Modelling Political Popularity: A Correction

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In this journal, Byers, Davidson and Peel (1997) present a model of the micro-foundations of the observed long memory property of opinion poll series. Specifically, such series are found to be fractionally integrated (I(*d*)) with parameter *d* in the region of 0.7, and hence to be nonstationary although 'mean reverting'. This finding has been documented in a range of studies with poll data from the UK, the US and elsewhere. Box-Steffensmeier and Smith (1996) made the observation independently for US aggregate partisanship data, and more recently a special issue of *Electoral Studies* (Vol.19 No. 1, March 2000), has been devoted to the phenomenon, and includes our own follow-up study (Byers, Davidson and Peel, 2000) in which we analyse 26 poll series from eight countries.

Our original paper presented a model in which the fractional integration property arose through the aggregation of heterogeneous poll responses, exploiting a well-known result due to Granger (1981). However, a step is missing from our aggregation argument, and the purpose of the present note is to remedy this omission. On page 475, we assert that if $\bar{X}_t$ denotes the mean of *N* binary responses in an opinion poll, and the log-odds in favour of the response "Yes" by the *i*th respondent is

$$Y_t^i = \log\left(\frac{p_t^i}{1-p_t^i}\right)$$

then $\log\{\bar{X}_t/(1-\bar{X}_t)\}$ converges in probability to the same limit as the unobserved variable

$$\bar{Y}_t = N^{-1}\sum_{i=1}^{N} Y_t^i$$

as $N \to \infty$. This is strictly incorrect, as will be clear from the following analysis. However, we show that the approximation invoked in the paper nonetheless remains valid, for the purpose of showing that the time-dependence profiles of the unobserved and the observed series will be similar.

We observe first that is not the closeness of $\text{plim}\,\bar{Y}_t$ to $\text{plim}\log\{\bar{X}_t/(1-\bar{X}_t)\}$ that matters for our conclusion, but rather, the closeness to linearity of the relation between them. Assume that $\text{plim}\,\bar{Y}_t = E(Y_t^i) = a_t$ (say) where *E*(.) denotes the expectations operator derived from the poll sampling distribution conditional on the "common" stochastic components. In other words, let a_t be the true "Granger aggregate" process. Then, the question at issue is the functional relation between a_t and $\log\{E(p_t^i)/(1-E(p_t^i))\}$ = $g(a_t)$ (say) where

$$E(p_t^i) = E\left(\frac{e^{Y_t^i}}{1+e^{Y_t^i}}\right).$$

If $g(.)$ were linear, substituting $g(a_t)$ for a_t would amount to no more than changing the units of measurement and/or origin. In particular, the time series properties of a_t, after centring, would be preserved under g.

Write

$$Y_t^i = a + \sigma x$$

for some $x \sim (0,1)$, so that

$$g(a) = \log\left\{\frac{E(e^{a+\sigma x}/(1+e^{a+\sigma x}))}{1-E(e^{a+\sigma x}/(1+e^{a+\sigma x)}))}\right\}$$

$$= a + \log\left\{\frac{E(e^{\sigma x}/(1+e^{a+\sigma x}))}{E(1/(1+e^{a+\sigma x)}))}\right\}.$$

Note that $E(e^{\sigma x}/(1+e^{a+\sigma x}))$ is monotone decreasing in a with range (0, $E(e^{\sigma x})$), and $E(1/(1+e^{a+\sigma x}))$ is similarly monotone decreasing with range (0, 1). While the actual form of the relation depends on the distribution of x, it is very plausible to assume that in most cases $g(a)-a$ is smooth and monotone in the relevant region, with slope depending on σ. Moreover,

$$\frac{e^{\sigma x}}{1+e^{\sigma x}} - \frac{1}{2}$$

is an odd function of x, and hence has a mean of 0 if x is symmetrically distributed around 0. It follows that $g(0) = 0$ in the case where x has a symmetric distribution.

We illustrate the approximations involved with some calculations for leading cases. Since $e^{-4}/(1+e^{-4})$ = 0.017 and $e^{4}/(1+e^{4})$ = 0.98, the interval $-4 \le a \le 4$ contains most points of interest. Figure 1 plots $g(a)$ against a for four cases, estimating the expected values by the averages of 10,000 random drawings. The cases are

1. $x \sim N(0,1)$, $\sigma = 1$
2. $x \sim 2^{-1/2}(\chi^2(1)-1)$, $\sigma = 1$
3. $x \sim N(0,1)$, $\sigma = 5$
4. $x \sim 2^{-1/2}(\chi^2(1)-1)$, $\sigma = 5$

In Case 1, around 95% of the p_t^i are between 0.12 and 0.88, whereas in Case 3 most of the p_t^i are close to 0 or 1, simulating a highly 'committed' population of voters. The latter case is deliberately exaggerated. Cases 2 and 4 are also not motivated by considerations of realism; they are chosen to see how far a high degree of skewness affects matters. Again, the distributions chosen are deliberately extreme cases.

Only in Case 4 is there noticeable nonlinearity, and even here, the fractional time series model should approximate the observed series perfectly well. The main effect, removed by centring, would be some downward translation of the series. In the other cases, the only effect that would be observed in estimation is some shrinkage of the shock variance, which is not the subject of any theoretical restriction.

A Monte Carlo experiment with $N = 1000$, confirming that long memory property of the aggregated process predicted by the model of Granger (1981) is indeed preserved under the log-odds transformation, is reported by Dolado, Gonzalo and Mayoral (2001), who apply the aggregation model in an analysis of Spanish political popularity data.

References:

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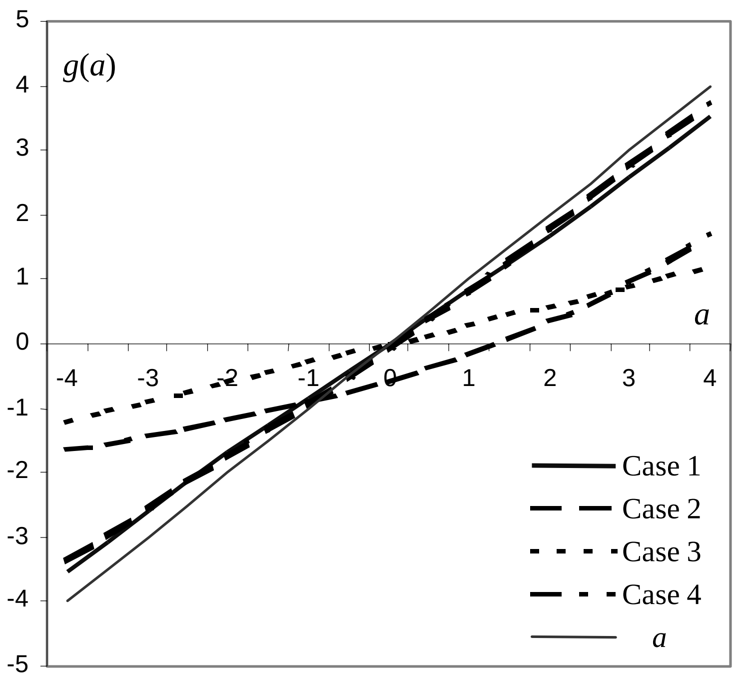

Figure 1.