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A non-linear error correction mechanism based on the bilinear model[1]

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Abstract

This paper proposes a new non-linear error correction mechanism motivated by the bilinear model, which can approximate a wide class of non-linear cointegrated systems, and in particular can capture abrupt changes in the speed of adjustment in the face of shocks. Illustrative empirical results are reported for two data sets. © 1998 Elsevier Science S.A.

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1. Introduction

It is by now well known that systems in which non-stationary variables are cointegrated can be described by error correction mechanisms (see, e.g., Granger, 1986; Engle and Granger, 1987). In the great majority of papers that have reported error correction mechanisms, an implicit assumption is that the adjustment to equilibrium is a constant proportion of the error. More recently, a number of authors (e.g., Escribano and Mira, 1996) have discussed non-linear error correction mechanisms. This approach involves introducing non-linear functions of the deviations from equilibrium, in the dynamic model. A popular case is the 'threshold' model, in which the speed of adjustment to equilibrium switches, depending on the magnitude of the deviation (Balke and Fomby, 1996; Michael et al., 1994). Such mechanisms can be motivated by transactions costs (Dumas, 1992) or lumpy costs of adjustment (Bertola and Caballero, 1990). The ECM in such cases, whilst globally stable, can admit unit root or explosive behaviour for small deviations from equilibrium.

Whilst these specifications of the ECM have many potential applications (see Granger and Terasvirta, 1995, for a survey) there may be circumstances in which non-linearity cannot be modelled in this way. For example, in the modelling of spot and forward exchange rates or the term structure of interests rates, a time varying risk premium could result in a different form of non-linear dynamics

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[1]The data used in this study are available from the authors on request.

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(Backus and Gregory, 1993). Models such as those of Begg (1984), Boldrin and Woodford (1990), Van der Ploeg (1987) and De Grauwe et al. (1993) show how variables can be described by complex, possibly chaotic trajectories. Threshold ECMs may well not capture the behaviour of risk premia or ‘abrupt’ behaviour. The purpose of this article is to propose an alternative form of non-linear ECM, which can capture abrupt changes in adjustment speeds, such as amplitude ‘spiking’ associated with market ‘bubbles’ and ‘crashes’.

2. Some analysis

The model is motivated by consideration of the bilinear form for a univariate process, given by:

$$y_t = \alpha_0 + \sum_{j=1}^{m} \beta_{0j} u_{t-j} + \sum_{i=1}^{p} \left(\alpha_i + \sum_{j=1}^{m} \beta_{ij} u_{i-j} \right) y_{t-i} + u_t, \tag{1}$$

where u_t is usually assumed i.i.d.[2] The bilinear model provides a direct generalisation of the ARMA model. Bilinear models have an unconditional moment structure very similar to ARCH, and consequently may easily be mistaken for ARCH (Weiss, 1986). More importantly, it has been demonstrated (Brockett, 1976) that the bilinear form can approximate any non-linear dynamic model to an arbitrary degree of accuracy over a finite time interval. The covariance stationarity of the process (which requires the existence of all moments of the u_t) depends on complex restrictions on the parameters analogous to the ARMA case; see Tong (1990). When the model borders on the non-stationary, the trajectories can exhibit pronounced volatility. Bilinear models are discussed further in, e.g., Granger and Andersen (1978) and Subba Rao and Gabr (1974).

An integrated form of (1) can be considered quite simply, by letting the stationary process y_t represent the difference of a measured process Y_t, which is accordingly $I(1)$. If we also introduce an exogenous $I(1)$ variable X_t which is cointegrated with Y_t with cointegrating vector $(1, -a)$, a representation of the dynamic relationship which shares essential features with (1) is the class of bilinear ECM models. In full generality, we can write this as

$$\Delta Y_t = \alpha_0 + \sum_{j=1}^{m} \beta_{0j} u_{t-j} + \sum_{i=1}^{p} \left(\alpha_i + \sum_{j=1}^{m} \beta_{ij} u_{t-j} \right) \Delta Y_{t-i} + \sum_{i=0}^{q} \left(\gamma_i + \sum_{j=1}^{m} \delta_{ij} u_{t-j} \right) \Delta X_{t-i}$$
$$+ \sum_{i=1}^{r} \left(\theta_i + \sum_{j=1}^{m} \lambda_{ij} u_{t-j} \right) z_{t-i} + u_t, \tag{2}$$

where $z_t = Y_t - aX_t$.[3] The extension to a cointegrating dynamic system incorporating a similar equation for ΔX_t is implicit here, as is the extension to more variables, but for present purposes we confine attention to (2).

As well as specialising to the form (1) for ΔY_t when Y and X are unrelated, this model encompasses the standard linear ECM, in which all the β_{ij}, δ_{ij} and λ_{ij} are set to 0. Another specialisation, which could capture the kind of behaviour we presently have in mind, is to have z_t itself follow (1). This

[2]Time varying volatility processes can however be allowed; see Lane et al. (1996).
[3]m represents the maximum lag here. This need not be 1, but some coefficients could be zero.

form is obtained from (2) by setting $\gamma_0 = a$, and deleting all other terms in ΔX_{t-i} and ΔY_{t-i}. In fact, restrictions of the latter type are often not accepted in the linear ECM setup. We have no reason to impose them here, but by viewing (2) as a generalisation of (1) applied to z_t, we infer that the model can approximate arbitrary non-linear dynamics for the deviations from equilibrium. However, our model is neither contained in, nor contains, the model class featuring non-linear transformations of z_{t-1} as the error correction term, such as the threshold ECM. As such, it deserves separate consideration.

3. Empirical results

We illustrate the potential applicability of the bilinear ECM with two data sets. The first is annual real consumers' expenditure and real gross national product at market prices for the UK for the period 1830–1990, obtained from Mitchell (1990) and from Economic Trends. Standard unit root tests do not reject the hypotheses that the logarithms of real consumption (C) and real income (Y) are $I(1)$ processes, and also that C and Y are cointegrated, with the cointegrating vector estimated by least squares as (1, −0.918). As Eqs. (3)–(5), we report three versions of an error correction model for C. These are respectively the linear case, the bilinear case, and an alternative non-linear specification including squares of lagged terms. The included terms in (4) were selected by a preliminary specification search for the best parsimonious version of (2). We include matching lags in (3) and (5) for ease of comparison, although extra lags do not appear significant in these equations and there is no evidence of serial correlation.

All the parameters in these equations, including the cointegration parameter, are estimated jointly by maximum likelihood. The data were expressed in mean deviations, so that z_t has a sample mean of 0 by construction[4]

$$\Delta C_t = \underset{(0.0033)}{0.018} - \underset{(0.054)}{0.0073} z_{t-1} + \underset{(0.20)}{0.31} \Delta C_{t-1} - \underset{(0.13)}{0.18} \Delta C_{t-2} - \underset{(0.14)}{0.14} \Delta Y_{t-1} + \hat{u}_t, \tag{3}$$

$$z_t = C_t + 2.36 - \underset{(1.93)}{1.17} Y_t,$$

$$R^2 = 0.0998, \quad \log L = -373.1,$$

$$Q(\text{levels}) = 0.53(4), 5.09(12), \quad Q(\text{squares}) = 35.9(4), 39.76(12),$$

$$\begin{aligned} \Delta C_t = {} & \underset{(0.0028)}{0.022} - \left(\underset{(0.028)}{0.031} - \underset{(0.011)}{0.079} \hat{u}_{t-1} \right) z_{t-1} + \left(\underset{(0.20)}{0.31} + \underset{(0.024)}{0.88} \hat{u}_{t-1} \right) \Delta C_{t-1} \\ & - \underset{(0.069)}{0.24} \Delta C_{t-2} - \underset{(0.071)}{0.21} \Delta Y_{t-1} + \hat{u}_t, \end{aligned} \tag{4}$$

[4]Note that in (4) and (5) the intercept term of z_t is not absorbed into the equation intercept, but is a distinct parameter. The optimisation algorithm failed to converge when we attempted to fit this parameter directly, apparently due to scaling problems. The reported intercept of the cointegrating relation is calculated as $\bar{C} - \hat{a}\bar{Y}$, imposing the restriction $E(z_t) = 0$ which accords with the usual rationale for these models. We also performed the relevant variants of the Engle and Granger (1987) two-step method for each model by including the OLS cointegrating residual in the equations, and this gave fairly similar results in each case.

$$z_t = C_t - 0.901 - \underset{(0.02)}{0.88} Y_t,$$

$$R^2 = 0.340998, \quad \log L = -349.2,$$

$$Q(\text{levels}) = 2.05(4), 8.23(12), \quad Q(\text{squares}) = 0.60(4), 2.60(12),$$

$$\Delta C_t = \underset{(0.0043)}{0.019} - \underset{(0.018)}{0.0076} z_{t-1} + \underset{(0.101)}{0.025} z^2_{t-1} + \underset{(0.12)}{0.41} \Delta C_{t-1} - \underset{(0.016)}{0.045} (\Delta C_{t-1})^2 - \underset{(0.11)}{0.23} \Delta C_{t-2} - \underset{(0.094)}{0.17} \Delta Y_{t-1} + \hat{u}_t, \tag{5}$$

$$z_t = C_t + 4.75 - \underset{(1.01)}{1.38} Y_t,$$

$$R^2 = 0.214, \quad \log L = -361.8,$$

$$Q(\text{levels}) = 1.15(4), 6.26(12), \quad Q(\text{squares}) = 13.74(4), 18.23(12).$$

The reported standard errors in parentheses are heteroscedasticity-consistent estimates. R^2 is the squared correlation of the actual and fitted values. Two cases of the Box-Pearce Q statistic, for 4 and 12 autocorrelations, are reported for both the levels and the squares of the model residuals. If we assume two and 10 degrees of freedom respectively for the former statistics (given two lags of the dependent variable) there is no evidence of level autocorrelation in any of these equations. The Q for the squares provides a test for ARCH-type persistence in volatility, and in this case there is a sharp contrast between the models. The additional parameters in Eqs. (4) and (5), relative to Eq. (3), are in both cases jointly significant by the likelihood ratio, but the bilinear model (4) is plainly superior to both (3) and (5) in several respects. It has a substantially larger likelihood and R^2, and it alone shows no evidence of ARCH in the residuals. Also, not least, the cointegration parameter estimate is both well determined and theoretically plausible in (4), in marked contrast to the other cases. This finding is indicative of the potential difficulty of estimating the cointegrating parameter without a correct specification of the short-run dynamics

Our second data set consists of monthly observations on the spot (S) and forward rate (F) for the dollar/yen exchange rate over the period 1974–1993, obtained from the Harris data tape. In this case, standard unit root tests do not reject the hypotheses that S and F are $I(1)$ and cointegrated with cointegrating vector $(1, -1)$. The standard approach of regressing the exchange rate appreciation on the lagged forward premium yields Eq. (6). We hypothesise that a time varying risk premium related to the shocks impinging on the system is an omitted variable in this context. The same equation augmented by a bilinear term is given in (7)

$$\Delta S_t = \underset{(0.0021)}{0.0078} - \underset{(0.66)}{1.73} (F - S)_{t-1} + \hat{u}_t, \tag{6}$$

$$R^2 = 0.028, \quad \log L = 516.25,$$

$$Q(\text{levels}) = 8.65(4), 17.71(12), \quad Q(\text{squares}) = 12.28(4), 23.36(12),$$

$$\Delta S_t = \underset{(0.0023)}{0.0050} - \underset{(0.64)}{1.86}(F-S)_{t-1} + \underset{(1.26)}{3.35}\Delta S_{t-1}\hat{u}_{t-1} + \hat{u}_t, \tag{7}$$

$$R^2 = 0.052, \quad \log L = 516.99,$$

$$Q(\text{levels}) = 6.23(4), 16.07(12), \quad Q(\text{squares}) = 6.11(4), 17.30(12).$$

Note that the additional term is significant by the t test, the fit is improved, and the ARCH in the residuals becomes insignificant at the 5% level. Also note that because ΔS_t is considerably more volatile than the forward premium, $\hat{u}_{t-1}$ is in this case similar to ΔS_{t-1}, and for this reason (7) gives very similar results to the model analogous to (5) above, including the square of the lagged dependent variable.

4. Conclusion

There is growing interest in modelling adjustment to equilibrium in cointegrated systems in a non-linear manner. The purpose of this article has been to propose a new form of non-linear ECM motivated by the bilinear form. This form would appear to have potential applicability when non-linear behaviour is thought a priori to be important, but models such as threshold or ESTAR fail to capture it parsimoniously. Models in which time varying risk premia are important or variables display abrupt changes are natural candidates.

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