Chapter 6

Determination of The Recorded Magnetisation
Transitions from Captured Isolated Pulses Using
Inverse Filtering

6.1. Introduction

The cross-track average magnetisation transition in magnetic media has been approximated by an arctangent function. This function has been widely used in the record (Williams and Comstock, 1971; Maller and Middleton, 1974) and replay (Miyata and Hartel, 1959; Speliotis and Morrison, 1966) theories and helped in linking the performance of a recording system with the characteristics of the head and the bulk properties of the medium. In addition, utilisation of the arctangent function in signal and noise analysis yielded close agreement with experiment (Middleton, 1966; Middleton and Wisely, 1978; Fung et al, 1997; Middleton et al, 1998) indicating the validity of the arctangent function in approximating the actual recorded magnetisation transition in magnetic media. Other mathematical functions have been used to represent the cross-track average magnetisation transition including the tanh and error function transitions. These were thought to be a closer approximation to the actual recorded transitions in low moment high coercivity media due to their sharp corners as found from analysis of the zig-zag recorded patterns in metallic disks (Arnoldussen and Tong, 1986). The tanh and error function transitions were used extensively in noise analyses and enjoyed good agreement with experimental observations (Barany and Bertram, 1987a, 1987b; Xing and Bertram, 1997; Aziz et al, 1999a).
From Chapter 3, the replay signal due to an arbitrary transition function can be written using the reciprocity principle as (3.31):

\[
e_x(x) = -\mu_0 v W \int_{y=\delta}^{\infty} \int dx \frac{dM_x(x)}{dx} \frac{H_x(x+y)}{I} dy dx
\]  

where \( \mu_0 \) is the permeability of free space, \( v \) the linear velocity of the disk, \( x = vt \) and \( W \) is the written track width. The inner integral in (6.1) is a correlation integral (Mallinson and Minuhin, 1984); taking the Fourier transform of (6.1) and integrating through the depth of the medium, assuming that the magnetisation does not vary with thickness, yields:

\[
E_x(k) = \mu_0 v WN \eta \delta \cdot D(k) \cdot [H_s(k)^*] \left[ e^{-|k|\delta} \right] \left[ \frac{1 - e^{-|k|\delta}}{|k|\delta} \right]
\]  

where \( E_x(k) \) is the Fourier transform of the longitudinal component of the isolated replay pulse, \( H_s(k)^* \) is the complex conjugate of the head surface field transform (gap loss function), \( D(k) \) is the Fourier transform of the normalised magnetisation derivative i.e. \( D(k) = \mathcal{F} \left\{ \frac{1}{M_r} \frac{dM_x(x)}{dx} \right\} \), \( k \) is the wavenumber (spatial frequency) given by \( k = 2\pi/\lambda \) where \( \lambda \) is the wavelength, and the last two terms in square brackets are the head spacing and thickness losses.

If the Fourier transform of a measured isolated replay pulse, \( S(k) \), was computed, then it is possible to determine the Fourier transform of the magnetisation derivative by dividing the replay signal transform by the loss terms in (6.2), i.e.:

\[
\tilde{D}(k) = \frac{S(k)}{L(k)}
\]  

where \( \tilde{D}(k) \) is the estimated value of the Fourier transform of the magnetisation derivative and \( L(k) \) contains the head losses, i.e.:

\[
L(k) = C \cdot [H_s(k)^*] \left[ e^{-|k|\delta} \right] \left[ \frac{1 - e^{-|k|\delta}}{|k|\delta} \right]
\]  

where \( C = \mu_0 v WN \eta \delta \). By inverse transforming the magnetisation derivative transform and integration, the recorded magnetisation function can be determined. In signal processing theory, the recorded transition can be thought of as the input to a filter whose impulse response is determined by the replay head field function. The
output replay signal is then simply the convolution of the input response and the impulse response of the filter. The mathematical operation for attempting to remove this convolution operation to restore the input waveform is termed deconvolution (Brigham, 1988) or, more generally, inverse filtering in the case where the gap-loss function contains imaginary components (asymmetrical head). This operation is manifested in equation (6.3).

In practice, there are several difficulties with this simple idea and equation (6.3) cannot be implemented directly. These problems centre around the effects of noise on the filtering process and the effects of errors in modelling the head field spectrum. Noise includes that due to the head and electronics in addition to media noise. If the noise is considered additive, then the Fourier transform of the measured signal, $S(k)$, includes the original signal plus the noise, i.e.:

$$S(k) = E_x(k) + N(k)$$

where $N(k)$ is the Fourier transform of the noise and $E_x(k)$ is defined in (6.2). Consequently, direct implementation of (6.3) leads to an enhancement of the noise at the gap-nulls and at short wavelengths where $E_x(k) \rightarrow 0$.

The inverse filtering technique has been used by a number of authors to determine the magnetisation transition function in longitudinal (Wells, 1985; Weismehl et al., 1988) and in perpendicular thin-film media (Vos et al., 1986). The differences in these publications were mainly concentrated in the noise removal process. In one instance, optimal filtering has been used to remove the unwanted noise from the replay pulse voltage with no clear indication of the optimisation process involved (Wells, 1988). Others have introduced a noise level in the head field spectrum to prevent the enhancement of the noise at short wavelengths and to reduce the noise in the calculated magnetisation spectrum (Vos et al., 1986). To minimise the errors in the modelling of the head field transform, micro-loop techniques have been used to determine the actual field from a thin-film head, and polynomial fitting algorithms were used to suppress the noise associated with the replay signal (Weismehl, 1988).

In this chapter, the Wiener optimal filter, based on least-squares minimisation, will be utilised as the means of removing the unwanted noise from the replay pulse spectrum.
After establishing the optimal filter formula, the technique will be used on a simulated isolated pulse with added random noise and the recovered transition will be examined. The effects of the errors in the modelling and in the parameters of the gap loss function on the estimated transition shape will be analysed. Experimental results of thin-film disk media will then be presented and observations will be made. Application of the inverse filter technique in separating the two transition noise modes will be illustrated. Finally, the effect of replay filter bandwidth on the filtered replay signal shape will be discussed.

The following analyses will consider only the longitudinal component of the magnetisation transition assuming negligible contribution from the vertical component due to the high vertical demagnetising fields. Furthermore, the recorded magnetisation will be assumed uniform throughout the thickness of the medium. Since interest is concentrated on the mathematical shape of the estimated magnetisation transition, all the constants of proportionality will be ignored and scaling to any vertical axis will therefore be arbitrary. Filtering of the noise will take place in the Fourier domain. Small letters will be used to describe the spatial domain signals and their capital counterparts will be used to indicate the Fourier transformation.

6.2. The Wiener Optimal Filter

In principle, in order for the inverse filtering technique to work, any nulls of the head field spectrum must be matched by nulls of the readback transform. However, since there is noise associated with the readback signal, the nulls in the replay transform will be washed out. One method of reducing the unwanted noise in the Fourier domain is by employing the Wiener optimal filter.

The spatial measured readback signal $s(\bar{x})$ including the noise can be written as:

$$s(\bar{x}) = e_\lambda(\bar{x}) + n(\bar{x})$$  \hspace{1cm} (6.5)

where $e_\lambda(\bar{x})$ is the ideal replay pulse as defined in (6.1) and $n(\bar{x})$ is the additive noise including the contribution of the head, electronics and medium noise. Our problem is
to find the optimal filter, $\Phi(k)$, which when applied to the measured signal spectrum
$S(k)$ (convolved with the measured signal in the space domain), and then divided by
head loss terms, produces $\tilde{D}(k)$ that is as close as an approximation as possible to the
uncorrupted magnetisation derivative transform $D(k)$, i.e.:

$$\tilde{D}(k) = \frac{S(k)}{L(k)} \cdot \Phi(k) \quad (6.6)$$

To decide how close $\tilde{D}$ is to $D$, least-squares minimisation was used for all values of $k$
(Press et al, 1995) in which the function $\gamma$ is minimised where:

$$\gamma = \int_{-\infty}^{\infty} [\tilde{D}(k) - D(k)]^2 \, dk$$

with

$$\tilde{D}(k) = \frac{[E_x(k) + N(k)]}{L(k)} \cdot \Phi(k) \quad \text{and} \quad D(k) = \frac{E_x(k)}{L(k)}$$

so that

$$\gamma = \int_{-\infty}^{\infty} \left[ \frac{[E_x(k) + N(k)]}{L(k)} \cdot \Phi(k) - \frac{E_x(k)}{L(k)} \right]^2 \, dk \quad (6.7)$$

Assuming that the signal $E_x(k)$ and noise $N(k)$ are uncorrelated (Wells, 1985; Press et
al, 1995), i.e. $\int S(k)N(k) \, dk = 0$, then equation (6.7) (after manipulation) reduces to:

$$\gamma = \int_{-\infty}^{\infty} \frac{[E_x(k)]^2 |\Phi(k) - 1|^2 + |N(k)|^2 |\Phi(k)|^2}{|L(k)|^2} \, dk \quad (6.8)$$

Minimising equation (6.8) with respect to $\Phi(k)$ at every value of $k$ leads to:

$$\Phi(k) = \frac{|E_x(k)|^2}{|E_x(k)|^2 + |N(k)|^2} \quad (6.9)$$

Equation (6.9) is the optimal filter formula. An interesting observation is that when
dividing the numerator and denominator in (6.9) by $|N(k)|^2$, the optimal filter response
can be written in terms of the narrowband signal-to-noise ratio, SNR$_b$, i.e.:

$$\Phi(k) = \frac{1}{1 + 1/|\text{SNR}_b(k)|^2}$$

where $N(k)$ includes the system and medium noise. Hence the filter response changes
according to the narrowband SNR of the captured waveform and is unity when the
SNR is sufficiently high. Furthermore, if the narrowband SNR of the medium noise is
known by theory or measurement as a function of \( k \), then the optimal filter response can be determined to minimise that noise in the absence of system noise.

Now \( |E_x(k)|^2 \) and \( |N(k)|^2 \) cannot be found separately in (6.9). However, they can be evaluated approximately from the PSD (Power Spectral Density) of the measured replay pulse. Evaluating the Fourier transform of (6.5) and squaring ignoring the first order term gives the measured signal PSD approximately as:

\[
|E_x(k)|^2 \approx |S(k)|^2 + |N(k)|^2 
\]  

and hence the ideal replay pulse spectrum can be approximated by:

\[
|E_x(k)|^2 \approx |S(k)|^2 - |N(k)|^2 
\]

Substituting back in (6.9) yields the Wiener filter response as:

\[
\Phi(k) = \frac{|S(k)|^2 - |N(k)|^2}{|S(k)|^2} 
\]  

where \( S(k) \) is the Fourier transform of the captured replay pulse. The only unknown in the filter response of (6.11) is the value of the noise spectrum \( N(k) \). Since \( S(k) = E_x(k) + N(k) \), then \( N(k) \) can be defined as the value of the measured signal spectrum at the first gap null where the ideal signal level \( E_x(k) \) is zero and \( S(k) \rightarrow N(k) \). The first gap null can easily be determined from available gap loss expressions (Chapter 2). The noise floor level can also be defined, with less certainty, at short wavelengths where \( E_x(k) \rightarrow 0 \) and the measured signal spectrum approaches the noise floor.

From (6.11), it can be seen that the Wiener filter response \( \Phi(k) \rightarrow 1 \) when the measured replay pulse spectrum is well above the noise level and \( \Phi(k) \rightarrow 0 \) where the signal spectrum is at the noise floor and the noise is dominant. Thus, the inverse filter algorithm can be described as:

\[
\tilde{D}(k) = \frac{S(k)}{L(k)} \Phi(k) = \begin{cases} 
\frac{S(k)}{L(k)} & \text{if } |S(k)| > |N(k)| \\
0 & \text{if } |S(k)| \leq |N(k)| \text{ or } H_x(k) = 0 
\end{cases} 
\]  

(6.12)
As a result of the least-squares minimisation, the quality of the results obtained by optimal filtering differs from the true optimum by an amount that is second order in the precision to which the optimal filter is determined (Press et al, 1995).

The magnetisation distribution estimate $\tilde{M}(x)$ can be evaluated from $\tilde{D}(k)$ by a number of methods. The direct method involves computing the inverse Fourier transform of (6.12) thus obtaining the magnetisation gradient where direct integration (through interpolation) can be performed to obtain $\tilde{M}(x)$. Direct integration is, however, rather slow and its accuracy is limited by the interpolation scheme used. A quick and convenient method involves multiplying $\tilde{D}(k)$ with the Fourier transform of the unit Signum function (i.e. equivalent to a convolution of the magnetisation derivative with the unit Signum in the spatial domain) and then evaluating the inverse Fourier transform of the product (Wells, 1985). Application of the Wiener filter and the various techniques for computing the magnetisation transition will be demonstrated next in an illustrative example.

### 6.3. Illustrative Example

Figure 6.1 illustrates an ideal replay pulse from an arctangent transition correlated with a Karlqvist type replay head calculated for a thin medium using (3.24). Random noise with Gaussian distribution was added to simulate the additive noise associated with an actual measured pulse. The main stages of the filtering process will be illustrated graphically using the computed replay pulse in the spatial and Fourier domains. The Karlqvist sinc gap-loss function in equation (2.28) (corrected for the first gap null) will be used in this example. Since interest is mainly concentrated in the shape of the mathematical functions involved, any proportionality constants will be omitted. Scaling of any vertical axis is therefore arbitrary.
The next stage is to evaluate the Fourier transform of the replay pulse. The replay pulse spectrum was computed using a Fast Fourier Transform (FFT) algorithm. A magnified section of the calculated magnitudes ideal and noisy replay pulses spectra, $|E_x(k)|$ and $|S(k)|$, along with the head field spectrum, $|H_x(k)|$ (gap loss times the spacing loss) are demonstrated in Figure 6.2.
The spatial replay pulse and head field functions were sampled using 512 points, i.e. \( N = 512 \), with sampling interval \( T = 0.01 \mu s \) (= 0.01\( \mu m \) for unit velocity). This ensured that there is no aliasing in the calculated transforms of Figure 6.2. The frequency axis of the measured spectra was evaluated according to \( f = n / NT \), where \( n \) is the sample number. The wavenumber, \( k \), is then calculated using \( k = 2\pi / \lambda \) where \( \lambda = v / f \) is the wavelength.

From Figure 6.2, it can be seen that the noise is mainly concentrated in the floor of the spectrum. In order to calculate the required optimal Wiener filter response, the noise floor level in the spectrum must be specified. The noise level was determined to be equal to the average of noisy signal level \( |S(k)| \) at and beyond the first gap null of the gap loss function as shown by the straight line in Figure 6.2. Having defined the noise level, the ideal replay response, \( E_s(k) \), can then be approximated by calculating:

\[
|E_s(k)|^2 = |S(k)|^2 - |N(k)|^2
\]

and the Wiener filter response is then computed from (6.11). Figure 6.3 compares the noisy replay pulse transform with the filtered signal response as modified by multiplying by the Wiener filter response, \( \Phi(k) \).

![Figure 6.3 Comparison between the filtered and unfiltered replay pulse spectra. The Wiener optimal filter response is shown on the secondary y-axis.](image-url)
It can be seen from Figure 6.3 that the filter response approaches unity when the signal is above the noise level, and becomes zero at and below the specified noise level. Hence, the cut-off wavelength (or wavenumber) and the roll-off rate with $k$ of the Wiener filter are adjusted according to the specified noise level in the signal. The Wiener optimal filter causes a gradual decrease of the filtered signal response with wavelength thus minimising the effect of truncation in the Fourier domain when transforming back to the space domain (Brigham, 1988) (reducing the ripple in the restored space domain waveforms). The filter also attenuates the short wavelength noise components (hence including transition width noise in an actual replay pulse) thus smoothing the output replay pulse. A magnified section of the filtered replay pulse is shown in Figure 6.4 evaluated by computing the Inverse Fast Fourier Transform (IFFT) of the product $|S(k)|\Phi(k)$ and is compared with the unfiltered signal.

![Figure 6.4](image)

Figure 6.4 Effect of filtering out the high frequency components of the simulated noisy signal.

Clearly, the noise has been reduced significantly by the application of the optimal filter. Moreover, the pulse width and height were successfully preserved after filtering in this case. However, care must be taken when setting the noise level in the filter response to account for the effect of the cut-off wavelength of the filter on the
recovered magnetisation transition length and hence on the characteristics of the filtered replay pulse as will be discussed later.

After filtering out the unwanted noise in the Fourier domain, the modified replay spectrum is then divided by the head field transform (gap loss times the spacing loss terms). This generates the magnetisation gradient transform. Computing the IFFT produces the spatial magnetisation gradient distribution. The magnetisation transition derivative is then integrated to yield the magnetisation transitions. Integration was performed by convolving the transition derivative with the unit Signum function which is defined by:

$$\text{sgn}(x) = \begin{cases} 
1 & x \geq 0 \\
-1 & x < 0 
\end{cases} \quad (6.13)$$

The convolution process was carried out in the Fourier domain by multiplying the magnetisation derivative transform with the Fourier transform of the Signum function. The computations were performed using an FFT algorithm by restructuring of the sampled spatial waveforms (Brigham, 1988). Convolution by restructuring of the sampled data points for both spatial functions involved the following steps:

1. Shifting both the spatial magnetisation gradient and the unit Signum function to the origin and sampling:

$$\tilde{dM}_n(x)/dx = d\tilde{M}_n(nT)/dx \quad n = 1, 2, \ldots, K$$

$$\text{sgn}(x) = \text{sgn}(nT) \quad n = 1, 2, \ldots, L$$

where $K = 512$ and $L = 1024$ respectively. The sample number, $L$, for the unit Signum function was chosen to be greater than that of the magnetisation derivative to maintain the value of the magnetisation derivative integral within the spatial extent of the replay pulse.

2. Choose a period $N$ for both spatial functions that satisfy the relationships:

$$N \geq K + L - 1 \quad \text{and} \quad N = 2^\gamma \quad \text{where} \ \gamma \text{is an integer.} \ \text{Hence,} \ N \text{was selected to be 2048 samples thus satisfying the above conditions.}$$

3. Augment with zeros the sampled functions of step 1 to prevent the wrap around effect of the convolution process:

$$\tilde{dM}_n(nT)/dx = 0 \quad n = K+1, K+2, \ldots, N$$

$$\text{sgn}(nT) = 0 \quad n = L+1, L+2, \ldots, N$$
4. Compute the FFT of the restructured sampled functions of step 3. Care must be taken when calculating the FFT of a non-causal function (defined for positive and negative time) such as the unit Signum function. The spatial waveform must be restructured in a form that implies periodicity and the value at the discontinuity must be set to 0 as shown in Figure 6.5(a) (a non-zero constant value at the discontinuity in this case produces a non-zero real part of the computed spectrum of the unit Signum function which is not acceptable since the Signum function is odd and it Fourier transform is purely imaginary and odd (Brigham, 1988)).

Figure 6.5 Sampled waveforms required for the convolution process: (a) unit Signum function and (b) the computed magnetisation derivative.
5. Evaluate the product of the computed transforms, i.e. \( \tilde{M}(k) = \tilde{D}(k) \cdot \text{sgn}(k) \).

6. Finally, compute the IFFT of \( \tilde{M}(k) \) and take the first \( K \) samples to represent the restored magnetisation transition, \( \tilde{M}(x) \).

The magnetisation derivative and the results of integration are demonstrated in Figure 6.6. The transition width of the computed transition was calculated from the slope at the centre of the transition and was found to be 0.11\( \mu \text{m} \) as compared to the actual value set for the computation of the simulated noisy pulse (0.1\( \mu \text{m} \)). It can be seen that the computed transition length differs by only 10% from the actual value indicating the applicability of the inverse filter technique in restoring the shape and length of recorded magnetisation transitions. The rms error between the actual arctangent and computed transitions was found to be less than 5% as calculated using (2.16) (Wells, 1985).

![Figure 6.6 Computed magnetisation compared with an ideal arctangent transition. Also shown is the magnetisation transition derivative before integration.](image-url)
6.4. Sensitivity of Inverse Filtering to Modelling Errors

As can be seen from Figure 6.6, there is close agreement between the magnetisation transition obtained by inverse filtering and the ideal arctangent transition. This is, in a way, is expected since the head field function and the head-to-medium parameters are known beforehand. However, in the case of a captured replay pulse, the degree with which the filtering process is effective depends upon how closely the simulated sensitivity function transform conforms to the actual head field and the accuracy of the head-to-medium parameters used in the modelling. Special attention is given to the head-to-medium separation error since its value is not readily available and only approximate values are given by the head manufacturer.

Modelling errors involve the effect of the parameters that are associated with the head field sensitivity function; namely the flying height, the gap length and the gap loss function on the computed magnetisation transition. The effects of varying the head flying height, \( d \), and the gap length, \( g \), were investigated and the findings are identical to the quantitative observations made by Wells (1985) from simulated pulses and will not be repeated here but, nevertheless, will be indicated. Error in the value of the gap length in the gap loss function and hence in the position of the gap null, will affect the position of the cut-off wavelength of the Wiener filter (Figure 6.3); introducing either ripple in the restored magnetisation waveform due to truncation when the gap length is larger than the actual length, or noise level dependent distortion when the applied gap length is less than the actual length. Record and replay theories predict that the transition length is a function of the flying height; this is truly the case when varying \( d \) in the simulated calculations. Underestimating the value of the flying height in the gap loss function causes a reduction in the slope of the computed magnetisation transition and hence an increase in the transition length. Overestimating the value of \( d \), on the other hand, causes ripple at the corners of the computed magnetisation transition. As a result, it was concluded that error in the value of the flying height has the most pronounced effect on the computed magnetisation transition and in particular on the calculated transition lengths. Therefore, accurate estimation of ‘\( d \)’ is an essential requirement for obtaining accurate results when using inverse filtering (Wells, 1985).
By examining Figures 6.2 and 6.3, it can be seen that using a more accurate expression for the gap loss function is inconsequential or will have little effect as long as the formula gives the correct location of the first gap null for the semi-infinite gapped head structure. This is because beyond the first gap null (where the available gap loss expressions differ), the Wiener filter response is usually zero. Consequently, when the different gap loss expressions of Fan (1961), Lindholm (1975), Ruigrok (1991) and Karlqvist were tested in the gap region, almost identical results of the computed magnetisation transitions were obtained.

6.5. Experimental Results

All the measurements were conducted on a Guzik S-1701MP spinstand with read and write analyser. Four 3.5inch diameter metallic disks were used in the experiments and their properties are listed in Table 6.1. The disks were rotated on an air bearing at 4577rpm (revolutions per minute). Two thin-film heads were used for recording and replay to minimise the error in the flying height between record and replay. The thin-film heads were flying above the disk in a slider assembly. The thin-film heads details are summarised in Table 6.2. Disks 1 and 3 are approximately 30nm thick with an overcoat thickness of 15nm. The thickness of the magnetic layer in disk 2 ≈ 30nm with an overcoat thickness of 12nm. Disk 4 is ≈ 40nm thick with an overcoat thickness of 20nm. More detailed description of the experimental apparatus and the record properties (record current, overwrite and replay amplifier bandwidth) are given in Chapter 7.

<table>
<thead>
<tr>
<th>Disk</th>
<th>$H_c$ (kA/m)</th>
<th>$M_r \delta$ (A)</th>
<th>$S^*$</th>
<th>$I$ (mA)</th>
<th>$v$ (m/s)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>Head 1</td>
<td>Head 2</td>
</tr>
<tr>
<td>1</td>
<td>159.155</td>
<td>0.021</td>
<td>0.92</td>
<td>7</td>
<td>7</td>
</tr>
<tr>
<td>2</td>
<td>159.155</td>
<td>0.01</td>
<td>0.92</td>
<td>7</td>
<td>5</td>
</tr>
<tr>
<td>3</td>
<td>159.155</td>
<td>0.019</td>
<td>0.917</td>
<td>7</td>
<td>-</td>
</tr>
<tr>
<td>4</td>
<td>95.5</td>
<td>0.052</td>
<td>0.86</td>
<td>6</td>
<td>-</td>
</tr>
</tbody>
</table>

Table 6.1 Experimental disks parameters.
Table 6.2 Thin-film heads parameters.

<table>
<thead>
<tr>
<th>Head</th>
<th>g (µm)</th>
<th>d (µm) @ 4577rpm</th>
<th>P (µm)</th>
<th>W (µm)</th>
<th>n (turns)</th>
<th>Throat height (µm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.22</td>
<td>~0.031</td>
<td>3.5 (symmetrical)</td>
<td>5</td>
<td>42</td>
<td>2.25</td>
</tr>
<tr>
<td>2</td>
<td>0.18</td>
<td>~0.026</td>
<td>3.5 (symmetrical)</td>
<td>4</td>
<td>42</td>
<td>2</td>
</tr>
</tbody>
</table>

The Tektronix 2440 digital storage scope was used to capture and digitise the isolated replay pulses with sampling rates up to 500MSamples/s within 200MHz bandwidth with an 8-bit A/D converter (vertical resolution of 256 samples). The track was DC erased prior to writing of an ‘all ones’ NRZI pattern. The square wave record frequency was chosen to be small (0.5MHz) for isolated pulse measurements to eliminate overlap and non-linear effects between neighbouring transitions. In this experiment, 256 isolated replay pulses were captured per head and disk combination from a single location on the written track to average out the background noise and to reduce the quantisation noise in the measured waveforms. Each captured waveform contains 1024 points. Although replay was made using thin-film heads, the experimental waveforms were captured within the limits of the gap region where the transition exists to obtain the highest resolution possible for the computation of the magnetisation transitions. Therefore, the negative undershoots at the pole corners of the thin-film replay heads will not appear in the measured replay signal plots that follow.

6.5.1. Removal of Unwanted Noise from the Captured Pulses

Noise in the measured replay pulses includes the head, electronics and medium noise. The head and electronics noise is known to be stationary with Gaussian distribution (Tang, 1985; Carley and Moon, 1987) and hence can be minimised by capturing a number of isolated replay pulses at a given position on the track and averaging. Due to the presence of time base jitter in the measured replay pulses (due to system or medium noise), averaging alone will yield incorrect results, and alignment of all the measured events (replay pulses) is therefore necessary. A wide range of experimental and signal processing techniques has been used to compensate for the time base jitter.
One of the available techniques is the centre of energy alignment; where the centre of energy of each captured pulse is computed and the corresponding waveform is then displaced to coincide with this position prior to averaging. The centre of energy is defined as the point where one half of the energy of a waveform is on either side of it. Provided that the signal-to-noise ratio of the measured signal is sufficiently high such that the noise energy is negligible compared to the signal energy, it has been shown that the centre of energy of a waveform is insensitive to time base jitter and hence alignment according to the centre of energy can greatly reduce the effect of time base jitter in a measured signal (Armstrong et al, 1991). In the experimental results presented in this section, the captured replay pulses are aligned according to their centre of energy before computing the average replay pulse voltage.

Figure 6.7(a) illustrates an isolated replay pulse after alignment and averaging of individual pulses for head 1 and disk 3. Also shown in the figure, is the average noise voltage as calculated by subtracting the averaged signal from each captured waveform and then averaging the individual noise voltages. Figure 6.7(b) illustrates the average noise voltages computed with and without alignment of individual events. Averaging alone without alignment produces a noise voltage that does not show the expected character of the residual non-stationary transition noise component obtained when using alignment. This indeed is a proof of the validity of the centre of energy alignment technique in minimising the time base jitter in captured waveforms.
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(a) Graph showing average voltage (mV) vs. distance (x (µm)) for signal and noise.

(b) Graph showing average noise voltage (Normalised) vs. distance (x (µm)) for no alignment and with alignment.
The noise voltage waveform obtained in Figure 6.7 represents the non-stationary transition noise voltage after removing the stationary noise components (head and electronic noise) and agrees with the theoretical noise voltage plots illustrated in Figure 5.3; in particular the distinctive shape of the jitter noise voltage. The noise voltage autocorrelation, $R_n$, in Figure 6.7(c) was calculated following the procedure outlined by Tang (1985) to find:

$$ R_n(x_1, x_2) = \frac{1}{m} \sum_{i=1}^{m} e_i^n(x_1) e_i^n(x_2) $$

where $e^n(x)$ is the measured noise voltage, $x_1$ and $x_2$ are two spatial points in a noise voltage waveform and the average is taken over all the noise voltages in the 256 captured waveforms (i.e. $m = 256$) for head 1 and disk 3. The shape of the noise autocorrelation function agrees with that obtained by Tang (1985) and Xing and Bertram (1997) from captured pulses (thus verifying the centre of energy alignment.
procedure) and indicates the non-stationary nature of transition noise. The diagonals along $x_1 = x_2$ give the noise voltage variance or the square of the noise voltage in Figure 6.7(a) (inverse Fourier transform of noise power spectral density). Pulse amplitude variation (transition width noise) is also present but seems to contribute negligibly to the total transition noise voltage. Transition width noise contribution is implied by the asymmetry of the measured noise voltage about the x-axis in addition to the slight displacement of the zero crossing of the noise voltage from the position where the peak in the replay signal occurs (Figure 5.3). The same characteristics of the noise voltage illustrated in Figure 6.7 have been observed on all the disks in this experiment.

The effect of transition jitter noise is inconsequential to inverse filtering as it only affects the position of the written transitions and not their shape or width. Transition width noise, on the other hand is expected to influence the width of the recorded transitions. Since transition width noise is a short wavelength phenomenon (Figure 5.5), then its effect would be attenuated naturally by the action of the Wiener optimal filter (Figure 6.3). By raising the chosen noise level of the optimal filter, the cut-off wavenumber of the filter response is then reduced and further attenuation of the short wavelength components of the isolated replay spectrum will occur.

However, reducing the cut-off wavenumber of the optimal filter response would enhance the effect of truncation in the Fourier domain and produce unacceptable ripple in the restored magnetisation and magnetisation derivative waveforms in the spatial domain. This limits the advantage of the Wiener filter of attenuating the short wavelength components of the replay pulse transform and hence on reducing transition width noise. In general, studies and experimental observations have shown that at small write frequencies (low packing densities), the effect of transition width noise is negligible compared to the jitter noise and its effect becomes more important only at high packing densities (Moon et al., 1988; Yuan and Bertram, 1992; Slutsky and Bertram, 1994; Xing and Bertram, 1997). This is also indicated from Figure 6.7 where jitter noise voltage appears to be dominant. Therefore, the effect of transition width noise will be ignored in the calculations of the magnetisation transition shape.
An alternative method to alignment and averaging for removal of signal and quantisation noise involves using a high order moving average filter. This method works by least-squares fitting a polynomial of specified order within a limited window size to a single replay pulse waveform to smooth out the noise and at the same time preserve the pulse width and height. By controlling the polynomial order and window size, different levels of smoothing can be achieved. As a result, the FFT of the smoothed waveform can be used directly in the inverse filter formula (6.3) without the need for further filtering except for a rectangular window to suppress the noise enhancement at the gap nulls and at short wavelengths. High-speed numerical algorithms are available for the generation of the polynomial coefficients (Savitzky-Golay smoothing filter (Press et al., 1995)). Hence, this method does not require a large number of isolated replay pulses for averaging thus avoiding the need for alignment and at the same time minimising the filtering in the Fourier domain to produce the magnetisation derivative waveform. The disadvantage of this method, however, lies in the choice of the window size and polynomial order to achieve the best inverse filter results. Unless iterative methods are used, the process of determining the best parameters is done by eye. This can be laborious in some instances. Another disadvantage is that only one isolated replay pulse is used for this filtering process; this pulse may not represent the average magnetisation transition shape on that particular location on the track. In addition, an anomaly on the particular instant that the pulse was captured could happen yielding a spurious transition shape and width.

6.5.2. Computed Magnetisation Transitions

For the convolution process, the magnetisation derivative waveform had 1024 samples (K = 1024) and the unit Signum function was sampled using 2048 points (L = 2048) to avoid the periodicity effect of the FFT algorithm, and hence the sampling period was chosen to be \( N = 4096 \) points. The computed Fourier transform of the average replay pulse was divided by the Fourier transform of the head field function of a semi-infinite gapped head (product of the gap loss, spacing loss and thickness loss terms). The gap loss function of Lindholm (1975) (equation 2.27) was used in the calculations. The
value of the flying height incorporated the thickness of the carbon overcoating for each disk.

The computed magnetisation transition for head 1 and disk 3 is illustrated in Figure 6.8(a). Also shown in the figure, is the estimated transition width parameter, $\tilde{\alpha}$, evaluated from the slope of the computed transition at the zero-crossing (transition centre) according to:

$$\frac{d\tilde{M}_x}{dx} = \frac{2M_o}{\pi\tilde{\alpha}}$$

(6.14)

where $M_o$ is the peak value of the computed magnetisation transition. The transition centre position was defined by the peak position of the isolated replay pulse shown in Figure 6.7.

In Figure 6.8(b), the computed magnetisation transition is compared with the least-squares fitted arctangent and tanh transitions defined in Table 3.1. The constant of proportionality and the transition width parameter were allowed to vary in the least-squares fitting algorithm. The best-fit transition width parameters for the arctangent and tanh transition are given in the legend of Figure 6.8(b). It can be seen that the calculated transition is not symmetrical about the zero crossing; it resembles closely the arctangent transition on the right-hand-side of the zero-crossing and the approaches the tanh transition on the left-hand-side of the zero-crossing. The asymmetry in the determined transition shape is in agreement with the calculations of Wells (1985) on thin-film disks.
Figure 6.8 (a) Computed magnetisation transition for head 1 and disk 3 where the transition length is determined from the slope of the transition at the zero crossing, (b) computed magnetisation from the same head and disk compared with the least-squares fit of the tanh and arctangent transition with the best fit ‘a’ parameters.

\[ M(x) = \frac{1}{a} \tanh \left( \frac{x}{a} \right) \]

\[ \bar{a} = 0.089 \mu m \]
The alignment and averaging procedures for head 1 and disk 3 were found to improve the signal-to-noise ratio of the captured isolated pulses dramatically such that inverse filtering can be performed in the absence of the Wiener optimal filter. In this case, the Wiener filter response was replaced with the simple rectangular window function, \( W(k) \), to prevent the enhancement of the noise at the gap null and at short wavelengths. The inverse filter formula then becomes:

\[
\tilde{D}(k) = \frac{S(k)}{L(k)} W(k)
\]

where \( W(k) \) is defined as:

\[
W(k) = \begin{cases} 
1 & k < k_g \\
0 & k \geq k_g 
\end{cases}
\]

and \( k_g \) is the wavenumber at the first gap null which is approximated by \( k_g \approx 2\pi/(1.136g) \). Figure 6.9 compares the Wiener optimal filter response for head 1 and disk 3 with the simple rectangular windowing function of equation (6.16).

Figure 6.9 Wiener filter response compared with the rectangular window function for head 1 and disk 3.

Applying the simple rectangular windowing function, however, introduced some ripple in the restored magnetisation transition and caused a slight increase in the transition length due to the sharp truncation. Nevertheless, the rms error between the computed transition using the Wiener optimal filter and with the rectangular window was less than 7%.
Figure 6.10 demonstrates the calculated magnetisation transitions for the remaining disks using head 1 and by application of the Wiener optimal filter. The transition widths were calculated from the slope of the transition centres using (6.14).

\[
\tilde{\alpha} = 0.092 \mu m \\
\tilde{\alpha} = 0.079 \mu m
\]
From Figure 6.10, it can be seen that the lengths of the computed magnetisation transitions decrease with the reduction in $M_r \delta$ and the increase in disk coercivity in agreement with the predictions of the slope theory in Chapter 4 (Williams and Comstock, 1971). To illustrate this point, Figure 6.11 compares the computed transition lengths using inverse filtering for head 1 and the four disk media listed in Table 6.1 with the transition lengths evaluated according to the full Williams and Comstock (1971) expression of equation (4.26) for the arctangent and tanh transitions.
The calculated transition lengths from inverse filtering do not agree with the theoretical values but nevertheless follow the same trend with dependence on media properties. Since the record slope theory ignores the micromagnetic details of the media that are included in the replay signal, then its is expected that the comparison between the two values would only be proportional (the computed transition lengths are approximately two orders of magnitude greater than the theoretical transition lengths obtained from the slope theory). The approximate slope theory (Chapter 4, equation (4.26)) produces the write limited transition length and does not include the effect of relaxation in increasing the transition widths in the absence of the record head. Although remagnetisation under the replay head tends to reduce the transition lengths after relaxation (Maller and Middleton, 1974), it appears from Figure 6.11 that the replay head shunting effect is small and the resulting transition lengths are limited by the relaxation process.

Figure 6.11 indicates the validity of the inverse filtering technique in detecting the change in the media properties on the computed transition lengths. Disks 1, 2 and 3
feature similar magnetisation distributions with sharp corners simulating a tanh or an error function transition, while disk 4 exhibits closely an arctangent type magnetisation transition. The length of the written transition in disk 4, however, exhibits deviation from the Williams and Comstock (1971) theoretical transition length trend. This can be explained by observing Figure 6.12 which illustrates the average isolated pulse of disk 4, its Fourier transform and the corresponding roll-off curve (computed by superposition of 200 alternating captured replay pulses).

The isolated replay pulse of disk 4 in Figure 6.12(a) exhibits a constant peak value within a specific range. This implies that the replay signal is the convolution (or correlation) of the head field function with a rectangular function (magnetisation derivative) in the space domain which produces a sinc function (and hence nulls) in the Fourier domain as indicated in Figures 6.12(b) and 6.12(c). This suggests that the

![Figure 6.12](image-url)
recorded transition is a ramp function (its derivative is a rectangular function) defined as:

\[
M(x) = \begin{cases} 
-M_r & -\pi < -\pi a/2 \\
2M_r/x & -\pi a/2 \leq x \leq \pi a/2 \\
M_r & x > \pi a/2 
\end{cases}
\]

The Fourier transform of the derivative of the ramp magnetisation function is therefore:

\[
F\left\{ \frac{dM(x)}{dx} \right\} = 2M_r \frac{\sin(k\pi a / 2)}{(k\pi a / 2)}
\]

where the first null occurs at a wavelength \( \lambda = \pi a \). From Figure 6.12(b), the first null in the spectrum occur at a wavelength \( \lambda = 0.72 \mu m \), and in Figure 6.12(c) the first null in the roll-off curve occur at a record wavelength \( \lambda_o = 0.73 \mu m \) indicating that the transition width \( \tilde{\alpha} = 0.23 \mu m \). This value is slightly less than the value quoted in Figure 6.11. The discrepancy is due to the effect of the Wiener filter in smoothing the sharp transition corners (such that it appears as an arctangent transition) and increasing the transition length slightly. Although the inverse filter process was less efficient in preserving the transition shape and in particular the transition corners in this case, it nevertheless predicted the transition length within a 16% error margin.

To demonstrate the sensitivity of the computed transition lengths to the head-to-medium parameters, disk 1 was recorded and replayed with head 2 (having smaller flying height and narrower gap length compared to head 1). The resulting magnetisation transition is compared with that produced using head 1 for the same disk (disk 1) in Figure 6.13. The inverse filter algorithm was able detect the changes in the head-to-medium parameters as indicated by the smaller transition length obtained when using head 2 for the same disk.
For very small flying heights, however, another problem arises from the negative excursions that occur in the replay pulse for low moment media, limiting the sensitivity of the inverse filter to changes in the medium and head properties. Figure 6.14 illustrates the replay pulse for head 2 and disk 2 where the negative excursions in the replay pulse tails can be seen clearly. The negative values affect the long wavelength behaviour of the Fourier transform of the replay pulse causing a mismatch with the gap loss function of the semi-infinite gapped head used in the calculations. Consequently, the computed magnetisation derivative will also contain negative values on either side of the transition centre. This will affect the integration process causing the bending in the tails of the computed magnetisation transition as illustrated in Figure 6.14. Moreover, due to the poor agreement between the replay pulse and head spectra, the filtering process will yield an incorrect value of the transition length.

Figure 6.13  Sensitivity of inverse filtering to changes in record and replay flying height and gap length for disk 1.
6.6. Separation of Transition Noise Modes

It was shown in Chapter 5 that the two transition noise modes (transition position jitter and transition width fluctuation) are orthogonal. In particular, the Fourier transform of the transition noise is given by:

\[ M_{\text{ns}}(k) = k M_x(k) \cdot \left[ j x_n - \frac{a_n}{a} \right] \]  \hspace{1cm} (6.17)

Therefore, the real part will yield the Fourier transform of position jitter noise and the imaginary part will produce the Fourier transform of transition width noise \((M_x(k)\text{ is imaginary})\).

The computed magnetisation from inverse filtering effectively represents the mean of a number of cross-track averaged magnetisation transitions as determined from the average isolated replay pulse. By computing the magnetisation transition using inverse filtering for each captured replay pulse and subtracting from the mean, the noise in the transition of each individual pulse can be determined. The mean transition noise is then obtained by averaging the transition noise waveforms. Figure 6.14 illustrates the

![Figure 6.14 Effect of negative excursions in replay pulse on computed magnetisation shape and width for head 2 and disk 2.](image)
average transition noise obtained by averaging 256 computed magnetisation transitions for disk 3 using head 1.

![Graph](image)

**Figure 6.15** Average magnetisation transition noise for disk 3 replayed with head 1.

The ripple in the noise waveform in Figure 6.15 is due to the filtering and truncation processes in the Fourier domain. If it is assumed that only transition noise is present (system noise is suppressed by averaging) and that both noise sources are orthogonal, then according to equation (6.17) it is possible to separate the two noise modes by Fourier transforming the waveform in Figure 6.15 and separating the real and imaginary parts. By acquiring the real IFFT of the real part of the noise spectrum, the jitter noise mode will be obtained. Moreover, by taking the imaginary IFFT of the imaginary part of the spectrum, the transition width noise mode will be determined (imaginary part of the IFFT was taken since the transition width noise mode is odd). The imaginary part of the transition noise spectrum (and hence the transition width noise waveform) essentially reflects any irregularity or asymmetry in the average noise waveform in Figure 6.15. The two noise modes separated from Figure 6.15 are shown in Figure 6.16.
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The calculated transition noise modes in Figure 6.16 do agree with the theoretical noise plots in Chapter 5 (Figure 5.2). It can also be seen from Figure 6.16 that the transition jitter noise seems to be dominant (relative to the same scaling in Figure 6.15) as observed from the average noise voltage waveform in Figure 6.7.

Transition noise modes in disk media were previously separated from noise voltage waveforms (not magnetisation waveforms) (Yuan and Bertram, 1992). With inverse filtering, it is possible to determine the transition noise modes from the average magnetisation transition noise knowing that the two noise modes are orthogonal. This method, however, is susceptible to the effects of the filtering process in the form of oscillations in the spatial noise waveforms. Furthermore, due to the Wiener filter nature of attenuating the short wavelength components, it is expected that the magnitude of transition width noise in Figure 6.16 would be slightly underestimated.
6.7. Effect of Filter Bandwidth on the Shape of the Replay Pulse

It can be seen from equation (6.2) that in addition to the differentiating action of the replay head, the detected flux is also subject to head loss terms or filters that ‘smear’ the output replay signal. The bandwidths of these filters are controlled through parameters (such as the gap length, head-to-medium separation and medium thickness) that affect the characteristics and shape of the readback pulse. Consequently, the shape of the readback pulse from a replay channel will be influenced by a number of bandlimited filters in addition to the head loss functions.

To demonstrate the effect of channel filters on the replay pulse, consider a simple filter response which can be defined, by analogy to the spacing loss function, as:

\[
\omega(k) = e^{\frac{k_l}{k_c} \ln \sqrt{2}}
\]  

(6.18)

This filter has a response of one at long wavelengths and falls exponentially to zero at short wavelengths. \(k_c\) defines the –3dB, corner or cut-off wavenumber of the filter. Other filter functions are equally valid and the above was chosen for convenience as it allows the replay voltage expression to be derived in the usual manner. The replay signal transform for a thin medium and an arctangent transition after modification by the filter response of (6.18) becomes:

\[
E_x(k) = \mu_o vW N \eta M_r \delta(jk) \left( e^{-k(a+d+\ln(\sqrt{2})/k_c)} \right) \left[ \frac{\sin(kg/2)}{(kg/2)} \right]
\]  

(6.19)

Evaluation of the inverse Fourier transform of (6.19) yields the readback voltage as given by equation (3.24) with an additional factor \(\ln(\sqrt{2})/k_c\) added to the effective spacing \((a+d)\). Hence, the pulse width at 50% peak amplitude, using (3.27), becomes:

\[
PW_{50} = 2 \sqrt{(a+d + \ln(\sqrt{2})/k_c)^2 + (g/2)^2}
\]  

(6.20)

It can be seen from equation (6.20) that reduction of the cut-off wavenumber causes an increase in the replay pulse width. Thus, in general, reduction of the cut-off or corner frequency of a filter in the replay channel will cause an increase in the replay pulse width and consequently a further decrease in the peak signal roll-off at high packing densities.
In the case of inverse filtering investigated here, the cut-off wavenumber can be changed by varying the selected noise level in the Wiener filter response. This was achieved through multiplying N(k) by a constant factor and increasing the value of this factor in step increments. The filtered pulse width at each step increment was then measured and plotted relative to the original pulse width without filtering. The measured pulse widths for the four disks using head 1 as a function of reciprocal cut-off wavenumber are plotted in Figure 6.17.

![Figure 6.17 Effect of Wiener filter bandwidth on the width of the filtered replay pulse.](image)

Increasing the noise level in the Wiener filter formula (6.11) effectively reduces the cut-off wavenumber of the filter response causing further attenuation of the short wavelength components of the replay spectra. As a result, the width of the filtered replay pulses (after computing the IFFT) would increase as shown in Figure 6.16 as predicted in equation (6.20). From Figure 6.16, it can be seen that disk 4 is less sensitive to the decrease in the filter bandwidth; this can be explained by examining equation (6.20) in the case where the transition length is large compared to the corner wavenumber of the applied filter.
The additional pulse broadening introduced due to filter bandwidth limitations becomes more apparent at high packing densities when transitions overlap causing a further decrease in the signal amplitude. Figure 6.18 illustrates the roll-off curves computed by summing 200 isolated pulses for disks 3 and 4 replayed with head 1. The Wiener filter was applied to each replay pulse before summing. The corner wavenumber was changed in the Wiener filter by increasing the noise level in the filter response formula (6.11).

In addition to the increase in the slope of the roll-off curve with the reduction of the cut-off wavenumber at high packing densities, there is also ripple in the peak signal at low packing densities due to the sharp truncation in the isolated pulse spectra in the Fourier domain.
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6.8. Summary

Inverse filtering is the process by which the input signal (magnetisation) is restored from the smearing effect of the replay head filters in the replay signal. This technique has been used to compute the magnetisation transition shape in longitudinal (Wells, 1985; Weismehl et al, 1988) and perpendicular thin-film media (Vos et al, 1986).

The Wiener optimal filter formula was determined by minimising the difference between the actual and computed magnetisation derivative transform in the presence of noise. The filter response changes the cut-off wavenumber and the short wavelength roll-off rate according to the specified noise level in the replay signal spectrum. It provides an enhanced noise filtering through its gradual and smooth truncation compared to the rectangular window function thus reducing the ripple in the computed spatial waveforms.

Figure 6. 18 Effect of limited filter bandwidth on the roll-off curve behaviour for (a) disk 3 and (b) disk 4 for different cut-off wavenumbers.
Integration of the smoothed magnetisation derivative waveform was achieved through the spatial convolution with the unit Signum function. Convolution was made using the Fast Fourier Transform through restructuring of the sampled waveforms (Brigham, 1988).

Centre of energy alignment of experimental isolated pulses was used to minimise the time base jitter associated with the measurements. In addition to the alignment procedure, averaging successfully reduced the stationary head and electronics noise associated with individual captured pulses in the averaged replay signal. The remaining non-stationary noise voltage indicated that position jitter noise is dominant.

Magnetisation transitions were calculated from the average replay pulses using the Wiener optimal filter. For high coercivity media, the computed transitions were not symmetrical about the magnetisation axis; with an arctangent function on one side and a tanh function on the other side as found elsewhere (Wells, 1985; Weismehl et al., 1988). In the low coercivity disk, the transition was symmetrical and closely resembles the arctangent function.

The computed transition lengths have showed the same dependence on the demagnetisation ratio as predicted by the slope theory. In addition, the inverse filtering technique accompanied by the Wiener filter was able to detect changes in the transition length as reflected by the change in head parameters.

It was possible to determine the average transition noise from individual transitions computed using inverse filtering. With the knowledge that the main magnetisation transition noise modes are orthogonal, the transition position jitter and width variation noise modes were separated from the net average transition noise.

Using a simplified expression for an arbitrary filter response, the effect of filter bandwidth on the replay pulse width was investigated. Reduction in the cut-off wavenumber of a filter leads to an increase in the width of the filtered replay pulse. This has been verified by applying the Wiener optimal filter with varying corner wavenumbers (by varying the noise level) on actual replayed pulses and measuring
their PW$_{50}$. The consequence of bandwidth limiting of replay filters on the roll-off curve was also demonstrated. The reduction in filter bandwidth leads to an increase in the slope of the roll-off curve and hence further reduction of signal amplitude with increased packing density.