Bend Example

The 90 pipe bend is a simple example of how the MAARP process can be used to build a model that we could not previously attempt under the single direction approach in the CNC machining centre. This model has been attempted several times using the single direction approach and has failed on each attempt due to the overhang structure.

Figure 7(a) shows the initial pipe bend represented by 1214 facets. Analysis of this model, construction of the micro-planes, building the hash table, and identifying and ordering the loops, takes approximately two seconds on a standard desktop PC [11]. Figure 7(b) shows the identified loops at the end of the process. The thicker lines represent the loops that identify the boundaries between segments. The first loop is at the bottom of the vertical cylinder. The second loop is the top of the same cylinder. The third is the first loop of the curved section of the bend. The bend is then traced upwards to the first loop of the horizontal cylinder and finally the closing loop at the end of that cylinder.

The remaining step is to slice each of the segments with respect to the normal identified by their starting loops. Slicing of the model takes less than two seconds on the same PC.
9 Conclusions

Unlike many other RP methods, MAARP attempts to build models using the forth and fifth axis of a CNC machining centre to overcome the problems of overhangs between one layer and the next. The MAARP algorithm identifies planes across models which can then be used to segment the model into smaller pieces. The decomposed segments of the model are then independently sliced with respect to their starting planes, thereby reducing the possibility of the overhang structure problem. The time taken to perform this type of search, sort and slice using simple methods is prohibitive.

10 Future Directions

MAARP is designed for CNC machining centres containing forth and fifth axes. Some machining centres only contain four axes and therefore cannot use the MAARP algorithms for building. However, an appropriate set of constraints would allow many models to be constructed within a four axis-machining centre. The constraints would naturally reduce the set of geometries that could be built.

Complex sections of the model are presently still impossible for the MAARP software to solve, as there are too few, or no planar slices, to locate segments between. Models with complex geometries may show several sections which cannot be enclosed by a known top and bottom loop, and which cannot therefore be constructed. Future work will use the loops topologically to produce a more complete order so that such models can be built.

CNC machining centres offer the possibility of constructing a model using more than one material by using additional deposition nozzles from the tool carousel. This method could be useful for creating different parts of the model from different materials, or for use as support structures in areas where model geometry prohibits the use of rotation to solve the overhang structure problem.

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References


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