Corrigendum

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In equation (2.32) of Davidson (2002), it is implicitly assumed that $\mathbf{\Phi}_{t-m}$ is $\mathcal{F}_{t-m}^{t+m}$-measurable. Therefore this relation cannot be used to analyse the subsequent examples (Sects. 2.5-2.6) where $\mathbf{\Phi}_t$ depends on x_t. The conditions of Proposition 2.5, in particular, require modification.

For simplicity, consider model (2.28) of the paper with $p = 1$, where $\phi_t = \phi(x_t)$, and assuming $|\phi_t| \leq 1$. Also assume a Lipschitz restriction, with $|\phi(x_1) - \phi(x_2)| \leq B|x_1 - x_2|$ for all x_1, x_2 and some $B > 0$. Note that where ϕ_t depends on a set indicator, such a condition is satisfied by a 'smoothed indicator' similar to (22.15) of Davidson (1994). Construct a $\mathcal{F}_{t-m}^{t+m}$-measurable approximation to x_t by setting $u_{t-j} = 0$ for $j > m$, i.e.

$$\hat{x}_t^m = u_t + \sum_{j=1}^{m} \prod_{k=1}^{j} \hat{\phi}_{t-k} u_{t-j}$$

where $\hat{\phi}_{t-k} = \phi(\hat{x}_{t-k}^{m-k})$. With the convention that an empty product equals 1, note that

$$\begin{aligned} x_t - \hat{x}_t^m &= \sum_{j=1}^{m} \left(\prod_{k=1}^{j} \phi_{t-k} - \prod_{k=1}^{j} \hat{\phi}_{t-k} \right) u_{t-j} + \prod_{k=1}^{m+1} \phi_{t-k} x_{t-m-1} \\ &= \sum_{j=1}^{m} \left(\prod_{i=1}^{j-1} \hat{\phi}_{t-i} \sum_{k=j}^{m} \prod_{i=j+1}^{k} \phi_{t-i} u_{t-k} \right) (\phi_{t-j} - \hat{\phi}_{t-j}) + \prod_{k=1}^{m+1} \phi_{t-k} x_{t-m-1}. \end{aligned} \tag{1}$$

Since $|\phi_{t-j} - \hat{\phi}_{t-j}| < \min\{2, B|x_{t-j} - \hat{x}_{t-j}^{m-j}|\}$ by assumption, an argument similar to Davidson (1994) Theorem 17.15 yields, for any $r > 1$,

$$E|\phi_{t-j} - \hat{\phi}_{t-j}|^{2(r+1)/(r-1)} \leq 2^{4/(r-1)} B^2 E|x_{t-j} - \hat{x}_{t-j}^{m-j}|^2.$$

Using this relation, double applications of the Minkowski and Holder inequalities to (1) yield

$$\|x_t - \hat{x}_t^m\|_2 \leq C_1 \sum_{j=1}^{m} \left(\sum_{k=j}^{m} \mu_1^{k-1} \right) \|x_{t-j} - \hat{x}_{t-j}^{m-j}\|_2^{(r-1)/(r+1)} + C_2 \mu_2^{m+1} \tag{2}$$

where $C_1 = 2^{2/(r+1)} B^{(r-1)/(r+1)} \|u_t\|_{2r}$, $C_2 = \|x_t\|_{2r}$ and $\mu_1 < 1$ and $\mu_2 < 1$ are numbers satisfying

$$\mu_1^{k-1} \geq \left\| \prod_{i=1}^{j-1} \hat{\phi}_{t-i} \prod_{i=j+1}^{k} \phi_{t-i} \right\|_{2r(r+1)/(r-1)}, \qquad \mu_2^{m+1} \geq \left\| \prod_{k=1}^{m+1} \phi_{t-k} \right\|_{2r/(r-1)}.$$

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Since $\|x_t - E_{t-m}^{t+m}x_t\|_2 \leq \|x_t - \hat{x}_t^m\|_2$, a sufficient condition for L_2-NED can now be stated as follows: solving (2) as an equality to yield a bounding sequence, there exists $r > 1$ such that the sequence of solutions for $m = 1, 2, 3, \ldots$ approaches 0 at the requisite rate. In model (2.33), for example, equation (A.5) could be used to estimate μ_1 and μ_2. The introduction of a smoothed indicator of the set $(-a, a)$ will not alter the essentials of the argument leading to (A.5).

References

Davidson, J. (1994) Stochastic Limit Theory, Oxford University Press.

Davidson, J. (2002) Establishing Conditions for the Functional Central Limit Theorem in Nonlinear and Semiparametric Time Series Processes, Journal of Econometrics 106, 243-269.