Who Should Provide Public Goods?

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March 2000

Abstract: Private provision of public goods is generally inefficient and government provision must be financed by distortionary taxation. Given these observations, how should provision be organized? The paper investigates the interaction between tax instruments, preferences and provision. It is shown that purely government provision is rarely optimal. Instead, provision should either be entirely private, with appropriate price intervention, or combine public and private provision.
Acknowledgements: Thanks are due to Richard Cornes, Todd Sandler and seminar audiences at Exeter, Iowa State, UBC, Toronto, Western Ontario, QMW, the APET meeting in Tuscaloosa and the Public Finance Weekend at Essex.

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1. Introduction

This paper asks a simple question: who should provide public goods? It may seem surprising that we do not yet have a satisfactory answer to what is one of the core issues of public policy. We know the quantity of public goods that should be provided (Samuelson (1954)), how successful private provision would be under a variety of motives for giving (Andreoni (1990), Cornes and Sandler (1996)), and how taxes interact with provision when either the government (Atkinson and Stern (1974)) or private individuals (Itaya, de Meza and Myles (1996)) provide the public goods. But there has been no analysis of how private and public provision should be best combined. This is a significant gap in our understanding of a major policy question.

In contrast to the paucity of theoretical work, the issue of who should provide public goods is one which has received considerable airing in political circles. There has been increasing discussion of the extent to which it is possible to rely on private individuals to provide public goods for themselves and a retrenchment of the government from the provision of numerous goods and services. This has left a situation in which some public goods are provided entirely by the government, some by private contributions alone and others through a mix of the two. For example, medical research is supported by a combination of private and public finance.

Defence provision is almost entirely government finance. Public sector broadcasting in the US is supported by voluntary contributions but is publicly financed in the UK. As well as investigating the theoretical problem, an analysis of who should provide may also shed light on the reasons underlying these differences in financing methods.

For a pure public good, the Samuelson rule provides a characterization of Pareto optimal allocations. Such allocations represent the first-best and are a guide to the quantity that should be provided but say little about the best mechanism for provision. Except in special cases when the government is the only provider of public goods, Samuelson-rule allocations will only be attainable if the government is able to levy optimal lump-sum taxes. As is well known from the work of Mirrlees

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1 Research into the treatment of illnesses such as cancer is financed by government grants, profit-making companies and charities. A successful treatment may be excludable but is a public good in that a breakthrough is non-rival and all members of the population may potentially benefit.

2 Such as when incomes are fixed so that linear taxes can be non-distortionary.
(1986), implementation of such taxes face insurmountable problems. At the opposite extreme, it is also known that total reliance on private provision will lead to an allocation that is not Pareto optimal. This result was initially derived in models (such as Chamberlin (1974) and McGuire (1974)) that assumed individuals cared only about private consumption and the quantity of the public good. With such preferences, a number of very clear-cut conclusions such as the irrelevance of income redistribution and the fact that in a large economy there is one-to-one crowding out of private by public contributions. However, empirical and experimental evidence conflicts with these findings. This has lead to the development of the warm glow formulation of preferences in which individuals derive utility from the act of giving itself. This model still retains the inefficiency but eliminates most of the contentious results. It does this, though, at the expense of not providing any clear-cut findings.

In the existing literature Atkinson and Stern (1974) provide a characterization of optimal commodity taxes when it is assumed that the government is the sole provider of a public good. In the same situation, Boadway and Keen (1993) analyze the optimal structure of income taxation. At the other extreme, Itaya, de Meza and Myles (1996) consider both sets of tax instruments when public goods are provided by private contributions alone. The important feature of the latter analysis are the corner solutions for individual choices and the consequent motive that emerges for inequality-worsening income redistribution. The role of corner solutions will become apparent in Section 3.

As we have already noted, what is missing is any integration of these alternative strands. This is what we attempt in the present paper. In general terms the underlying issue can be introduced as follows. We know that private contributions generally lead to an inefficient outcome even with warm glow preferences. We also know that the government can only finance public provision through distortionary taxes. Simple intuition would then suggest that the optimal policy will balance these effects and lead to a mixture of public and private provision. It may further be expected that in some cases one of these effects may more than offset the other so that a corner solution with only private or only public provision.

Unfortunately, the analysis does not prove to be this simple. It is necessary to distinguish between a number of different forms of preferences and assumptions on consumers' knowledge about the government's reaction to changes in their behaviour. These yield widely differing conclusions. In particular, we need to address the
validity of the "see-through" assumption and whether giving obtains a warm glow. And, if a warm glow is obtained, is this quantity-driven or value-driven? It is looking at these alternative cases and assessing their relevance that forms the paper's contents. Having determined the nature of the correct policy for each situation, we then discuss the relevance of each and methods for inferring from data which is most appropriate.

For the greatest part of the paper we adopt the standard model of a benevolent government that acts as correctly as possible given its limited information. This idealistic view of government may be far too generous. Indeed, a primary reason for supporting the private provision of public goods may be that the government cannot be trusted to “get it right”. It may be grossly inefficient or supply the wrong public goods. Alternatively, the government may be unable to set anything like the correct taxes or may fail to collect them effectively. We do offer comments on some of these observations below, but all of them should be borne in mind when assessing our results.

The major finding that emerges from the analysis is the extent to which it is desirable to rely on private financing of public goods. In some cases all finance should be private and in most others there should be a mix of public and private. Viewed differently, it appears to be preferable for the government to provide incentives, via the price mechanism, for individuals to voluntarily supply public goods rather than to supply the public goods itself. The common textbook perception is that the market failure resulting from the existence of public goods naturally implies government provision. This is not the case.

Section 2 discusses the ingredients of the model. Section 3 analyses provision of public goods under the see-through assumption. Non-see-through is analyzed in Section 4. Extensions of the basic model are described in Section 5. Section 6 discusses the results. Conclusions are given in Section 7.

2. Key ingredients
Here we define and discuss alternative specifications of preferences and assumptions on consumers' information about government behaviour employed in the following sections. Some suggestions about interpretations and appropriate applications are also made.

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3 This assumption is defined in Section 2.
4 The consequences of an inefficient government are developed further in Myles (2000).
To fix ideas, consider an economy with a single private good, a single public good and \( H \) consumers each with a fixed income \( M^h \). The consumption of the private good by consumer \( h \) is denoted by \( x^h \) and the contribution to the public good by \( g^h \). Government provision of the public good is denoted \( \Gamma \) and total provision, \( \Gamma + \sum_{h=1}^{H} g^h \), by \( G \). The production technology is linear and both the public and private good have a per-unit production cost of one. The consumer prices of the private and public good are \( q_x = 1 + t_x \) and \( q_G = 1 + t_G \) respectively, where \( t_x \) and \( t_G \) are the commodity taxes. At these prices, the value of the public good provision by \( h \) is \( \nu^h = q_G g^h \).

2.1 Preferences

The first distinction that needs to be drawn is between different forms of preferences. As will become clear, the nature of preferences is of major importance in determining the properties of equilibrium and optimal policy.

2.1.1 Instrumental preferences

The early literature on the private provision of public goods focused on the case in which consumers cared about consumption of the private good and the total provision of the public good. In this case, the utility function can be written

\[
U^h = U^h(x^h, G).
\]  

The form of preferences in (1) will be termed instrumental preferences in what follows. This terminology is adopted because the consumer’s utility is derived solely from the total quantity of the public good and there is no benefit per se from the level of individual contribution.

The implications of this form of preference have been extensively studied and surveys of the literature can be found in Cornes and Sandler (1996), Itaya, de Meza and Myles (1999) and Myles (1995). The essential consequence of this formulation is that the Nash equilibrium reached in the absence of any government intervention is not Pareto optimal. Furthermore, the equilibrium is invariant to reallocations of income that do not alter the set of contributors, in large populations any government provision crowds-out private provision on a one-for-one basis and, if there are income differences, only the richest consumer contributes. Empirical and experimental

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5 A general equilibrium interpretation of the model would have \( M \) as a labour endowment which is supplied inelastically,
evidence has provided conflicting evidence on whether these results hold in practice. These have lead to questions about the appropriateness of this model.

These observations point to the conclusion that instrumental preferences lead to private provision only in small populations, say within the family. This statement is based upon the observation that a positive level of provision is only arise likely to arise if a contributor captures a substantial fraction of the benefits. This condition is easily met when the population is small but only the richest can be non-negligible when the population is large. It is this property that leads to the conflicts between this model and data, since it is an empirical fact that charitable donations are significant for many consumers even in large populations. Instrumental preferences therefore have a role to play in some applications of the private contribution model. As a representation of the motivation for the bulk of personal charitable donations, they are of questionable validity.

2.1.2 Warm glow preferences
One response to these difficulties has been the introduction of warm glow preferences. With such preferences, a contribution to the public good generates a utility benefit from both the addition to the quantity of the public good and from the act of charity itself. Essentially, contributing makes consumers feel good about themselves and generates a “warm glow” of self-approval. Such preferences ensure that contributions remain positive in large populations since the warm glow is essentially a private good and remains significant even if the addition to the stock of public good is negligible. Furthermore, neither the crowding-out or invariance results apply in an economy with warm glow preferences. Despite this, the equilibrium will not in general be Pareto optimal since the externality effect of contributions is not taken into account by consumers.

The form of warm glow introduced by Andreoni (1990) can be written as

$$U^h = U^h(x^h, g^h, G),$$

(2)

where the extent of the glow depends on the quantity of the public good contributed. We choose to term this the quantity warm glow. An obvious alternative to (2) is the value warm glow given by

6Another has been to consider alternative forms of conjecture in the game played between the consumers. This line of reasoning does not appear to have been successful, see Myles (1995) for further discussion.
In the preferences described by (3), it is the market value of the contribution that is relevant for the warm glow whereas in (2) it is what is purchased with the contribution that counts.

The two formulations of the utility function suggest rather different motivations for donations. According to (2), what matters to the individual is the benefit to others reflecting something of a utilitarian outlook. In contrast, the preferences represented by (3) are more self-centred in embodying the view that making a sacrifice for others is appropriate irrespective of the practical good it does. There is further discussion of these issues in Section 6.

2.2 See-through

A further issue is the extent to which consumers take into account the implications of their choices on government revenue and, hence spending, decisions. In a large economy it is reasonable that if a consumer buys more of a highly taxed good they ignore the feedback arising from the fact that more of the public good will be provided by the government or else some tax will be cut or subsidy raised. In a small economy rational consumers will anticipate significant feedbacks.

When consumers anticipate a significant reaction by the government, we say that the see-through assumption applies (see Boadway, Pestieau and Wildasin (1989)). In a small economy with a government that is committed to providing the public good, the application of economic rationality makes it hard to avoid concluding that this assumption must be satisfied. Despite this, it is not impossible to conceive of a situation in which consumers act in ignorance or else feedback effects are small. In such cases we say non-see-through holds. When a large economy is considered, as in the Mirrlees (1971) model of income taxation, the see-through assumption is not an issue since the effect of a change in action by a single consumer has a negligible effect upon the government’s behaviour. In a large but finite economy, non-see-through is approximately valid in the same way the competitive assumption can only be approximately true in a finite economy.

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7 Of course, this is not full utilitarianism even in reduced form due to the asymmetric treatment of the donors contribution.

8 The issue of see-through is circumvented in Diamond and Mirrlees (1971) by the assumption that government revenue is spent on a good which does not directly affect utility.
As will be seen below, the policy choice that arises when the see-through assumption is imposed is quite distinct from that when it is not. This shows that the choice of which to adopt is not an inconsequential modelling issue. In assessing the results we derive, it is worth holding in mind the suggested settings for each.

3. Provision with see-through
Throughout we adopt the standard assumption that consumers are engaged in a Nash contribution game with each maximizing utility taking the contributions of the others as fixed. Although we have noted inconsistencies between some of the models using this assumption and the data, it appears to be the structure of preferences that is at fault, not the underlying nature of the game. Where we differ from existing treatments is that we permit the government to levy taxes and to supply the public good - if it chooses to do so. The intention is to determine the optimal mix of government and private contribution to the public good and to characterize optimal pricing policy. In contrast to much of the tax literature, the latter is given lower prominence than the division of funding.

3.1 Instrumental preferences
With the see-through assumption and instrumental preferences, it is clear that Bernheim's (1986) equivalence result applies. That is, if all consumers contribute towards provision of the public good, the equilibrium of the provision game is independent of any tax system which does not change the set of contributors. To show this, denote the tax on income by $T(M^h)$ so the budget constraint of $h$ is

$$M^h - T(M^h) = q_x x^h + q_G g^h.$$  \hspace{1cm} (4)

Government provision of the public good is

$$\Gamma = \sum_{h=1}^{H} \left[ T(M^h) + [q_x - 1] x^h + [q_G - 1] g^h \right].$$ \hspace{1cm} (5)

Using (4) and (5), total provision of the public good is

$$G = \Gamma + \sum_{h=1}^{H} g^h = \sum_{h=1}^{H} \left[ M^h - x^h \right].$$ \hspace{1cm} (6)
In response to a change in tax rates, the consumers can reallocate their expenditures between the public and private goods so as to keep $x^h$ and $G$ constant. The equilibrium established under one tax system can then be maintained under any other.

This equivalence only holds if all consumers are contributors before and after any tax change. To see this, assume that some consumers do not contribute and choose the labelling of consumers so that $g^h = 0$ for $h = 1, \ldots, \eta$. Then

$$G = \sum_{h=1}^{\eta} T(M^h) + \sum_{h=\eta+1}^{H} M^h - \sum_{h=1}^{\eta} x^h + q_h \sum_{h=1}^{\eta} x^h = \sum_{h=1}^{H} [M^h - x^h], \quad (7)$$

as before. What is now different is that the consumers who do not contribute cannot maintain the constancy of $x^h$ as the tax system changes. For $h = 1, \ldots, \eta$,

$$x^h = \frac{M^h - T(M^h)}{q_x}, \quad (8)$$

which is determined directly by the tax system so the Bernheim equivalence result breaks down when there are non-contributors. This shows immediately that if tax policy is to affect resource allocation, there must be some non-contributors before or after the policy change.

Following these preliminaries, it is possible to analyze the question of provision. To make the issues as clear as possible, assume that all consumers have identical preferences. Now let the division between contributors and non-contributors be temporarily fixed. Using (7) and (8), the objective function of a contributor in the Nash game can be written as

$$\max_{\{x^h\}} U \left( x^h, \sum_{h=1}^{H} M^h - \sum_{h=1}^{\eta} x^h + \sum_{h=\eta+1}^{H} x^h \right). \quad (9)$$

The equilibrium of the game is then a set of consumption levels for the contributors given by

$$x^{h'} = x \left( \sum_{h=1}^{\eta} \frac{M^h - T(M^h)}{q_x} \right), \quad h' = \eta + 1, \ldots, H. \quad (10)$$
It should be noted that in equilibrium consumption levels are the same for all contributors - some further consequences of this observation are explored in Itaya, de Meza and Myles (1997).

From these calculations, it can be seen that the equilibrium is independent of $q_G$. In fact, with this formulation the only role for $q_G$ is to determine the division between government and private provision with the total level of provision being independent of this division. What lies behind this is that as $q_G$ is varied (holding $q_s$ constant), contributors to the public good keep their expenditure upon it constant since they reason that the share of taxation in the price becomes government provision. In addition, changing $q_G$ does not affect the set of contributors.

For the remainder of this section, attention will be focused upon linear tax systems only. A discussion of the role of nonlinear taxation is given in Section 5.1. With linear taxes the argument of the function in (10) can be written

$$\frac{M^h - T(M^h)}{q_s} = \frac{M^h[1 - \tau]}{q_s} = \frac{M^h}{q_s},$$  \quad (11)

where $\tau$ is the tax rate on incomes and $q_s = q_s(1 - \tau)$. It follows from (11) that it is sufficient to focus upon taxes on the private good alone.

Bearing this in mind, assume that the tax on the private good is denoted by $t$, so $q_s = 1 + t$, and that the public good is untaxed so $q_G = 1$. The decision problem of the government is to choose the tax rate, $t$, to maximize a utilitarian social welfare function.\footnote{In particular it is shown that utility levels are equalized despite income differences, so private provision leads to an egalitarian outcome.} In so doing, the government must take into account the fact that the decision of a consumer on whether or not to be a contributor is endogenously determined by the tax rate. As the tax rate changes, so does the set of contributors. To describe the solution to this optimization, the approach is taken of characterizing what happens as the tax rate is raised from zero upwards. A distinction has to be drawn between the case in which all consumers contribute when the tax rate is zero and that in which some are non-contributors at all tax rates. The analysis begins with the former.

Choose the labelling of the consumers so that $M^1 < M^2 < \ldots < M^H$.\footnote{The assumption that the social welfare function is utilitarian is not critical.}
Lemma 1. Assume that $g^h > 0$ for all $h$ when $t = 0$. Then there is a tax rate $t^1 > 0$ such that:

(i) welfare is constant for $t \in [0, t^1]$;
(ii) consumer 1 ceases contributing at $t^1$;
(iii) the right-derivative of social welfare (with respect to $t$) is positive at $t^1$.

Proof. Since all consumers are contributors when the tax rate is zero, the equivalence result applies for the range of taxes up until one of the consumers becomes a non-contributor. Let the tax rate at which this happens be denoted $t^1$. Then it follows that social welfare is constant on the range $[0, t^1]$. It is straightforward to show that the first consumer to decide to become a non-contributor is the lowest-income consumer. To see this, note that with all contributing, the solution to the Nash game is dependent on $t$ alone, so $x^h = x(t)$, again employing the fact that consumption levels are the same for all contributors. Non-contribution begins when $[1 + t]x(t) = M^h$, which must happen first for the lowest-income consumer.

When consumer 1 has ceased contributing, the social welfare function is

$$W = U \left( \frac{M^1}{1 + t}, \frac{tM^1}{1 + t} + \sum_{h=2}^H M^h - \sum_{h=2}^H x^h \right) + \sum_{h=2}^H U \left( x^h, \frac{tM^1}{1 + t} + \sum_{h=2}^H M^h - \sum_{h=2}^H x^h \right).$$

(12)

Taking the derivative of (12) with respect to $t$, using the envelope theorem for individual choice and evaluating at $t^1$ gives

$$\frac{\partial W}{\partial t} \bigg|_{t^1} = [H - 1]U_G \left[ \frac{M^1}{1 + t} \right] - [H - 1]x'.$$

(13)

The term $x'$ can be evaluated by returning to the Nash game being played. The contributors each solve

$$\max_{\{x^h\}} U \left( x^h, \frac{tM^1}{1 + t} + \sum_{h=1}^H M^h - \sum_{h=1}^H x^h \right).$$

(14)

Since the solution is symmetric between the contributors (so $x^h = x$ all $h$), the first-order condition can be used to calculate

$$x' = -\frac{M^1}{[1 + t]} \left[ \frac{U_{sG} - U_{GG}}{U_{ss} - HU_{sG} + [H - 1]U_{GG}} \right].$$

(15)
Substituting (15) into (13) shows that
\[
\frac{\partial W}{\partial t} = \frac{[H - 1]U_G M^1}{[1 + t]^2} \left[ \frac{U_{xx} - U_{xG}}{U_{xx} - HU_{xG} + [H - 1]U_{GG}} \right],
\] (16)
which is positive under standard assumptions on preferences. ||

Consequently, the gradient of social welfare as a function of the tax rate is zero until \( t^1 \) and then is kinked upwards at this tax rate. The existence of this kink shows that social welfare is greater for tax rates immediately above \( t^1 \) than it is for those below. Hence it is always optimal to set a tax rate that forces the lowest income consumer to become a non-contributor. The argument of Lemma 1 can be extended to the more general conclusion of Lemma 2.

**Lemma 2.** There is a kink in the welfare function at each tax rate at which an additional consumer becomes a non-contributor.

**Proof.** This is a direct generalization of the calculations in the proof of Lemma 1. ||

The consequence of Lemma 1 is that the lowest-income consumer should be forced into being a non-contributor. Lemma 2 shows that the social welfare function is kinked upward each time a critical tax rate is reached at which a further one becomes a non-contributor. However, this does not imply they should all be forced into non-contribution. The reason for this is that the gradient prior to the kink may be negative \( i.e. \) social welfare has reached a maximum between a pair of kinks. Hence the optimum policy is to crowd out some consumers with government provision but not necessarily to crowd out all. Provision is either a mix of public and private or entirely public.

These possibilities cannot be explored fully at the level of generality used so far. Instead a numerical simulation is used below to illustrate the working of the model. Before proceeding, it is worth noting one further result. Even though incomes are fixed, the first-best cannot be attained in this model. This arises because the marginal rates of substitutions for those consumers crowded out cannot be manipulated by the use of a single tax instrument in a sufficient way to obtain efficiency. Looking somewhat ahead, it will be seen that this result is a product of the
see-through assumption. Without this, the first-best can be achieved with instrumental preferences.

Consider now the situation in which some consumers do not contribute even at a zero tax rate. There is now no range of tax rates where social welfare remains constant. Otherwise, it remains true that there is an upward kink each time a tax rate is reached at which a consumer ceases to contribute. However, a zero tax rate may be optimal. Consider a situation in which consumer 1 is the only non-contributor at a zero tax rate, then

\[
\frac{\partial W}{\partial t}
= M^1[U_G(t) - U_x(t)] + \frac{(H - 1)U_G M^1}{1 + t} \left[ \frac{U_{xx} - U_{xG}}{U_{xx} - H U_{xG} + (H - 1)U_{GG}} \right].
\]  

(17)

The second term in (17) is positive as already noted but the first, which relates to the marginal utilities of consumer 1, is negative since consumer 1 obtains a strictly higher marginal utility from consumption of the private good. It may therefore be possible for welfare to be decreasing as the tax rate moves away from zero. A corner solution with private provision being optimal is therefore possible if income disparities are sufficiently great that there are a significant number of non-contributors in the absence of taxation. Since this result is dependent on income disparities, the role that redistributive income taxes may play is important discussion of this is given in Section 5.1.

This is as far as results can go using the general structure so we turn to a numerical example to obtain further insights. Consider two consumers with preferences represented by

\[
U = \log x^h + \log G
\]

(18)

Letting \( M^2 = \rho M^1 \), \( \rho > 1 \), then both will contribute with zero taxes if \( \rho < 3 \). With only consumer 2 contributing, social welfare is

\[
W = 4\log[1 + \rho]M^1 - \log 4 + 3\log \frac{3}{8},
\]

(19)

and, with no consumers contributing, is

\[
W = 2\log[1 + \rho]M^1 + \log M^1 + \log \rho M^1 + 4\log \frac{1}{2}.
\]

(20)
Social welfare is illustrated as a function of the tax rate in Figure 1a for $M^2 = 1.5M^1$ and for $M^2 = 2.5M^1$ in Figure 1b. These two cases illustrate the two possible outcomes. In Figure 1a, welfare is maximized when the government is the sole supplier of the public good. In this case it should raise its provision until both consumers are crowded-out from contributing. Conversely, Figure 1b illustrates the possibility that it may be optimal to have some private contribution. The low-income consumer, 1, is crowded-out but the high-income consumer continues to contribute. Using the welfare functions given in (19) and (20), it can be seen that the break between the two cases occurs when $\rho = 2.3$. These results show that, for this specification at least, a mix of public and private contribution can be justified by sufficient income inequality. With limited income inequality, all provision should be by the government.

**Figure 1:** Social Welfare

What is most important to be concluded from this section is how the see-through assumption supports government provision. The ability of consumers to see what the government is doing allows them to undo its endeavours but only if all are contributing. The increase in the tax rate that induces some to be non-contributors then makes it impossible for them to fully offset government behaviour and this allows the government to influence the economy. Therefore, government provision of the public good is justified. Since some crowding-out must occur, the invariance does not apply and the tax system will be distortionary. For some specifications, the optimal tax rate on the private good will be such as to crowd-out all private provision. The reason that this does not occur in all cases is distributional: once a consumer is crowded out their utility falls as the tax rate is raised. Hence the optimal tax rate trades off utility loss to the low income against utility gains (through increased public good) for the higher income.

Because of the see-through assumption, the price of the public good does not affect whether contribution takes place or not. When the optimum policy is to crowd-out all private provision, the price of the public good plays no role. For some income distributions public goods may be supplied by the higher income consumers. If so, all
that the price of the public good does determine the mix between public and private contributions; total provision is independent of price.

3.2 Warm Glow
The warm glow model has received increasing attention since its introduction by Andreoni (1990). As noted in Section 2, the important property of this model is that a contribution to the public good generates a private utility benefit, in addition to any return derived from increased provision of the public good. This private benefit is motivated through feeling good about the act of contributing. The important property of the model is that it gives a reason for contribution even in a large population where the marginal effect of an individual contribution is negligible.

Two versions of the warm glow model will be considered. The first, introduced by Andreoni, involves the donor deriving utility from the amount of public good contributed. In the second it is from the value of contribution.

3.2.1 Quantity Warm Glow
Assume that the government levies taxes upon the private and the public good. Under the see-through assumption, the derivations used above show that total supply of the public good is \( G = \sum_{h=1}^{H} [M_h - x^h] \) and the equilibrium of the private provision game is given by a demand for the private good of the form \( x^h = x^h(q_s, q_G) \). The important aspect of the warm glow is that, in contrast to the model without, non-contribution to the public good will not necessarily arise when income differentials increase. The reason for this is the warm glow is essentially a private good and which a consumer may not wish to see become zero. Although contributions could be zero, it will be assumed that they are not. A formal justification for this position would be to assume that the utility function satisfies the Inada conditions.

With quantity warm glow and see-through, social welfare can be written as

\[
W = \sum_{h=1}^{H} U \left( x^h(q_s, q_G) \right) \sum_{h=1}^{H} [M^h - x^h(q_s, q_G)] \frac{M^h - q_s x^h(q_s, q_G)}{q_G},
\]

(21)

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11 We assume that the warm glow does not extend to the public good provision financed by the taxes paid by an individual.
The nature of the policy resulting from the maximization of (21) can be derived by considering the effect of a variation in the price of the public good upon social welfare. Differentiating (21) and employing the envelope theorem,

\[
\frac{dW}{dq_G} = -\sum_{h=1}^{H} U_G(h)\sum_{h'=h}^{H} \frac{\partial x^{h'}}{\partial q_G} - \sum_{h=1}^{H} U_G(h) \frac{M^h - q^h}{q_G^2} < 0, \tag{22}
\]

if \(\frac{\partial x^h}{\partial q_G} > 0\). Consequently, as long as public good provision is positive, a reduction in the consumer price of the public good always raises welfare.

The mechanism that is operating here is that a subsidy to the public good substitutes private contributions for public. This generates more of a warm glow. The resulting reduction in revenue for the government implies that it makes a smaller contribution to the public good. However, total supply of the public good actually rises since consumption of the private good falls. This result is a variant on that found in Andreoni (1990).

To incorporate the above reasoning into an optimal policy, account must be taken of the fact that the government’s budget must be balanced. Clearly, since more government supply has no direct effect on welfare but only an indirect cost via the higher taxes needed to finance it, there should be no government supply of the public good. So, with quantity warm glow and see-through, all public goods should be privately provided. The intuition behind this conclusion is that private provision generates both a private and public return whereas public provision does not provide the private return. It is therefore always best to use the tax system to encourage private provision rather than to fund public provision.

The optimal prices can be shown to satisfy

\[
q_s - q_G = \sum_{h=1}^{H} g^h \sum_{h'=h}^{H} \left[ q_G U_G(h) \frac{\partial x^{h'}}{\partial q_s} + x^h U_s(h) \right] - \sum_{h=1}^{H} x^h \sum_{h'=h}^{H} \left[ q_G U_G(h) \frac{\partial x^{h'}}{\partial q_G} + g^h U_s(h) \right] - \sum_{h=1}^{H} \sum_{h'=h}^{H} \left[ q_G U_G(h) \frac{\partial x^{h'}}{\partial q_s} + x^h U_s(h) \right] - \sum_{h=1}^{H} \sum_{h'=h}^{H} \left[ q_G U_G(h) \frac{\partial x^{h'}}{\partial q_G} + g^h U_s(h) \right]. \tag{23}
\]
Although (23) is not particularly informative in the general case, consider what happens if there are two consumers with identical incomes. The demands, demand derivatives and marginal utilities are then equal for both consumers and (23) reduces to

$$q_x - q_G = \frac{U_x q_G}{U_g}.$$  

(24)

Using the first-order condition for consumer choice, $U_x - U_G - \frac{q_x U}{q_G} = 0$, (24) becomes

$$U_x - U_G - 2U_g = 0,$$  

(25)

which is the optimality condition for this economy. Therefore, if there are no income differentials, the commodity taxes can achieve efficiency even if there is a warm glow from contributions.

3.2.2 Value warm glow

One interpretation of the warm glow is that it reflects a feeling of self-congratulation for acting in a selfless way. Following this view, the actual level of purchases of public good made with any contribution need not be relevant; all that matters is the fact that the contribution was made. This reasoning leads into the value warm glow model where it is the value of the contribution to the public good that enters utility. Expressed alternatively, it is the value of what is sacrificed to finance the charity rather than what is received that generates a warm glow. This formulation therefore suggests that gross donations will be insensitive to their tax treatment. It will now be shown how this provides very different policy implications to the quantity warm glow model.

Under the see-through assumption, the total quantity of the public good is again

$$\sum_{h=1}^{H} \left[ M^h - x^h \right].$$

Using the budget constraint $M^h = q_x x^h + v^h$, preferences with the value warm glow, with the value denominated in the same units as income, can be written as

\[12 \text{ A further possibility would be that it is sacrifice as a proportion of income, } M^h/v^h, \text{ that is the determinant of utility. This would capture a number of ancient charitable traditions but is not pursued here for reasons of space.} \]
The nature of the optimal policy can be seen immediately from (26). The Nash game between the consumers will result in equilibrium choices that depend only (taking incomes as given) upon \( q_s \): \( x^h = x^h(q_s) \). This is because contributions now have unit price. Hence, of the policy variables \( q_s \) and \( q_G \), only \( q_s \) appears in the welfare function. The price of the consumption good is therefore chosen to maximize welfare. As far as this optimization is concerned, the price of the public good is a matter of indifference. The only role it plays is to determine the division of public good provision between the public and private sector. Welfare is independent of this division. This leads to the conclusion that with value warm glow preferences and see-through, welfare is independent of the price of the public good and the division of provision between the private and public sectors.

Maximizing social welfare, the optimal price for the private good satisfies the equation

\[
-x^h \sum_{h=1}^{H} U_s(h) - \sum_{h=1}^{H} \sum_{j=1}^{H} U_q(h) \frac{\partial x^h}{\partial q_s} = 0 ,
\]

so that it balances the decreased private contribution that is possible as \( q_s \) rises with the increased total provision of the public good as private consumption falls.

This conclusion is worth contrasting to that for the quantity warm glow. In that case, it was the price of the public good that was of utmost importance. Here it is the price of the private good that is fundamentally important with the price of the public good an irrelevance. Given such divergent findings from slight variations in the form of preferences, it becomes important to consider which is the most relevant description. More is said about this later.

4. Non-see-through
Non-see-through implies that consumers take no account of how their actions affect the government budget constraint. This may be because they are “small” relative to the economy and non-see-through is approximately true in the way that competitive behaviour is approximately true in a large economy. Alternatively, it may hold simply through the ignorance of the consumers or because the government can run a
budget deficit or surplus in the short run. Whichever is the case, the analysis remains the same.

The analysis of non-see-through bears a closer relation to the standard literature on optimal taxation than does see-through. This is not surprising since the assumption also underlies that analysis – though it is never made explicit. In any model of taxation a change in consumer behaviour that alters tax payment will affect government revenue. Only in a continuum model can the effect of a consumer be truly negligible. From this perspective, the analysis of see-through can be viewed as providing insight into the issues that would arise if that assumption were not imposed.

4.1 Instrumental Preferences
With instrumental preferences, the decision problem facing a consumer is

\[
\max_{\{x^h, g^h\}} U\left( x^h, \Gamma + g^h + \sum_{j=1}^{H} g_j^j \right) \quad \text{s.t.} \quad M^h = q_x x^h + q_G g^h, \quad (28)
\]

where \( \Gamma \) is taken as given. The Nash equilibrium choice of contributors is determined as

\[
x = x(q_s, q_G, \Gamma). \quad (29)
\]

This is common for all contributors. Non-contribution arises when

\[
M^h < q_x x(q_s, q_G, \Gamma). \quad (30)
\]

For the consumers for which (30) is satisfied, the level of private good demand is given by

\[
x = x(q_s). \quad (31)
\]

It is interesting to analyze how the contribution decision depends upon the prices. To this end, assume that demand in the contribution case (29) is independent of \( \Gamma \) and \( q_G \), as for example it would be with Cobb-Douglas utility. If demand was unit elastic the non-contribution decision would then be independent of \( q_s \) and the set of contributors would be determined by income alone and unaffected by any change in prices. Alternatively, if demand is inelastic then a rise in \( q_s \) would make the set of contributors smaller. The converse would occur if demand were elastic. Since
dependence upon $q_G$ and $\Gamma$ would allow an even broader range of outcomes, these observations show that there is no simple relationship between prices and non-contribution.

Returning to (29) the choice for a contributor is dependent upon both prices. This allows the government to exert greater control over the economy by combining taxes on both the private good and the public good.

**Theorem 1.** With instrumental preferences and non-see-through, the Samuelson rule can be attained.

*Proof.* Define $MRS^h \equiv \frac{U_x}{U_y}$. The assumptions on technology ensure that the marginal rate of transformation ($MRT$) between the public and private good is equal to 1. The Samuelson rule is then

$$\sum_{h=1}^{H} MRS^h = MRT = 1.$$  \hfill (32)

It will now be shown that this can be achieved whether or not there are any non-contributors.

Assume that there are no non-contributors. Since the optimization of each consumer involves setting $MRS^h = \frac{q_G}{q_x}$, choosing the price ratio so that

$$\frac{q_G}{q_x} = \frac{1}{H},$$  \hfill (33)

ensures that (32) is satisfied.

If there are some non-contributors, the argument is modified as follows. Each $MRS^h$ is a continuous function of the consumer prices of the two goods. This applies whether the consumer is a contributor or not and at the point at which non-contribution begins (although $MRS^h$ is kinked as a function of prices at the non-contribution point). Assuming that $\lim_{x \to 0} U_x(x, G) = \infty$ and $\lim_{x \to \infty} U_x(x, G) = 0$, then $\sum_{h=1}^{H} MRS^h$ is zero as $q_x \to \infty$ and infinite as $q_x \to 0$ with $q_G$ adjusted to maintain a given level of revenue for the government. Since $\sum_{h=1}^{H} MRS^h$ is the sum of continuous functions, it is itself continuous and there must exist a value of $q_x$ (and an
implied value of \( q_G \) where it is equal to one. At this point the Samuelson rule is satisfied. ||

This theorem has straightforward implications for the division of provision that can be determined as follows. Set prices as described in (33). If all consumers are contributors at these prices, then the Samuelson rule is attained with purely private provision\(^{13}\). Conversely, if some are non-contributors at these prices the prices need to be adjusted as described in the second part of the proof. Since an increase in \( q_x \) always reduces the welfare of the non-contributors, it will never be beneficial to raise a positive level of revenue. So again all provision will be private.

4.2 Warm glow
The first modification of these assumptions is to move to warm-glow preferences. Only the quantity warm glow will be considered. The reasoning for value warm glow is similar but does not yield such precise conclusions.

With quantity warm glow, the optimization problem of consumer \( h \) is

\[
\max_{\{x^h\}} U \left( x^h, \Gamma + \sum_{j=1}^{n} \frac{M^j - q_j x^j}{q_G} \right). \tag{34}
\]

This generates, via the Nash game, a demand function for the private good and a supply of the public good with the forms

\[
x^h = x^h(q_x, q_G, \Gamma), \quad g^h = g^h(q_x, q_G, \Gamma), \tag{35}
\]

and a level of social welfare

\[
W = \sum_{h=1}^{n} U \left( x^h(q_x, q_G, \Gamma), \Gamma + \sum_{j=1}^{n} g^j(q_x, q_G, \Gamma), g^h(q_x, q_G, \Gamma) \right). \tag{36}
\]

The optimal policy is determined by maximizing (36) subject to the constraints that

\(^{13}\) There is one exception to this conclusion. If all consumers are identical and there is a price for the public good above which they will not contribute, then the Samuelson rule can be attained by setting the public good price above this and financing the optimal level of provision through a tax on the private good. This solution cannot apply when there are income differentials.
Simple economic reasoning suggests that the solution to this optimization should have $\Gamma = 0$: a reduction in government provision allows subsidization of private provision which raises the return from the warm glow. This is the same reasoning as used in Section 3.2.1. Actually establishing this for the general case is a difficult task but the following result can be proved.

**Theorem 2.** Assume that the private good and contributions to the private good are gross substitutes and that the public good is normal. Then, if there are two identical consumers, there will be no public provision and the first-best is attained.

**Proof.** Using the first-order conditions for consumer choice, the necessary conditions for $q_s$ and $q_G$ from the optimization in (36) and (37) are

$$
U_G q_s \frac{\partial g}{\partial q_G} - U_x g + \lambda \left[ g + \left[ q_G - q_s \right] \frac{\partial g}{\partial q_G} \right] = 0, \\
U_G q_s^2 \frac{\partial g}{\partial q_s} - U_x \left[ M - q_G g \right] + \lambda \left[ \left[ M - q_G g \right] + q_s \left[ q_G - q_s \right] \frac{\partial g}{\partial q_s} \right] = 0.
$$

Now assume that $\Gamma = 0$. Eliminating $\lambda$ between these conditions and using the budget constraint gives

$$
\left[ U_x \left[ q_G - q_s \right] + U_G q_s \left[ q_s - 1 \right] \frac{\partial g}{q_s} + \left[ q_G - 1 \right] \frac{\partial g}{q_G} \right] = 0.
$$

The gross substitutability and normality conditions imply that

$$
\left[ U_x \left[ q_G - q_s \right] + U_G q_s \right] = 0,
$$

or

$$
q_G = q_s \left[ 1 - \frac{U_G}{U_s} \right].
$$

Substituting this into the necessary conditions gives
\[ \lambda = U_x. \] 

(43)

Using these results the necessary condition for \( \Gamma \) is

\[ 2U_g - U_s + \mu = 0. \]

(44)

Using the optimal prices in the consumer's first-order condition shows that

\[ 2U_g - U_s + U_g = 0, \]

(45)

which is the necessary condition for the first-best outcome. Hence \( \mu = U_g > 0 \) so the assumption that \( \Gamma = 0 \) is consistent with the Kuhn-Tucker conditions.

Several points in the theorem deserve comment. Firstly, the attainment of the first-best is a consequence of the assumption of fixed income levels. If this was relaxed the taxes would distort the labour/leisure choice and potentially disrupt this conclusion. Secondly, a similar theorem cannot be established if there are income differences. However, the proof is easily generalized to any number of identical consumers. To see this, note that the necessary condition for individual choice is

\[ -U_s \frac{q_g}{q_s} + U_G + U_g = 0, \]

(46)

and that for the social optimum is

\[ -U_s + HU_G + U_g = 0. \]

(47)

Equating (46) and (47) determines a price ratio

\[ \frac{q_s}{q_G} = \frac{U_s - [H - 1]U_G}{U_s}, \]

(48)

that ensures the Samuelson rule is met with purely private provision.

5. Some extensions
The models that have been analyzed in the previous sections have been intentionally simple. This has emphasized the most important aspects of the problem without
introducing issues that detract attention from these. This section briefly considers a number of potential modifications to the basic model.

5.1 *Endogenous income and income taxation*

Two additional issues are raised when incomes are made endogenous. Firstly, there is the interaction between the supply of labour and the tax results identified above. Secondly, endogenous income potentially makes the study of income taxation interesting. With fixed incomes, any tax would just be a lump-sum. For warm glow preferences, endogenous income does not have particularly significant consequences. The optimal characterizations of policy already given will remain valid. The incorporation of labour supply will lead to some interactions between the prices of commodities and work effort that will modify the precise expressions but will not alter the broad policy proposals. The same general comment applies to instrumental preferences under see-through.

Where labour supply is most significant is the case of non-see-through. Under the fixed income assumption taxation can achieve an efficient Samuelson-rule equilibrium. When labour supply is endogenous this is not possible except in special cases such as Cobb-Douglas utility. Instead, what emerges is a form of Ramsey rule for setting the consumer prices. In turn, this provides a potential argument for positive provision by the government. The leads to something of a paradox: if taxes are distortionary then they should be used to finance public provision, otherwise individuals should contribute. Although this line of reasoning leads to an argument for government provision, this is perhaps the least convincing of the alternative models.

Similar comments apply when non-linear income taxes are considered. The most significant effect that income taxation, and income reallocation, can have is to affect the decision upon whether to contribute or not. The reasoning behind this statement is that the potential inefficiency of private provision is a consequence of the relative prices of the commodities and not one of income distribution. As a result, since non-contribution is not an important feature of warm glow preferences, income taxation has little importance in this framework. It can be used to achieve some income redistribution but does not address the basic problem of public good provision.

Now consider instrumental preferences and non-see-through. For the sake of argument, assume that preferences are linear in labour supply. Then it can be shown
that income taxes and commodity taxes play two distinct roles (for details see Itaya, de Meza and Myles (1996)). Commodity taxes should be set to ensure that the Samuelson rule is achieved. The income tax is then employed to reach the most preferred income distribution. Obviously, this direct separability will break down when the linearity is absent (because of the links between labour supply and prices already identified) but the essentially different roles for the two tax instruments will still remain.

The limited role that can be played by income taxes is nicely illustrated by the following example. Take the quantity warm glow with see-through and let there be two consumers with identical and fixed incomes. The government finances its provision of the public good through a lump-sum tax on the two consumers. For a typical consumer, the level of utility is

\[ U(M - 0.5\Gamma - g, g, 2g + \Gamma). \] (49)

The effect of an increase in government provision is given by

\[
\frac{dU}{d\Gamma} = \left[ U_g + 2U_g - U_x \right] \frac{dg}{d\Gamma} + U_g - 0.5U_x \\
= \left[ 1 + \frac{dg}{d\Gamma} \right] U_2 - 0.5U_3, \] (50)

where the second equality follows from the envelope condition for individual choice. Evaluating \( \frac{dg}{d\Gamma} \), it can be shown that for \( \frac{dU}{d\Gamma} < 0 \) it is sufficient that \( U_{11} + U_{21} - U_{31} < 0 \). When this condition is satisfied, the financing of public good provision by lump-sum taxation will reduce welfare. The outcome would be even worse if distortionary income taxes were employed.

With see-through, the situation is somewhat different. If all consumers contribute, then the equivalence result still applies so that the equilibrium is independent of taxation. Once the non-contribution threshold is passed for the first consumer, social welfare is again kinked. Since non-contribution arises when

\[ q_x \geq z^1 - T(z^1), \] (51)

the point at which non-contribution is reached can be affected by both commodity and income taxes. This observation can be used to understand the nature of the optimal
policy. First note that a high price for the private good reduces the welfare of the non-contributors so that it is something to be avoided where possible. Since the optimum is achieved when all consumers are non-contributors, the income tax should be used to reduce income inequalities so that non-contribution is reached at the lowest private good price. Consequently, with a nonlinear income tax all provision should be by the government.

5.2 Government inefficiency
The assumption that private contributions and government expenditure are equally efficient in generating public goods is one that is open to question. The economic analysis of bureaucracy suggests a number of reasons why the government may be inefficient at turning tax revenue into public goods. Accepting these, the question arises as to how such inefficiency affects the results described above.

In the case of value warm glow and see-through in which there was previously indifference between private and public provision, there would now be a clear preference for private provision. Where private financing was previously preferred, it will be even more so with government inefficiency.

For the remaining cases, the use of government financing will always be reduced and the mix will move in favour of private financing. With instrumental and see-through, this will increase the likelihood that there will be some residual private financing.

5.3 Public good differentiation
An alternative notion of government inefficiency is that it provides the wrong kind of public good. What we have in mind here is that the public good is available in a range of varieties (whether horizontally or vertically differentiated) and that the government may elect to supply a variety that is not the first choice of the consumers.

In this context, the case of vertical production differentiation seems the most relevant so that the public good is available in a range of quality levels. This opens up the possibility that consumers may choose to direct their contributions (if any) to purchases of different quality levels and public provision may be at a different quality level again. An example of this is schooling: state schools typically have higher pupil/staff ratios than private schools and poorer facilities generally.
To obtain insight into the consequences of vertical differentiation, assume that there are just two consumers. Let consumer $h$ contribute amount $g^h$ of quality $s^h$. Assume that the consumers care about the total quantity of public good provided and the (weighted) average quality and that preferences are separable. Hence

$$U^h = U\left(x^h, f\left(\frac{g^1 + g^2, g^1 s^1 + g^2 s^2}{g^1 + g^2}\right)\right).$$ (52)

Units of measurement are normalized so that the measure of quality, $s$, is also the price per unit. In the absence of taxation, the budget constraint of $h$ is then

$$M^h = x^h + s^h g^h.$$ (53)

The first-order conditions for optimization are

$$-U_x(h)\left[\frac{g^1 + g^2}{g^1 + g^2}\right] + U_G(h)f_2 = 0,$$ (54)

$$-U_x(h)s^h + U_G(h)f_1 + U_G(h)f_2 \left[\frac{g^1 s^h - s^h}{g^1 + g^2}\right] = 0, \ h = 1, 2, \ h \neq \tilde{h}.$$ (55)

Solving these conditions, it can be seen that $s^1 = s^2$. So that if both consumers contribute, they both contribute to the same quality level.

The finding that both contribute to the same quality level is a consequence of the same factors that are responsible for the invariance and crowding-out results in the standard instrumental model. The functioning of the private provision equilibrium is therefore little different to the standard analysis without the quality differential. Moreover, it also follows that the results on taxation will also be similar in the case of see-through. Welfare will remain constant until the low income consumer is crowded out and then it will begin to rise. Optimality will either be public provision alone or a mix of public and private. Without see-through, the important feature is that there are two dimensions of inefficiency of the private provision equilibrium: quantity and quality. Consequently, it will not be possible to achieve efficiency using only the a simple tax or subsidy on the public good. A non-linear pricing scheme that forces the correct quality to be chosen (by making the price of wrong qualities prohibitively high) could achieve efficiency.

5.4 Heterogenous tastes and peer pressure
Not everyone enjoys a warm glow from giving and some are consumed with fury when others are perceived not to have contributed their "fair" share. Evolution has bequeathed us a mix of emotions but distributed them across the population in unequal measure. Our analysis has only scratched the surface of the motivations for private contribution. A fuller account of the psychology underlying donation may well change some of our conclusions.

Peer pressure is undoubtedly important. How much one person contributes is influenced by the actions and reactions of others. A social-custom model of contribution may well be relevant here. In addition, contribution could also be motivated through the guilt associated with failing to do so. These could well provide arguments for public provision since enforced uniformity has its merits and can prevent the economy being trapped in a bad equilibrium. Indeed, it is difficult to believe that a modern society could provide a satisfactory flow of public goods just by the manipulation of prices though, as we argue, this could help. Evidence-based research in this area would be fascinating.

6. Discussion
The results derived in Sections 3 and 4 are easily summarized. With instrumental preferences and see-through, the optimum involves some government provision and possibly only government provision. The position is reversed with warm-glow preferences and see-through: entirely private provision is optimal (although the mix is irrelevant with value warm glow). In the case of non-see through, all provision should be private with instrumental preferences. The same is true of warm-glow preferences in some cases.

It can be readily seen from this summary that the answer to the question "Who should provide?" is highly sensitive to the specification adopted. In fact, the entire range of conclusions is possible from purely private provision being optimal through to purely government provision. Given this, the importance of determining which of these cases is valid is of significant importance.

The first point that can be addressed is the choice between the instrumental model and warm glow. The empirical and experimental evidence does not seem to support the instrumental model. But some of this evidence is inconclusive and the rejection is not complete. In fact, there may be some cases in which it is entirely appropriate. One that comes to mind is the case of federal authorities within a state contributing to a state-wide public good. The example of Bohm (1984) is worth
noting in this respect. In such cases the small numbers involved suggest that the see-through assumption is also likely to be valid. This leads to the focus upon the fact that public provision should be positive and, in some cases, mixed provision may be optimal.

When considering goods benefitting large numbers of consumers, only the warm glow model plausibly accounts for positive contributions. This as presented in the existing literature (such as Andreoni (1990)) has assumed what has been here termed the quantity form. Because of the nature of the warm glow, the value version is also a reasonable structure. This is a form that has not been considered before in the literature. These two forms of preferences have very distinct implications under the see-through assumption. With quantity preferences, then it is the price of the public good that is critically important and the government should seek to minimize this. In contrast, when it is value preferences, it is the price of the private good that matters. All that the public good price does is determine the allocation between public and private provision, and the level of welfare is independent of this division. Without see-through the outcomes are qualitatively similar with both having a mix of public and private provision.

But which form of warm glow is most appropriate? One line of argument that can give some insight into this issue is to consider contributions to charities. An important fact here is that the vast majority of individuals contribute to numerous charities. Now consider a charity with many donors so that the marginal effect of a single individual upon the total stock of public goods is negligible so the only reason for giving is some form of warm glow. The quantity warm glow can be interpreted as a concern for the recipients of charity. Let there be many charities which can be ranked by the consumer. If each consumer has a uniquely preferred charity then it follows that they will contribute only to that charity - since they are small relative to the economy, the marginal benefit of the recipients will not be affected by the contribution. Conversely, with value warm glow, the concern is with the sacrifice made. This implies that the consumer is indifferent as to how the money is divided between charities - and hence to how many charities they contribute to. In fact, a natural modification when there are many charities is that the warm glow is concave in donations so there is a strict preference for diversification. This line of reasoning provides support for the value warm glow.
An alternative way to distinguish between the two is to look for differences in response to incentives. In most tax codes contributions to charities are deductible from income taxation. With warm glow, this gives rise to the budget constraint

\[ M^h = \frac{q_x}{1-t}x^h + q_Gg^h. \]  

(56)

It can be seen from (49) that changes in the tax rate cause a substitution between the private good and contributions to the public good (as well as an income effect). For value warm glow, it can interpreted as the total sacrifice of private consumption that is relevant for the consumer. This is given by

\[ \tilde{\nu}^h = [1-t]M^h - q_xx^h = [1-t]^h. \]  

(57)

Hence the implied budget constraint is

\[ M^h = \left[ \frac{1}{1-t} \right] q_xx^h + \tilde{\nu}^h, \]  

(58)

so that a change in the tax rate causes no substitution between the choice variables. Data on contributions to charities could be used to test this prediction.

7. Conclusions
The paper has considered the combination of government and private financing of public goods under a range of assumptions. Although the precise outcome differs between these, taken as a whole they place much greater emphasis upon the value of private contributions than expected. This is not true in every case, but the only one in which purely government provision is optimal is the least empirically justifiable. Otherwise, optimality requires either completely private financing or a mix of the two. The motivation for this conclusion is that it is less distortionary particularly when consumers enjoy giving for the government to encourage private financing than it is to simultaneously crowd-out private financing and raise the revenue to provide the good itself.

The range of tax instruments has been limited to the choice of consumer prices. Our justification for this is that the inefficiency of private provision arises from pricing signals and hence these are the taxes best suited to overcome this. The role of income taxes is rather limited in this context since income distribution is not of particular significance. It can have some effects but these are minor compared to
the consequences of manipulating prices. Similarly, the extensions of the analysis that were considered did not significantly affect the results. Government inefficiency due to agency problems and administrative costs gives even more emphasis to private provision.

References


Figure 1a