BUFFER STOCK MODELS OF THE MONETARY SECTOR[1]

by JAMES DAVIDSON AND JONATHAN IRELAND

Introduction

Modelling the monetary sector has long been seen as problematic. Conventional demand for money functions have typically failed to cope with the experience of the past 15 years (Judd and Scadding, 1982) and models based upon the counterparts to monetary growth have fared little better. The current interest in buffer stock money (see Cuthbertson and Taylor, 1987, for a recent survey) can be seen as a response to the failure of the more traditional approaches. We believe that the issues involved in the empirical implementation of the buffer stock hypothesis are subtle and that the literature to date has not dealt with them satisfactorily. In spite of the substantial difficulties involved, we think that a system approach to buffer stock modelling (for example, Davidson, 1987a) may be unavoidable to ensure theoretical consistency.

This note is a progress report on continuing work concerned both with the specification of models of the monetary sector and linkages with the economy as a whole. One eventual aim of the work is to estimate a model of the monetary sector which could be incorporated into the National Institute's UK model.

The buffer stock hypothesis

Micro foundations and the aggregation problem

The buffer stock hypothesis is just a restatement of the traditional assumption of a transactions/precautionary motive for holding money balances. Agents are uncertain about the pattern of their future receipts and expenditures, and hold money balances to reduce the risk of shocks in one market restricting their ability to participate in other markets. Ordinary transactions change money holdings without reference to any 'desired' holding. From moment to moment, holdings are determined by the net volume of the agent's transactions in goods, services and non-money assets; yet holdings can only be adjusted towards a desired level by making *further* transactions. Since they would not generally be undertaken for any other reason (by definition), such adjustments are costly. There is no particular reason—in theory or casual introspection—to suppose that agents should be thought of as targeting a desired holding continuously.

A number of formal models (Miller and Orr, 1966; Akerlof, 1973, 1979; Milbourne, 1983) which address this problem have proposed that the agent's desired money holding is not a single valued function of the usual choice arguments but consists of a band of acceptable holdings. This band is defined by an upper and lower threshold such that once money holdings cross either boundary an adjustment to a 'return point' is triggered. Following Akerlof's (1979) distinction, we say that transactions aimed at such adjustment are *induced*; other transactions are called *autonomous*. Letting the jth individual's upper and lower thresholds be U_j, L_j and his return point z_j such that $L_j \leqq z_j \leqq U_j$, the consequence of this assumption is that any aggregate money stock which is the sum of points in the intervals $[L_j, U_j]$, $j=1 \ldots n$ (for an economy with n money holders) is an equilibrium. The set of equilibrium points so defined is called the 'money box', and in this theory we have to become accustomed to thinking of the equilibrium money stock as set-valued, not point-valued. A simple version of this model introduces the 'money demand' arguments—for example, permanent income, interest and inflation rates—as explaining z_j, and assumes the thresholds have the form:

$$L_j = a_j z_j, \; U_j = b_j z_j$$

where $0 \leqq a_j < 1 < b_j$; a_j and b_j being fixed parameters for individual j.

As monitoring the level of money balances relative to those variables which influence the 'desired' level (by assumption, in fixed relationship with z_j) is costly, it is likely that agents will monitor at regular intervals (that is, when they receive their bank statements) rather than continuously. If in addition we suppose that agents monitor their balances following a particularly large autonomous transaction, we can envisage three cases in which agents will make induced transactions:

(i) when money holdings are found to lie outside the acceptable band at a routine monitoring;

(ii) when a large autonomous transaction results in *ad hoc* monitoring and money balances cross the thresholds; and

(iii) when a change in the thresholds results in balances lying outside the new thresholds.

Denoting agent j's autonomous transactions during period t by TA_{jt}, induced transactions by TI_{jt} and period t's opening money balances (equal to period $t-1$'s closing balances) by M_{jt-1}, we can represent the expected level of his induced transactions by:

$$E(TI_{jt} \mid TA_{jt}, M_{jt-1}) = \alpha \left[E(z_{jt-1}) - M_{jt-1}\right] + \gamma E(\Delta z_{jt}) - \beta \, TA_{jt} \qquad (1)$$

noting that the expectation is taken over the distribution of individual attributes. The term in Δz_{jt} represents event (iii) in the simple model where the thresholds are multiples of the return point and α, β, and γ represent the probabilities of events (i), (ii) and (iii) occurring. Averaging equation (1) over M_{jt-1} and TA_{jt-1}, we find that in a steady state in which $E(\triangle z_{jt})$ and $E(TA_{jt})$ are zero, $E(z_{jt-1})$ equals $E(M_{jt-1})$ so that z_{jt} (as a function of the usual arguments) can be identified with a 'long-run demand for money'. (See Davidson, 1987b, for further details.)

Induced adjustments of money holdings by individual agents may or may not affect the size of the aggregate money stock. As the money stock is defined to be equal to the money holdings of the non-bank private sector (NBPS), only 'outside transactions'—those between agents within the NBPS and those outside it—will affect the size of the stock.(2) Transactions within the NBPS involve a redistribution of the money stock but do not change its size. They also redistribute disequilibrium since, for example, one individual's induced disbursement may represent another's autonomous receipt. We can imagine a succession of induced transactions, each triggered by the last: what is known as the 'hot potato' effect. On the assumption that a substantial proportion of transactions with an induced character are 'inside', we can view those markets in which money crosses sectoral boundaries—for example, the gilt-edged and foreign exchange markets and the market for bank loans—as adjustment bottlenecks; the rate of *aggregate* adjustment to equilibrium is expected to be generally slower than the average rate of *individual* adjustment (see Davidson, 1987a and 1987b, for further discussion on these points). This aggregation effect adds further sluggishness to aggregate adjustment, over and above individual buffering behaviour.

Empirical implementation

As the change in an individual's money balances is equal to the sum of his autonomous and induced transactions (that is, $\triangle M_{jt} = TA_{jt} + TI_{jt}$), adding agent j's autonomous transactions to both sides of equation (1) gives:

$$\triangle M_{jt} = \alpha(z_{jt-1} - M_{jt-1}) + \gamma\triangle z_{jt} + (1\text{-}\beta)(TA^{I}_{jt} + TA^{O}_{jt}) + e_{jt} \qquad (2)$$

where $E(e_{jt} \mid TA^{I}_{jt}, TA^{O}_{jt}, M_{jt-1}) = 0$

and autonomous transactions have been divided into inside (TA^{I}_{jt}) and outside (TA^{O}_{jt}) transactions. On aggregation over all NBPS agents we obtain the change in the aggregate money stock:

$$\triangle M_{t} = \alpha(z_{t-1} - M_{t-1}) + \gamma\triangle z_{t} + (1\text{-}\beta)(TA^{I}_{t} + TA^{O}_{t}) + e_{t} \qquad (3)$$

Finally, assuming that a proportion δ of induced transactions are inside, we can substitute TA^{I}_{t} out of (3) giving:

$$\triangle M_{t} = [\alpha(1-\delta)/(1-\delta\beta)][z_{t-1} - M_{t-1}] + [\gamma(1-\delta)/(1-\delta\beta)]\triangle z_{t} + [(1\text{-}\beta)/(1-\delta\beta)]TA^{O}_{t} + \epsilon_{t} \qquad (4)$$

This equation bears some resemblance to conventional demand for money functions, although it is linear (as opposed to log linear) and contains the term in TA^{O}_{t}. With $\alpha=\gamma$ the dynamics are similar to Goldfeld (1973, 1976) and with $\alpha\neq\gamma$ to Hendry and Mizon (1978).

'Shock absorber' and related approaches

A popular empirical approach to buffer stock models (for example, Carr and Darby, 1981) involves adding a measure of 'unanticipated' money ($M_{t} - E_{t-1} M_{t}$) to the conventional (that is, Goldfeld-type) demand for money equation. The justification is that under the buffer stock hypothesis agents will hold onto a proportion of any monetary shocks (hence the popular name 'shock absorber' models) and that under rational expectations these shocks are equal to the difference between actual balances and expected balances.

The Carr-Darby approach has been criticised (MacKinnon and Milbourne, 1984) on the grounds that if money is endogenous, its appearance on both sides of the estimating equation will lead to inconsistent least squares estimates. But apart from issues of econometric technique, it can be argued that there is a logical inconsistency in this approach. Given that $E_{t-1}M_{t}$ is measured as the predictions from a supply equation, the assumptions of rationality and endogeneity of money are incompatible. (See Cuthbertson and Taylor, 1987, p.113, on this point.)

There is nonetheless some resemblance between the shock absorber equation and our equation (4); both are versions of a conventional (for example, Goldfeld, 1973) money demand equation with an additional variable. But there are important differences between these variables and thus between the two equations. Unanticipated money consists of both demand and supply shocks while TA^{O}_{t} contains only supply shocks. Further, our interpretation of the buffer stock hypothesis does not suggest that TA^{O}_{t} need only incorporate unanticipated elements. We have already noted that money is considered a buffer stock in the sense of allowing transactions in different markets to be coordinated and it is clear that it is actual rather than expected balances which perform this function. For example, the knowledge that you will receive a tax refund next month does not help pay this month's unexpectedly large gas bill. (It may induce your bank manager to allow you a temporary overdraft, but the point is that this would raise your actual balances.) These points indicate that from the point of view of our model the shock absorber equation is mis-specified.

Single equation and systems approaches to modelling buffer stock money

A single equation model

Changes in the aggregate money stock are by definition the net of all the outside transactions—the inside transactions net to zero identically. Moreover, the induced and autonomous components (of the outside transactions of the NBPS) are not observable as such; the distinction is motivational and in principle any transaction may fall into either category. As a first approximation, however, we might attempt to categorise the reported components of the monetary aggregate according to their predominant character. Initial results suggest that this approach does not work, yet it is worth attempting given the complexities of the alternative (systems) approach.

The counterparts identity for M3 (see *Bank of England Quarterly Bulletin*, Statistical Annex, table 11.3) may be written in a stylised form as:

$$\triangle M3 \equiv TA^{O} + TI^{O} \equiv \triangle(\textit{bank lending}) + \textit{PSBR} + \textit{current account BoP} + \textit{capital account BoP} - \textit{net bond sales to NBPS} + \triangle(\textit{banks' non-deposit liabilities}).^{(3)} \quad (5)$$

By construction the right hand side terms in (5) are all aggregates of outside transactions. Bank loans are induced more or less by definition. The same could be said for bond sales to the NBPS if they did not contain a speculative element; we ignore this and treat bond sales as induced. The PSBR is taken as autonomous. As for the two categories of foreign flows, the most realistic approach may be to treat the current account as autonomous and—appealing to the monetary approach to the balance of payments (Johnson, 1972)—the capital account as induced. This classification may be wrong on two counts: the chosen proxies for TA^{o}_{t} may include induced transactions or exclude autonomous transactions. Both cases may result in correlation between the proxies and the error term.

Davidson (1987b) reports the results of some preliminary attempts to estimate a single equation model using equation (4) and the above classification of autonomous outside transactions. The components of TA^{o}_{t} enter the estimating equation separately to provide us with a test of the model; that the coefficients on each component should be equal and lie between zero and one. The return point z_t (which by our assumption determines the form of the individual's money box) is modelled as:

$$z_t = Y^P_t \exp\{b_O + \textstyle\sum^4_{i=0}[(c_i \log(Y/Y^P)_{t-i} + d_i(r^l - r^s)_{t-i} + e_i(r^s - \dot{p}\,)_{t-i}]\} \quad (6)$$

where Y_t is measured income, Y^P_t is a moving average measure of permanent income, r^l_t and r^s_t are representative long and short interest rates, and $\dot{p}_t$ is a measure of inflation. Equation (6) can be thought of as a long-run money demand function in the sense that average money holdings over a period of time are expected to correspond with the return point. As equation (4) is in nominal terms it may have heteroscedastic disturbances, which suggests dividing throughout by the current price level to give the final equation:

$$\triangle M_t/P_t = a_1[(z_{t-1} - M_{t-1})/P_t] + a_2(\triangle z_t/P_t) + a_3(PSBR_t/P_t) + a_4(CBAL_t/P_t) + u_t \quad (7)$$

Three versions of this model were estimated:

(i) imposing $a_3 = a_4 = 0$ which corresponds to the conventional money demand function apart from being non-linear;

(ii) with the proxies for TA^{o}_{t} ; and

(iii) using instrumental variables to overcome the problem that our chosen proxies for TA^{o}_{t} are likely to contain some induced elements giving rise to simultaneity bias.

The extent of bias should be indicated by the differences between the estimates in (ii) and (iii) or, more formally, by an exogeneity test on the autonomous terms.

As yet this work is incomplete, but the results obtained so far are not particularly favourable to the success of this approach. Using M3 data for 1966 I to 1984 II the sign of a_1 was found to be negative in both (ii) and (iii), implying that the model has an unstable long-run solution and cannot be correctly specified (although as might be expected, a_1 is positive for our version of the standard equation, (i)). Whether this result is due to the unsuitability of our proxies for the autonomous flows or to some other problem, probably connected with aggregation error, our findings point to the conclusion that this single equation approach to estimating buffer stock models is not satisfactory.

Systems estimation

It would seem that to distinguish between TA^{o}_{t} and TI^{o}_{t} it cannot suffice to disaggregate the money stock according to the counterparts identity (equation (5)); we must also model the money supply mechanism by modelling these components. The buffer stock hypothesis implies that money supply is equated with a single-valued money demand only in the sense of a presumed tendency of long time averages to correspond. But if supply is eventually responsive to the supply–demand gap so defined (that is, some outside transactions are induced), we should capture this effect by embedding a function of the gap in each equation. This approach makes it necessary to estimate the counterpart equations as a system. The money demand equation is undefined in the short run, but appears as a component of the long-run solution of the model. In this approach, money is the buffer and hence the residual asset of the system, determined by identity (5) plus the counterpart equations. The conventional short-run demand equation is to be seen in this context as a reduced form equation at best, but also generally mis-specified as the above analysis has shown.

One way to understand money disequilbrium is as a deviation of V from its desired level in the quantity theory identity $MV \equiv PY$. Given a money supply shock, V must give immediately to satisfy the identity, but movement to restore equilibrium can be either through counterveiling pressure on money itself or through pressure on nominal output (via the real balance effect) or on the arguments of long-run money demand (note, desired V is not a constant but is assumed to be a stable function of the money demand arguments). It is appropriate on these assumptions to embed the money disequilibrium term in equations for the price level, interest rates and a range of equations in the real part of the model. This allows the model to capture the transmission mechanism of monetary shocks to the real economy, an advantage of the systems approach over single equation methods of both the shock absorber and Davidson (1987b) variety.

These ideas have been applied by several authors, for example Jonson and Trevor (1979) for the Australian economy and Davidson (1987a) for the UK economy. Davidson's model is specified as an 'error correction system' in which a set of target relations $f(\boldsymbol{y}, \boldsymbol{x}) = \boldsymbol{0}$ represent a steady state, or long-run equilibrium. The model's dynamic equations represent the efforts of groups of agents to attain these targets and include an error correction component which, crudely, has endogenous variables being adjusted by a proportion of their deviations from their targets. The complete system can be written as:

$$A_1(L)\triangle \boldsymbol{y}_t + A_2(L)\triangle \boldsymbol{x}_t + C(L)f(\boldsymbol{y}_{t-1}, \boldsymbol{x}_{t-1}) = \boldsymbol{u}_t \quad (8)$$

where $A_1(L)$, $B_1(L)$ and $C(L)$ are matrix polynomials in the lag operator, $\boldsymbol{y}_t$ and $\boldsymbol{x}_t$ are vectors of endogenous and exogenous variables, and $\boldsymbol{u}_t$ is a disturbance vector. Note that the error correction component is represented by the term $C(L)f(\boldsymbol{y}_{t-1}, \boldsymbol{x}_{t-1})$ where for any $\boldsymbol{y}$ and $\boldsymbol{x}$ the deviations from the steady state targets are given by $f(\boldsymbol{y}, \boldsymbol{x})$. As the long-run demand

Table 1. Summary of systems results

	M3	M5
Coefficients of money demand equation		
Real short interest rate (δ_1)	0.013	0.012
	(0.003)	(0.002)
Long *less* short interest rate (δ_2)	0.002	0.002
	(0.005)	(0.005)
Logistic time trend	−0.401	0.857
	(0.178)	(0.225)
Coefficients of money disequilibrium terms in adjustment equations		
Bank loans to NBPS	−0.162	−0.174
	(0.028)	(0.028)
Market value public sector debt of NBPS	0.055	0.023
	(0.036)	(0.038)
Price of gilts	−0.031	0.001
	(0.043)	(0.042)
Real GDP	0.029	0.013
	(0.026)	(0.026)
GDP deflator	0.051	0.039
	(0.019)	(0.019)

Note: This table has been adapted from table 6.4 in Davidson (1987a). The estimates are computed by non-linear three stage least squares imposing cross-equation restrictions so that the money demand parameters are obtained by fitting six adjustment equations. In addition to those reported above, the full model contains equations for the current balance, the exchange rate and the NBPS's net capital liabilities overseas. The complete specification of the adjustment equations is given in table 6.3 in Davidson (1987a).

for money can itself be thought of as a target it is included in the error correction component of the model as follows:

$$M_t/P_t Y_t = \gamma_1 \exp\{\delta_1(r^s_t - 200.\triangle_2 \ln P_t) + \delta_2(r^l_t - r^s_t) + \delta_3 LT_t\} \quad (9)$$

where LT_t is a logistic time trend. The other targets specified in Davidson (1987a) include the ratio of bank advances to deposits, the market value of public sector debt held by the NBPS, purchasing power and uncovered interest parity.

This model was estimated using two alternative money variables (M3 and M5) over the period 1964 II to 1982 IV. See table 1 for the essential findings reported. The money disequilibrium term is most significant, unsurprisingly, in the bank loans equation and also, more interestingly, in the price equation. On the other hand it is insignificant in the real income equation. This supports a monetarist interpretation of the transmission mechanism.

Of course, there are a number of problems with the systems approach. Since the presence of cross-equation constraints enforces a full-information approach to estimation, the estimates of the long-run money demand function depend upon the specification of the remainder of the system; therefore the chances of inconsistent estimates are fairly high. Moreover, the computational burden is very heavy, and prohibitive for systems on the scale of the National Institute model. However, the recent literature on co-integrated processes (see, for example, Engle and Granger, 1987) suggests the possibility of a computationally simpler approach. The fundamental notion is that when data series are integrated—that is, when they contain trends and other evolutionary features such that only their differences can be treated as stationary processes—they can also be co-integrated, which means that certain linear functions of them may be stationary processes (for example, the difference between money supply and the long-run demand). Indeed, if this were not so it would be hard to claim the existence of a relationship of any meaningful kind. Systems of non-stationary variables can be represented as vector autoregressive processes containing unit roots, and the co-integrating vectors of the system (if any) are simple functions of the system parameters (see Davidson, 1987c). Engle and Granger show that it is possible to estimate the co-integrating vectors efficiently using ordinary least squares, and in principle it is also possible to test for the existence of such relationships, although the theory of inference for these models is non-standard. Some tests of this kind have been performed by Hall on some relationships in the National Institute model (Hall, 1986), and similar work is in progress on the monetary sector of the Danish economy (see Juselius, 1987).

NOTES

(1)We acknowledge the financial support of the ESRC Macro Economic Modelling and Forecasting Consortium.

(2)Just as induced transactions may be divided into 'inside' and 'outside' components, so may autonomous transactions. Only those autonomous transactions which are initiated outside the NBPS may be thought of as supply side changes. These changes consist of both anticipated and unanticipated elements.

(3)The two balance of payments terms in equation (5) do not correspond exactly with the external and foreign currency counterparts in *Bank of England Quarterly Bulletin*, table 11.3. See Central Statistical Office (1987), section 11, for a reconciliation.

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