Do unions reduce discrimination?
A model of Nash bargaining between a union and an employer with discriminatory tastes

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Abstract

Whilst there is a significant empirical literature on the effects of unions on pay discrimination, there is little by way of a theoretical treatment of this important topic. This paper offers a theoretical framework which integrates models of union–firm bargaining with the analysis of employer discrimination. Within the class of right-to-manage models of union–firm bargaining, we consider the bargain between a rent-maximising union and a utility-maximising employer with discriminatory tastes. The presence of a union reduces the wage gap between the different worker groups and the wage gap falls monotonically as union bargaining power increases. Increased discrimination by the firm leads the union to bargain a higher wage for the discriminated group. In contrast, positive discrimination by the union may lower the wage.

Keywords: Pay discrimination; Union–firm bargaining

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1. Introduction

There is a sizeable empirical literature on the effects of trade unions on the differences in pay which arise out of labour market discrimination. Sloane (1985) summarises studies carried out in Britain, Canada and the USA and concludes that on balance unions appear to “...change the relative wage differential in

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favour of black workers and against women in the USA (though in favour of women in Canada and Britain)". Freeman and Medoff (1984), using 1979 Current Population Survey data for the USA, find that the union wage effect is larger for nonwhites than for whites. In the UK, Main and Reilly (1992) consider the union wage effect by gender and find that the union wage mark-up is greater for women than for men. Lewis (1986) investigates the variation in the union wage gap across the US workforce and, with respect to differences by race, concludes that there is strong evidence to suggest that the union mark-up for non-whites exceeds that for whites. Taken together, the main thrust of these results suggests that the presence of unions in the economy causes a narrowing of wage differentials between white and non-white workers, at least for the USA. For the UK, unions seem to reduce pay differentials by gender — a result consistent with the more general finding that unions tend to narrow the overall dispersion of pay. Two well-known stylised facts of Scandinavian economies are their high rates of unionisation and the narrowness of their pay differences by gender. It is unlikely that these statistics are unrelated.

Against this backdrop of an extensive empirical literature there is a contrasting paucity of theoretical work on the impact of unions on the pay differences between groups when some groups are the subject of discrimination. As Lundahl and Wadensjö (1984) have observed, discrimination theories typically view wages as set by the market rather than through bargaining. In non-rigorous models unions can play either of two polar roles. Either they are bastions of white male workers controlling hiring decisions and thereby reducing employment opportunities to workers with unlike characteristics, or else they are progressive institutions motivated by the desire to equalise wages across workers with different characteristics. These descriptions are never incorporated into formal models and accordingly have never given rise to testable predictions.

The main aim of this paper is to develop a formal framework within which there is an integration of the theoretical work on employer discrimination with that on union–firm bargaining. Work in the latter area has progressed rapidly over the last decade (see Ulph and Ulph (1990) and Oswald (1985) for surveys) whilst there has been relatively little development of theoretical models of discrimination since Becker (1957) and Arrow (1973). The latters’ utility approach to employer discrimination continues to represent the standard treatment within mainstream economic analysis. The current paper focuses on a deficiency within the utility approach: that of the absence of any union with which the firm must bargain before wage and employment levels are determined. We attempt to correct for this omission by adapting the conventional union–firm bargaining model to accommodate the case in which the firm has Becker-like tastes for discrimination. Discrimination by the union, both positive and negative, is also considered but our central case focuses on a union with non-discriminatory preferences. Our main conclusion is that only weak conditions have to be satisfied for the presence of a union with bargaining power over
the wage rates paid by a discriminating firm to reduce the wage gap between the different worker groups and for increases in union bargaining power to reduce the wage gap monotonically. In particular, this is true when the union’s preferences are non-discriminatory so we can conclude that it is not the consequence of countervailing positive discrimination by the union. Amongst other results, we also find that as employer discrimination increases, the monopoly union bargains a higher wage for the group against which the firm is discriminating and as the positive discrimination of the union increases, it may actually bargain a lower wage for the group that is discriminated against.

The standard Becker model of discriminating tastes can be criticised not only for the absence of any union–firm bargaining but additionally on the grounds that (i) in the long-run discriminating firms will be driven out of business and (ii) the existence of tastes for discrimination is not explained but is merely assumed to be given exogenously. On the first of these two points, one response is to appeal to imperfect competition in the product market which allows the firm freedom to engage in discrimination. This is quite consistent with our model in which we assume that the marginal revenue product of labour is decreasing, although we do not need to specify whether this follows from falling marginal revenue or from diminishing labour productivity (or both).

On the issue of the exogeneity of discriminating tastes, it is usual for economists to take preferences as given. This is not always satisfactory, however, and in the case of discrimination it is arguable that attitudes will depend upon social norms which are themselves shaped, in part, by economic outcomes. One potentially fruitful approach to the problem of the endogeneity of discriminating attitudes would be through the economic analysis of conformity (see Jones, 1984) and social custom (see Naylor, 1989). Development along these lines is beyond the scope of the present paper but see Naylor (1993) for a further discussion.

Although discriminatory wage setting is illegal in most developed countries, different wage rates can be paid when there is occupational segregation. Additionally, as an anonymous referee observed, the law can be circumvented by the preferential promotion of favoured groups into jobs which are higher paying but involve essentially the same tasks. Indeed, this drawing of artificial differences between essentially the same tasks has received attention in the literature on discrimination. O’Donovan and Szyszczak (1988) have argued that industrial tribunals do not always appear to recognise the dangers of such artificial differences. They refer to the case of *Greendale v. Jarman and Flint Ltd.* in which an office cleaner compared her work with that of a man cleaning the warehouses. The claim was lost partly because “the office cleaners work in the comfortable surroundings of carpeted offices, very similar to the environment of one’s own home” (p. 132). Rubery and Wilkinson (1979) offer the example of a firm in which women, producing paper boxes, are paid less than men who produce cartons albeit under a similar process.
The next section describes the formal model, considers the objective of the firm and derives results required for later use. Section 3 describes the objectives of the union and presents the monopoly union outcome. Section 4 then analyses the generalised Nash bargaining solution to the right-to-manage model and presents the main results of the paper. Some of the consequences of introducing further groups of workers are considered in Section 5. Section 6 closes the paper with conclusions. All proofs are contained in the appendix.

2. The firm

The analysis of the firm is first introduced for a general utility function and the consequences of discrimination are discussed. Our approach is also contrasted to previous models in the literature. An additively separable utility function is then adopted and some comparative statics of the firm's choice are considered.

2.1. General utility

We adopt the standard Becker utility-based approach to employer discrimination in which the firm is assumed to possess discriminatory tastes towards one of the two groups of workers in its (potential) employment. Labelling the two groups of workers A and B, the firm is characterised as maximising a differentiable concave utility function, $U$, such that

$$u = U(H, B), (1)$$

where $H$ represents profits, $B$ the number of group $B$ workers employed and $U_H > 0, U_B < 0, U_{HH} < 0, U_{BB} < 0$. The negativity of $U_B$ embodies the notion of discrimination against the group $B$ workers. Utility is dependent on the number of group $A$ workers only through their effect on profits.

It is assumed that members of the two groups are perfect substitutes in revenue so that revenue is a function, $R(L)$, of employment of the two groups, $L = A + B$. Profits can be written as

$$\Pi = R(A + B) - w_A A - w_B B, \tag{2}$$

where, initially, $w_A$ and $w_B$ - the respective wage rates of $A$ and $B$ - are exogenously given to the firm. We consider below alternative processes for the determination of the wage rates. The firm then selects $A$ and $B$ to maximise utility. Substituting the expression for profits into the utility function yields

$$U = U(R(A + B) - w_A A - w_B B, B). \tag{3}$$
Maximising (3) subject to the constraints $A \geq 0, B \geq 0$ generates the following first-order conditions that describe the firm's behaviour:

$$\frac{\partial U}{\partial A} = U_{\Pi}[R'(A + B) - w_A] + \rho_1 = 0, \quad \rho_1 A = 0, \quad \rho_1 \geq 0,$$

and

$$\frac{\partial U}{\partial B} = U_{\Pi}[R'(A + B) - w_B] + U_B + \rho_2 = 0, \quad \rho_2 B = 0, \quad \rho_2 \geq 0. \quad (5)$$

Hence

$$w_A = R'(A + B) + \frac{\rho_1}{U_{\Pi}},$$

and

$$w_B - \frac{U_B}{U_{\Pi}} = R'(A + B) + \frac{\rho_2}{U_{\Pi}}. \quad (7)$$

In the case that both $A$ and $B$ are positive, so that $\rho_1 = \rho_2 = 0$, it follows that

$$w_B + \tilde{d}(A, B) = R'(A + B) = w_A, \quad (8)$$

where $\tilde{d}(A, B) = -U_B/U_{\Pi}$ is the marginal rate of substitution between $\Pi$ and $B$ for the firm. From the structure of utility it follows that $\tilde{d}(A, B) > 0$ for all $A, B$, and hence that $w_B + \tilde{d}(A, B) > w_B$. Essentially, the discriminating employer treats the marginal cost of employing workers from group $A, MCA$, as equal to their market wage, $w_A$, whilst regarding the marginal cost of employing an additional group $B$ worker, $MCB$, as the market wage, $w_B$, plus the discrimination factor, $\tilde{d}(A, B)$, i.e., $MCA = w_A$ and $MCB = w_B + \tilde{d}(A, B)$. Optimal employment levels of the firm are then chosen to equate $w_A$ to $w_B + \tilde{d}(A, B)$.

In the typical models in the literature, for instance Becker (1957), $\tilde{d}(A, B)$ is taken as a constant for each firm, $\tilde{d}(A, B) = \tilde{d}$, implying a segregation between group $A$ and group $B$ workers across firms. This follows because if $\tilde{d}$ is such that $w_B + \tilde{d} < w_A$, then $\rho_1 > 0$ and only group $B$ workers will be employed. If the inequality sign is reversed, $\rho_2 > 0$ and only group $A$ workers will be employed. In this situation a (random) mix of workers can occur only when there is strict equality between the two subjective prices of the two worker groups. There is evidence that, with respect to racial discrimination at least, employment tends to be segregated with some establishments employing largely black-only workforces and others employing whites-only. There is, however, little evidence of absolute segregation. Furthermore, if employment segregation occurs at establishment level only, our analysis would be relevant so long as bargaining takes place at company level. Finally, in the UK at least, unions are not organised on a racial basis. These observations tend to contradict the clear division of workers implied by the Becker analysis.
In contrast, from our assumptions \( \tilde{d}(A, B) \) can be shown to be increasing in \( B \) for given \( A \). To illustrate the consequences of this, let \( w_A \) and \( w_B \) be fixed exogenously at their competitive levels \( w_A^0 \) and \( w_B^0 \). We impose the assumption that \( w_A^0 > w_B^0 \) which is intended to capture discrimination in the wider labour market. It is implicit in Fig. 1 below, which illustrates the optimisation of the firm, that the values of \( R'(L) \), \( w_A^0 \) and \( w_B^0 \), and the behaviour of \( \tilde{d}(A, B) \) generate non-zero values of \( A \) and \( B \). If, conversely, either \( w_B^0 \) were lower relative to \( w_A^0 \) or the \( MCB \) increased more slowly in \( B \), then we could observe \( A = 0 \). Total employment would then be determined by the equality of \( R'(L) \) and \( MCB \). Otherwise, total employment is determined by the equality of \( R(L) \) and \( w_A^0 \).

Within such a regime as that depicted in Fig. 1, it is clear that:

(i) Changes in \( w_B^0 \) affect the distribution of employment between the two groups, but do not affect total employment, \( L \), in the firm.

(ii) Provided the derivatives of the utility function are finite, so that \( MCB \) is not vertical, \( B > 0 \) if \( w_A^0 > w_B^0 \).

(iii) Total employment, \( L \), is unaffected by changes in the discrimination factor, \( \tilde{d}(A, B) \), which keep the solution within the regime. This result is similar to (i), above.

(iv) An increase in labour demand will be absorbed entirely by an increase in employment of group \( A \) workers. Similarly, a reduction in demand, if sufficiently small, will have no effect on the employment of \( B \) workers. Contrary to expectations, the workers that are not discriminated against are the marginal workers.

2.2. Additively separable utility

Further results can now be derived by analysing the maximisation of the firm. To simplify the remaining analysis, the objective of the firm is assumed to be

![Fig. 1.](image-url)
additively separable in \( ii \) and \( B \). The resulting function is then transformed by taking the composition of the original utility function and the inverse of the separable profit component. Since \( U_{ii} \) is positive, this is simply a monotonic transformation of the original function, which is now additive in profit, and the transformation does not alter the firm's optimal choices. With this formulation, the firm becomes risk neutral.

The objective of the firm can therefore be written

\[
\max_{\{A,B\}} U = R(A + B) - w_A A - w_B B - d(B), \tag{9}
\]

where \( d(B) \) is termed the discrimination function and gives the reduction in the firm's utility due to the employment of the group \( B \) workers. To ensure that this is a strictly concave problem, the following assumption is maintained throughout:

Assumption 1. (i) \( R(0) = 0, \ R'(L) > 0, \ R''(L) < 0, \ R'''(L) < 0, \) (ii) \( d(0) = 0, \ d'(B) > 0, \ 0 < d''(B) \leq K, \ K \ finite, \ d'''(B) \geq 0. \)

Although it is unusual to restrict the third derivatives, such restrictions are sufficient (but not necessary) to sign a number of the expressions that occur below and their adoption greatly simplifies the presentation.

For the issues that will be addressed in this paper we shall be concerned with the regime in which the firm chooses to employ workers from both groups. Solving the maximisation, where it is assumed that \( w_A > w_B \) and an interior solution exists, gives

\[
R'(A, B) = w_A, \tag{10}
\]

\[
R'(A, B) = w_B + d'(B), \tag{11}
\]

which can be consistent only when \( w_A > w_B \).

From (10) and (11),

\[
\begin{bmatrix}
R'' \\
R'' - R'' - d''
\end{bmatrix}
\begin{bmatrix}
dA \\
dB
\end{bmatrix} = \begin{bmatrix}
dw_A \\
dw_B
\end{bmatrix}. \tag{12}
\]

Solving this equation

\[
\frac{dA}{dw_A} = \frac{1}{R'' - d''} < 0, \quad \frac{dA}{dw_B} = \frac{1}{d''} > 0, \quad \frac{dB}{dw_A} = \frac{1}{d''} > 0, \quad \frac{dB}{dw_B} = -\frac{1}{d''} < 0. \tag{13}
\]

From (13) an increase in \( w_B \) raises the employment of group \( A \) but reduces the employment of group \( B \) by an exactly equal amount. Total employment is therefore not affected although distribution between the groups is. An increase
in the wage rate of group A workers does reduce total employment since
\[
\frac{dL}{dw_A} = \frac{dA}{dw_A} + \frac{dB}{dw_A} = \frac{1}{R''} < 0. \tag{14}
\]

Now denoting the firm’s maximum value function by \(U(w_A, w_B)\), a further assumption is adopted.

**Assumption 2.** \(U(w_A^0, w_B^0) > 0\).

This assumption simply gives the problem that we study some content. It ensures that the firm generates a positive level of utility at the competitive wage rates so that when bargaining with the union takes place, there is some surplus to be shared.

Employing the envelope theorem shows
\[
\frac{\partial U}{\partial w_A} = U_a = -A, \quad \frac{\partial U}{\partial w_B} = U_b = -B. \tag{15}
\]

Using (13) and (14) the second derivatives of maximum value are
\[
U_{aa} = -\frac{\partial A}{\partial w_A} = \frac{1}{d''} - \frac{1}{R''}, \quad U_{ab} = -\frac{\partial A}{\partial w_B} = -\frac{1}{d''}, \quad U_{bb} = -\frac{\partial B}{\partial w_B} = \frac{1}{d''}. \tag{16}
\]

From (16), the maximum value function is strictly convex, a factor which greatly complicates the analysis below.

To complete this section, we calculate the effect of an increase in discrimination by the firm. This can be modelled by letting \(d(B)\) increase to \((1 + \varepsilon)d(B)\), with \(\varepsilon > 0\). Solving the resulting system and evaluating at \(\varepsilon = 0\), gives
\[
\frac{dA}{d\varepsilon} = \frac{d'}{d''} > 0, \quad \frac{dB}{d\varepsilon} = \frac{d'}{d''} < 0. \tag{17}
\]

The increase in discrimination therefore reduces employment of group B workers and increases that of group A. Total employment remains unchanged.

### 3. The union

We shall assume that a single union represents both groups of workers. To justify this we appeal to work by Horn and Wolinsky (1988) who develop a bargaining model for the case in which two groups of workers face a single employer. They derive a general principle which states that when the two types of labour are substitute factors in production, then it is in their interests to
coordinate their bargaining with the employer by forming a single encompassing union. The intuition for this result is that if the two groups of workers are highly substitutable then they could be easily divided-and-ruled in the event of a disagreement between the firm and just one of the groups: the firm's payoff in the event of such a disagreement would be high under separate bargaining as it would be able to continue production through the employment of the other group. This result is applicable to our model as we are assuming that workers from each group are perfect substitutes.

The bargaining structure is characterised by the right-to-manage class of models (see Nickell and Andrews, 1983) in which the union and the firm bargain over wages, leaving the firm free to choose employment levels so as to maximise firm-utility, given the bargained level of wages. This applies for both the monopoly union case and the generalised Nash bargain.

3.1. Union preferences

With respect to union preferences, we shall follow the approach of specifying a modified Stone–Geary utility function of the form $V = [w - w^0]L$. The most common interpretation of this utility function is one of rent-maximisation by the union. We extend this formulation by allowing the possibility that the union may also discriminate between the two groups of workers. Over the two groups, $A$ and $B$, of workers, union preferences are given by

$$V = [w_A - w^0_A]A + \mu[w_B - w^0_B]B. \quad (18)$$

If $\mu = 1$ the union places an equal weight on the rents accruing from each of the two groups and, being indifferent about the source of the rent, is described as non-discriminating. The union can be said to practice positive discrimination if $\mu > 1$ and negative if $\mu < 1$. Our main concern is to discover what happens to the wage gap relative to the competitive gap, $w^0_A - w^0_B$, when there is bargaining between the discriminating employer and the non-discriminating union but we will also consider changes in union discrimination.

Given the firm's selection of $A$ and $B$ conditional on the wages, it is possible to express the union's welfare level as a function of the wage rates. We denote this function by $V = V(w_A, w_B)$. Having done this, the following derivatives are implied by previous calculations:

$$\frac{\partial V}{\partial w_A} \equiv V_a = A + \frac{R'}{R''} + \frac{(\mu - 1)R'}{d''} - \frac{\mu d''}{d''} - \frac{w^0_A}{R''} + \frac{w^0_A - \mu w^0_B}{d''}, \quad (19)$$

$$\frac{\partial V}{\partial w_B} \equiv V_b = \mu B - \frac{(\mu - 1)R'}{d''} + \frac{\mu d''}{d''} - \frac{w^0_A}{R''} - \frac{w^0_A - \mu w^0_B}{d''}. \quad (20)$$
Differentiating these again,

\[ V_{aa} = \frac{2}{R''} - \frac{2}{d''} - \frac{(w_A^0 - \mu w_B^0 - \mu d')d''}{d''^3} - \frac{(R' - w_A^0)R''}{R''^3}, \]  

(21)

\[ V_{bb} = -\frac{(1 + \mu)}{d''} - \frac{(w_A^0 - \mu w_B^0 - \mu d')d''}{d''^3}, \]  

(22)

and

\[ V_{ab} = \frac{(1 + \mu)}{d''} + \frac{(\mu - 1)R'd''}{d''^3} + \frac{(w_A^0 - \mu w_B^0 - \mu d')d''}{d''^3}. \]  

(23)

3.2. The monopoly union model

As a prelude to analysing the outcome of the generalised Nash bargaining process, we first consider the monopoly union model in which the union has total control over setting the wage levels but is subject to the labour demand curve as the firm has sovereignty over employment. In our discrimination context, the monopoly union is assumed to choose \( w_A \) and \( w_B \) in the knowledge that the firm will then set the levels of \( A \) and \( B \) to maximise utility. Clearly, the monopoly union model is a special case within the right-to-manage framework and can be viewed as the limiting outcome of the bargaining model in which all power lies with the union. We choose to give it a separate treatment because of the importance of the monopoly union model in the literature and for the motivation it provides for the results of the generalised Nash bargain.

Considering the problem of the monopoly union, it can be expressed as

\[
\max_{(w_A, w_B)} V(w_A, w_B),
\]  

(24)

subject to the labour demand functions implied by (10) and (11). This maximisation has first-order conditions

\[ V_a = 0, \quad V_b = 0. \]  

(25)

As \( d'' \) is finite, the argument given in (ii) following Eq. (8) implies that \( B > 0 \). From (20) it follows that for \( \mu = 1 \), and by continuity for values of \( \mu \) in a neighbourhood around \( \mu = 1 \), that when the first-order condition (25) is satisfied

\[ w_A^0 - w_B^0 > d'. \]  

(26)

Substituting (26) into (21) to (23), at \( \mu = 1 \), and in a neighbourhood around 1, it can be seen that \( V_{aa} < 0, V_{bb} < 0, V_{ab} > 0 \) and \( V_{aa}V_{bb} - V_{ab}V_{ba} > 0 \). The second-order sufficient conditions for (24) are therefore satisfied.

Since the wage gap without union influence is given by \( w_A^0 - w_B^0 \) and with the union is equal to \( d' \) (since \( w_A = w_B + d' \) from (10) and (11)), (26) demonstrates...
that the outcome determined by the monopoly union always reduces the wage gap. In addition, if \( A \) is positive, (26) and (19) imply that \( R' - w_A^0 > 0 \) so, from (10), the union also raises the wage of group \( A \) workers above the competitive level. This reduction of the wage gap is a property of the rent-maximising behaviour of the union – not a consequence of any explicit positive discrimination since it holds at \( \mu = 1 \). The union does not reduce the wage gap to zero since, if it did, employment of group \( B \) would fall to zero by the complementary slackness conditions in (4) and (5), and the union would fail to acquire any rents from \( B \) employment. Consider, on the other hand, a union which set wages such that the wage gap was unchanged, and hence that the rent per worker was the same for both groups. Then, in terms of Fig. 1, the marginal labour cost curves would shift upwards by equal magnitudes and employment of group \( B \) would be unaffected. If the union now raised \( w_B \) by a small amount total employment would not be affected but the firm would substitute some \( A \) workers for an equal number of \( B \) workers. The effect of this substitution on union rents would be zero. However, the remaining \( B \) workers would be receiving higher wages and so net rents to the union are higher. Thus, the union optimally sets a wage gap less than the initial, unconstrained wage gap. It does not pay the union to continue to raise \( w_B \) to equate it to \( w_A \) as each successively displaced \( B \) worker has a higher rent than each successively appointed \( A \) worker. The union balances these two effects when there is still a positive wage gap, but one below the initial non-union level.

Next, consider the effects of changes in the competitive wage levels, in the degree of discrimination of the union and of a shift in the discrimination function of the firm from \( d(B) \) to \((1 + \varepsilon)d(B)\), \( \varepsilon > 0 \), which is the increase in discrimination introduced above (17). Total differentiation of (19) and (20) gives the equation system

\[
\begin{bmatrix}
V_{aa} & V_{ab} \\
V_{ba} & V_{bb}
\end{bmatrix}
\begin{bmatrix}
dw_A \\
dw_B
\end{bmatrix}
\]

\[
= \begin{bmatrix}
\left[ \frac{1}{R''} - \frac{1}{d''} \right] dw_A^0 + \left[ \frac{\mu}{d''} \right] dw_B^0 + \left[ \frac{(w_A^0 - \mu w_B^0)}{d''} + \frac{(\mu - 1)R'}{d''} \right] d\varepsilon + \left[ \frac{d' + w_B^0 - R'}{d''} \right] d\mu
\\n\left[ \frac{1}{d''} \right] dw_A^0 - \left[ \frac{\mu}{d''} \right] dw_B^0 - \left[ \frac{(w_A^0 - \mu w_B^0)}{d''} + \frac{(\mu - 1)R'}{d''} \right] d\varepsilon + \left[ \frac{R' - d' - w_B^0 - B}{d''} \right] d\mu
\end{bmatrix}
\]

(27)

Solving (27) and evaluating at \( \mu = 1 \), \( \varepsilon = 0 \), the effects of changes in the competitive wage and discrimination by firms upon the wage of the \( A \) group are given by

\[
\frac{dw_A}{dw_A^0} > 0, \quad \frac{dw_A}{dw_B^0} = 0, \quad \frac{dw_A}{d\varepsilon} = 0,
\]

(28)
and on the wage of the $B$ group by

$$\frac{dw_B}{dw_B^0} > 0, \quad \frac{dw_B}{d\epsilon} > 0, \quad \frac{dw_B}{dw_A^0} < 0. \quad (29)$$

The union therefore reacts to an increase in the competitive wage of one group by raising the bargained wage of that group. Thus if, for example, anti-discrimination legislation produces an exogenous increase in $w_B^0$, the wage gap in the unionised firm, anyway less than that where unions do not set wages, is reduced further. However, an increase in $w_A^0$ reduces $w_B$. The union counters an increase in discrimination within the firm by raising the wage of the group that is the object of the discrimination. Since the wage of the other group does not change, this reduces the wage gap.

More surprising is the effect of an increase in positive discrimination by the union. From (27) $dw_A/d\mu > 0$, so that the increase in $\mu$ raises the wage of the group that is not discriminated against. This occurs because an increase in $w_A$ raises the employment of group $B$ workers and this is now valued more highly than employment of group $A$. In contrast, $dw_B/d\mu$ cannot be unambiguously signed. However, it can be shown (see appendix) that when, $d'''' = R'''' = 0$, $dw_B/d\mu < 0$. In this case, the increase in positive discrimination actually reduces the wage of the workers who are more highly valued by the union. The ambiguity, and the reason for the potential negativity, is the new trade-off between the level of $w_B$ and the employment of group $B$ that arises when $\mu$ increases. Furthermore, it is also shown in the appendix that $dw_A/d\mu > dw_B/d\mu$ is always satisfied, so that an increase in positive discrimination increases the wage gap, the converse of what would naturally be expected.

4. The generalised Nash bargaining solution

We now collect together the analyses developed above and place the union and firm into a generalised Nash bargaining model in which the standard Nash bargaining solution is extended by the inclusion of a parameter, $\lambda$, that measures the relative weights given to the union and firm. This approach is adopted since it permits the analysis of variations in union influence on the bargained outcome. The generalised bargain is then studied from both an analytical and a numerical perspective. The aim of the analytical approach is to determine whether the presence of the union reduces the wage gap and therefore counters discrimination and, if it ever does, to characterise sufficient conditions for this to occur. The numerical results are intended to provide an illustration of the formal reasoning and to investigate the effect of other parametric variations.
Using the maximum value functions, the generalised Nash bargaining problem can be expressed as

$$\max_{(w_A, w_B)} V(w_A, w_B)^{\lambda} U(w_A, w_B)^{1-\lambda},$$

subject to the demand functions derived from (10) and (11), $w_A, w_B \geq 0$ and $A, B \geq 0$. The specification in (30) collapses to the unconstrained firm when $\lambda = 0$ and to the monopoly union when $\lambda = 1$. By varying $\lambda$ it is possible to move continuously between these extremes. It is implicit in (30) that the no-agreement utility level of the firm is set at zero and that the inclusion of competitive wage levels in (19) captures the no-agreement outcome for the union.

4.1. Analytical results

There are two major difficulties in analysing the solution to (30). Firstly, the solution will be discontinuous at $\lambda = 0$: at this value of $\lambda$ the objective function is unbounded as $w_A$ and $w_B$ decrease without limit. This implies that it will be necessary to restrict $\lambda$ to an open set not including 0. The second difficulty is that the convexity or concavity of the objective function with respect to $w_A$ and $w_B$ cannot be established. As $\lambda$ tends to 0 it would be expected that it should be convex, being in the limit the profit function of the firm. However, calculating the Hessian of $V^\lambda U^{1-\lambda}$ shows that this need not be the case since there are powers of $V$ and its derivatives that dominate those of $U$ for small $\lambda$. It follows that there will be few global results on the solution to (30) and that we must take care to consider possible corner solutions.

Proceeding with the analysis, the first step is to introduce the following reasonable assumption:

**Assumption 3.** There exist $\tilde{w}_A$ and $\tilde{w}_B$ such that (i) for all $w_A \geq \tilde{w}_A$, $A(w_A, w_B) = 0$ for all $w_B$; and (ii) for all $w_B \geq \tilde{w}_B$, $B(w_A, w_B) = 0$ for all $w_A$.

This assumption simply asserts that there will be some value of the wage rate for each group of workers above which the labour demand for that group becomes zero. We now define $\Omega \subset \Re^2_+$ by

$$\Omega = [w_A^0, \tilde{w}_A] \times [w_B^0, \tilde{w}_B].$$

$\Omega$ is clearly compact. Since the economics of the model make it clear that the union will never supply labour if the wage offered by the firm is below the competitive level, Assumption 3 implies that the choice variables in the maximisation in (30) can be restricted to $\Omega$ without altering the economic content of the problem. In addition, for given $\lambda$, $U^{1-\lambda} V^\lambda$ is a continuous function of $w_A$ and $w_B$ and therefore has a maximum on the compact set $\Omega$. In addition to the global maximum, there may be additional local maxima.
Differentiating (30) with respect to $w_A$ and $w_B$ generates the first-order conditions for an interior maximum:

\[ [1 - \lambda] U^{-\lambda} V^{\lambda} U_a + \lambda U^{1-\lambda} V^{\lambda-1} V_a = 0, \quad (31) \]
\[ [1 - \lambda] U^{-\lambda} V^{\lambda} U_b + \lambda U^{1-\lambda} V^{\lambda-1} V_b = 0. \quad (32) \]

Assuming that $U > 0$ and $V > 0$, an assumption justified for $\lambda \in (0, 1]$ by Lemma 1 below, (31) and (32) can be transformed by multiplying by $V^{1-\lambda} U^\lambda > 0$ to give

\[ \lambda V_a U + [1 - \lambda] V U_a = 0, \quad (33) \]
and

\[ \lambda V_b U + [1 - \lambda] V U_b = 0. \quad (34) \]

It is worth noting at this point that the results below are derived from (33) and (34) rather than from (31) and (32). The transformation involved in moving from one to the other clearly does not affect any of the conclusions. However, the Hessian of $V^\lambda U^{1-\lambda}$, which we denote $H$, is given by the Jacobian of (31) and (32) which differs in important respects from the Jacobian of (33) and (34), denoted $J$, that is employed in the comparative statics. In particular, in the limit as $\lambda$ tends to zero and the two wage rates tend to their competitive levels, $V$ will also tend to zero and $H$ will involve terms which are zero being raised to several different powers. This is not true of $J$ since the zero terms are eliminated by the transformation. The transformation therefore collapses at $\lambda = 0$ with wages at competitive levels.

Having already established by the compactness of $\Omega$ that for all $\lambda$ a maximum exists, we are now concerned with the dependence of the solution of the Nash bargain upon $\lambda$. For this purpose, denote each maximum of (30) conditional upon $\lambda$ as $w_A(\lambda)$, $w_B(\lambda)$. The first lemma rules out the possibility that the wage rates remain at the competitive levels for positive $\lambda$, that is for $\lambda \in (0, 1]$. Thus for all $\lambda$ in the open set $(0, 1)$, both $U$ and $V$ are positive. It is also shown that any maximum of (34), whether local or global, satisfies the first-order conditions with equality and is therefore not a ‘corner solution’. The proof of this, and all the following results, is contained in the appendix.

Lemma 1. For all $\lambda \in (0, 1]$, (i) $w_A(\lambda) = w_A^0$, $w_B(\lambda) = w_B^0$ cannot be a solution; (ii) at each local maximum the first-order conditions are satisfied with equality; and for $\lambda \in (0, 1)$ (iii) $U > 0$, $V > 0$.

Using the result in Lemma 1 that the first-order conditions are always satisfied, (33) and (34) imply that

\[ V_b U_a = V_a U_b, \quad \text{for all } 0 < \lambda \leq 1. \quad (35) \]
From (35), Theorem 1 determines the consequences of bargaining for the wage gap.

**Theorem 1.** For all \( \lambda \in (0, 1] \), the wage gap is reduced at each local maximum of the Nash bargain, and therefore at the global maximum, relative to the competitive wage gap.

Theorem 1 is the first of the two central results and states that whenever the union has any weight in the objective function for the Nash bargain, it always reduces the wage gap and therefore counters the discrimination. This provides the answer to the question posed by the paper.

Having concluded that the bargained outcome always reduces the wage gap, the remaining analysis is concerned with determining the relation of the wage gap to the weight given to the union in the Nash bargain. To do this it is first noted that at any local maximum of (30) the second-order conditions must be satisfied (or else it cannot be a local maximum) so that the principal minors of \( H \), denoted \( H_{aa} \) and \( H_{bb} \), are non-positive and \( |H| \geq 0 \) at each local maximum. These conditions also apply to the determinant of \( J \) and its principal minors since, by Lemma 1 \( U^\lambda V^{1-\lambda} > 0 \), and \( H_{ik} = U^\lambda V^{1-\lambda} J_{ik}, i, k = a, b \), \( |H| = [U^\lambda V^{1-\lambda}]^2 |J| \). Having noted this, Assumption 4 makes the weak restriction that \( |H| \) is strictly positive at each local maximum.

**Assumption 4.** At each local maximum \( |H| > 0 \).

From this assumption follows Lemma 2.

**Lemma 2.** If it is assumed that \( V \) and \( U \) are of class \( C^r \), \( r \geq 2 \), then \( w_A(\lambda) \) and \( w_B(\lambda) \) are continuous and at least once differentiable for \( \lambda \in (0, \lambda^*) \) in a neighbourhood of each local maximum.

As already noted, the non-concavity of the objective makes it possible for there to be multiple solutions to (30). Our response to this is to focus, for values of \( \lambda \) close to zero, upon the low wage outcome. This is justified by appealing to the fact that in the absence of the union the firm will pay the competitive wage levels and that adding an appropriately small amount of union power should not disturb this equilibrium too far. In a sense, we are imposing a degree of continuity upon the solution. This observation leads to Lemma 3.

**Lemma 3.** \( \lim_{\lambda \to 0} w_A(\lambda) = w_A^0 \) and \( \lim_{\lambda \to 0} w_B(\lambda) = w_B^0 \).

Therefore, as union power is reduced to 0 the bargained wages fall back to their competitive levels.
The second theorem concerns the effect of changing the share parameter $\lambda$ in
the Nash bargain and provides a sufficient condition for increased weight on the
union to reduce the wage differential.

Theorem 2. In a neighbourhood of any local maximum of (30), and therefore in
a neighbourhood of the global maximum, the wage gap is a monotonically decreas-
ing function of $\lambda$.

This theorem establishes that increasing the weight given to the union in the
bargain will reduce the wage gap further. It should also be noted from inequality
(A.2) of the proof that the conditions of Assumption 1 are strongly sufficient to
prove Theorem 2 and the necessary condition would be much weaker. As
a consequence, it can be seen as generally true that increasing the power of the
union in the Nash bargain will further reduce the wage gap. Additionally, for
$\lambda$ close to zero the application of Lemma 3 to (A.2) shows that the gap is always
decreasing with $\lambda$ since in the limit $R' - w_A^0 = 0$. In other words, starting from
the unconstrained firm outcome, increasing amounts of union power always
reduce the wage gap.

4.2. Numerical results

A numerical example is now analysed to illustrate the results proved above
and to investigate the consequences of variations in the degree of discrimination
by the firm and the union. These have already been analysed for the uncon-
strained firm and the monopoly union; the numerical results complete the study
for the intermediate case.

The example assumes a constant-elasticity revenue function, $R(L) = L^\beta$, with
$\beta < 1$ to ensure concavity, and a utility function for the firm given by
$U = \Pi - B^\alpha$, where $\alpha > 1$ as an implication of our earlier assumptions. Within
the region in which the firm employs workers from both groups, $L$ is chosen to
satisfy $w_A = R'(L) = \beta L^{\beta-1}$, $B$ to satisfy $w_A = w_B + \alpha B^{\alpha-1}$, and $A = L - B$. The
basic parameter values are $\beta = 0.4$, $w_A^0 = 0.4$, $w_B^0 = 0.2$, $c = 0$ and $\mu = 1$. The
competitive wage gap, without union intervention, is 0.2.

Tables 1 and 2 report the effect of variations in $\lambda$ for two values of $\alpha$ and
$Gap = w_A - w_B$. In both tables the wage gap decreases as the bargaining
power of the union increases and the extent to which the gap is reduced and
wages raised by the union influence is striking. The rate of change of wage gap
with respect to $\lambda$ appears to be fairly constant, though diminishing as $\lambda$ ap-
proaches 1. The wage gap is reduced, but the union pay-off increased, as the
discrimination function becomes more convex. In addition, all variables are
monotonic with respect to $\lambda$: wages and union utility are increasing whilst the
wage gap and firm utility are decreasing. These results confirm the finding that
an increase in the weight given to the union decreases the wage gap.
Table 1
\( \alpha = 2.5, \mu = \varepsilon = 0 \)

<table>
<thead>
<tr>
<th>( \lambda )</th>
<th>0.001</th>
<th>0.01</th>
<th>0.02</th>
<th>0.04</th>
<th>0.06</th>
<th>0.08</th>
<th>0.1</th>
<th>0.2</th>
<th>0.4</th>
<th>0.6</th>
<th>0.8</th>
<th>1</th>
</tr>
</thead>
<tbody>
<tr>
<td>( U )</td>
<td>0.622</td>
<td>0.616</td>
<td>0.610</td>
<td>0.598</td>
<td>0.587</td>
<td>0.576</td>
<td>0.566</td>
<td>0.520</td>
<td>0.451</td>
<td>0.398</td>
<td>0.357</td>
<td>0.329</td>
</tr>
<tr>
<td>( V )</td>
<td>0.001</td>
<td>0.006</td>
<td>0.012</td>
<td>0.023</td>
<td>0.033</td>
<td>0.042</td>
<td>0.051</td>
<td>0.084</td>
<td>0.119</td>
<td>0.135</td>
<td>0.141</td>
<td>0.142</td>
</tr>
<tr>
<td>( w_A )</td>
<td>0.401</td>
<td>0.404</td>
<td>0.410</td>
<td>0.420</td>
<td>0.431</td>
<td>0.442</td>
<td>0.453</td>
<td>0.511</td>
<td>0.621</td>
<td>0.758</td>
<td>0.891</td>
<td>1</td>
</tr>
<tr>
<td>( w_B )</td>
<td>0.201</td>
<td>0.214</td>
<td>0.225</td>
<td>0.246</td>
<td>0.265</td>
<td>0.284</td>
<td>0.302</td>
<td>0.384</td>
<td>0.525</td>
<td>0.667</td>
<td>0.807</td>
<td>0.920</td>
</tr>
<tr>
<td>( Gap )</td>
<td>0.200</td>
<td>0.190</td>
<td>0.185</td>
<td>0.174</td>
<td>0.166</td>
<td>0.158</td>
<td>0.151</td>
<td>0.127</td>
<td>0.104</td>
<td>0.091</td>
<td>0.084</td>
<td>0.079</td>
</tr>
</tbody>
</table>

Table 2
\( \alpha = 3, \mu = \varepsilon = 0 \)

<table>
<thead>
<tr>
<th>( \lambda )</th>
<th>0.001</th>
<th>0.01</th>
<th>0.02</th>
<th>0.04</th>
<th>0.06</th>
<th>0.08</th>
<th>0.1</th>
<th>0.2</th>
<th>0.4</th>
<th>0.6</th>
<th>0.8</th>
<th>1</th>
</tr>
</thead>
<tbody>
<tr>
<td>( U )</td>
<td>0.634</td>
<td>0.628</td>
<td>0.622</td>
<td>0.610</td>
<td>0.598</td>
<td>0.587</td>
<td>0.577</td>
<td>0.529</td>
<td>0.457</td>
<td>0.403</td>
<td>0.360</td>
<td>0.332</td>
</tr>
<tr>
<td>( V )</td>
<td>0.006</td>
<td>0.006</td>
<td>0.012</td>
<td>0.024</td>
<td>0.034</td>
<td>0.044</td>
<td>0.053</td>
<td>0.087</td>
<td>0.126</td>
<td>0.142</td>
<td>0.149</td>
<td>0.150</td>
</tr>
<tr>
<td>( w_A )</td>
<td>0.401</td>
<td>0.404</td>
<td>0.408</td>
<td>0.418</td>
<td>0.428</td>
<td>0.438</td>
<td>0.449</td>
<td>0.505</td>
<td>0.623</td>
<td>0.750</td>
<td>0.885</td>
<td>1</td>
</tr>
<tr>
<td>( w_B )</td>
<td>0.202</td>
<td>0.214</td>
<td>0.225</td>
<td>0.247</td>
<td>0.267</td>
<td>0.286</td>
<td>0.305</td>
<td>0.388</td>
<td>0.533</td>
<td>0.672</td>
<td>0.815</td>
<td>0.933</td>
</tr>
<tr>
<td>( Gap )</td>
<td>0.199</td>
<td>0.190</td>
<td>0.183</td>
<td>0.171</td>
<td>0.161</td>
<td>0.152</td>
<td>0.144</td>
<td>0.117</td>
<td>0.091</td>
<td>0.078</td>
<td>0.071</td>
<td>0.066</td>
</tr>
</tbody>
</table>

Table 3
\( \alpha = 2.5, \varepsilon = 0, \lambda = 0.4 \)

<table>
<thead>
<tr>
<th>( \mu )</th>
<th>1</th>
<th>1.1</th>
<th>1.5</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>( U )</td>
<td>0.451</td>
<td>0.453</td>
<td>0.466</td>
<td>0.477</td>
<td>0.493</td>
</tr>
<tr>
<td>( V )</td>
<td>0.119</td>
<td>0.122</td>
<td>0.131</td>
<td>0.139</td>
<td>0.150</td>
</tr>
<tr>
<td>( w_A )</td>
<td>0.629</td>
<td>0.629</td>
<td>0.618</td>
<td>0.609</td>
<td>0.600</td>
</tr>
<tr>
<td>( w_B )</td>
<td>0.525</td>
<td>0.508</td>
<td>0.447</td>
<td>0.397</td>
<td>0.339</td>
</tr>
<tr>
<td>( Gap )</td>
<td>0.104</td>
<td>0.120</td>
<td>0.171</td>
<td>0.213</td>
<td>0.261</td>
</tr>
</tbody>
</table>

Table 3 reports the effect on an increase in positive discrimination by the union. This can be seen to reduce the wage of the group \( B \) workers and to increase the gap between the wages. As in the monopoly union outcome, a union that weights returns to group \( B \) workers higher than those to group \( A \) succeeds in raising the employment of the group \( B \) but does not reduce the wage gap. The mechanism behind this was described in section 3: increasing \( w_A \) raises employment of group \( B \) workers which is more highly valued than employment of group \( A \). This offsets the gain from reducing the wage gap and, in the case of Table 3, is the dominant force.

The effects of an increase in discrimination by the firm are reported in Table 4. Since the effect of changes in \( \varepsilon \) are small relative to the accuracy of the grid-search algorithm, the table gives the derivatives of the wage rates with respect to \( \varepsilon \) since these can be calculated exactly once the optimal wages are known. For the higher values of \( \lambda \) an increase in \( \varepsilon \) raises the wage of the group \( B \) workers relative to that of the group \( A \), as would be expected given the finding
for the monopoly union. The converse occurs when $\lambda$ becomes small and the bargain reflects primarily the preferences of the firm. For most values of $\lambda$, an increase in $\varepsilon$ reduces the gap between the wages.

5. Further groups of workers

In many situations discrimination will involve more than two kinds of workers. For instance, discrimination may occur across gender and across racial backgrounds which, depending on the fineness of the racial categorisation, will give at least four distinct groups of workers.

The first possibility to consider is that all the $n$ types of worker are represented by a single union. If the firm discriminates to varying degrees between the different worker groups on the basis of their characteristics, the utility function of the firm would be

$$U = R(A_1, \ldots, A_n) - \sum_{i=1}^{n} w_i A_i - \sum_{i=1}^{n} d(A_i; \alpha_i).$$

(36)

where $\alpha_i$ is the vector of characteristics of group $i$ and $d(A_i; \alpha_1) = 0$ for the group 1 workers who are not discriminated against. Correspondingly, the union would have preferences

$$V = \sum_{i=1}^{n} \mu_i [w_i - w_i^0] A_i.$$  

(37)

The problem analysed in (30) would then be extended to the bargain over the $n$-vector of wages. Although of considerably greater algebraic complexity, there is no conceptual distinction between this problem and that analysed above. Indeed, it is straightforward to show that, even when $\mu_i = 1$ for all $i$, the monopoly union will always set wages so as to reduce the wage gap for each group of workers relative to the wage of group 1 workers.
To prove this result, note that when workers from all groups are employed the optimal choice of employment for the firm satisfies the conditions

\[ R\left( \sum_{i=1}^{n} A_i \right) - w_1 = 0, \]  
\[ R\left( \sum_{i=1}^{n} A_i \right) - w_j - d'_j(A_j) = 0, \quad j = 2, \ldots, n, \]  

where \( d'_j(A_j) \) is shorthand for \( \partial d(A_j; x)/(\partial A_j) \). The employment levels follow from (38) and (39) as

\[ A_1 = R^{-1}(w_1) - \sum_{j=2}^{n} d_j^{-1}(w_1 - w_j) = A_1(w_1, \ldots, w_n), \]  
\[ A_j = d_j^{-1}(w_1 - w_j) = A_j(w_1 - w_j), \quad j = 2, \ldots, n, \]  

where \( d_j^{-1} \) denotes the inverse of the function \( d_j \). The objective of the monopoly union is

\[ \max_{\{w_1, \ldots, w_n\}} V(w_1, \ldots, w_n) = \sum_{i=1}^{n} [w_i - w_i^0] A_i. \]  

The first-order necessary condition for the choice of \( w_j, j = 2, \ldots, n \), follows from (40), (41) and (42) as

\[ \frac{[w_1 - w_i^0]}{d''_j} + A_j - \frac{[w_j - w_j^0]}{d''_j} = 0, \]  

since \( \partial d_j^{-1}/\partial w_j = 1/d''_j \). When \( A_j > 0 \), (43) and the assumption that \( d''_j > 0 \) imply

\[ w_1 - w_j < w_1^0 \quad w_j^0, \quad j = 2, \ldots, n. \]  

The inequality in (44) proves that the monopoly union reduces the wage gap. An insight into the outcome of the generalised Nash bargain for \( n \) groups of workers can be obtained from Table 5. This reports the results for a bargain over the wages of four groups of workers in which the firm's revenue is determined as \( \Sigma_i A_i^\lambda \) and, for groups 2 to 4, the discrimination function is \( A_i^\lambda \). The initial wages are given by \( w_1^0 = 0.5, w_2^0 = 0.4, w_3^0 = 0.3 \) and \( w_4^0 = 0.2 \). The results show that the wage gap between the discriminated groups and group 1 workers is reduced for all values of \( \lambda \). The reduction is again monotonic in \( \lambda \). Furthermore, the reduction in wage gap between group 1 and group \( i, \Delta g_i, \) is greatest for those groups most discriminated against. The bargained outcome therefore always leads to a narrowing of the dispersion of wages.
Table 5
\(x_2 = 2, x_3 = 3, x_4 = 4, \beta = 0.5\)

<table>
<thead>
<tr>
<th>(\lambda)</th>
<th>(w_1 - w_2)</th>
<th>(\Delta g_2)</th>
<th>(w_1 - w_3)</th>
<th>(\Delta g_3)</th>
<th>(w_1 - w_4)</th>
<th>(\Delta g_4)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.1</td>
<td>0.091</td>
<td>0.009</td>
<td>0.168</td>
<td>0.032</td>
<td>0.232</td>
<td>0.068</td>
</tr>
<tr>
<td>0.25</td>
<td>0.079</td>
<td>0.021</td>
<td>0.130</td>
<td>0.070</td>
<td>0.166</td>
<td>0.134</td>
</tr>
<tr>
<td>0.5</td>
<td>0.064</td>
<td>0.036</td>
<td>0.095</td>
<td>0.105</td>
<td>0.112</td>
<td>0.188</td>
</tr>
<tr>
<td>0.75</td>
<td>0.056</td>
<td>0.044</td>
<td>0.077</td>
<td>0.123</td>
<td>0.088</td>
<td>0.212</td>
</tr>
</tbody>
</table>

As an alternative approach to the bargain with many groups, it can be assumed that the workers are grouped into employment categories according to one distinguishing characteristic (such as gender) and that discrimination takes place across the dimension of a second characteristic (such as race). In the case of four distinct groups, the revenue function for the firm becomes

\[
R = R(A_1, A_2, B_1, B_2),
\]

where \(x_i\) is the number of workers with gender \(x\) and racial characteristic \(i\) employed by the firm. Now assume that the groups are allocated to tasks within the firm according to one of the distinguishing criteria but within each pair remain perfect substitutes in revenue (for example, the males are allocated one pair of tasks, females another) and that task has a union that represents the workers undertaking it. For the bargain between the firm and one of the unions, if the wages determined by the other union are taken as given when bargaining, the resulting generalised Nash bargain will still take the form of (30) with the other pair of wages as parametric. The preceding results can then be applied directly and it remains true that each union will close the wage gap for the workers it represents.

6. Conclusions

We have shown how the generalised Nash bargaining model can be employed to analyse the previously ignored question of the effect of unions on wages and employment in the presence of employer discrimination. Our main result is to show how the presence of a union which has bargaining power over the wage causes a reduction in the wage gap between a discriminated and a non-discriminated group. This has been shown to be true for the monopoly union model and, by Theorem 1, for the generalised Nash bargaining model. As union bargaining power increases the wage gap falls monotonically provided only very mild restrictions are satisfied. This result arises with a union whose objective is simple rent-maximisation. Other results show that, in the case of a monopoly union model, an increase in discrimination by the firm leads the union to set
a higher wage for the group against which the firm is discriminating. The results have also been shown to generalise when the number of groups of workers is increased.

Appendix

Increase in \( \mu \) for monopoly union

From (27) \( \frac{d w_A}{d \mu} = \frac{(V_{ab}V_{b\mu} - V_{bb}V_{a\mu})}{(V_{aa}V_{bb} - V_{ab}^2)} \), where \( V_{aa}V_{bb} - V_{ab}^2 > 0 \). Eqs. (22) and (23) imply that \( V_{ab} = - V_{bb} \) so the sign of \( \frac{d w_A}{d \mu} \) is given by that of \( - V_{bb}[V_{b\mu} + V_{a\mu}] \). This is positive since \( V_{bb} < 0 \) and \( V_{b\mu} + V_{a\mu} = w_0^0/d'' > 0 \).

Similarly, \( \frac{d w_B}{d \mu} = \frac{(V_{ab}V_{a\mu} - V_{aa}V_{b\mu})}{(V_{ab}V_{bb} - V_{b\mu}^2)} \). Evaluated at \( \mu = 1 \), \( e = 0 \), the numerator of this expression can be calculated using (21), (23) and (27) to be given by \( 2[R - w_A^0]/R'' - 2[w_A^0 - w_B^0 - d']/d'' + [R - w_A^0]^2 R''' / R''^3 - [w_A^0 - w_B^0 - d']^2 d''/d''^3 \). This cannot be signed in general. When \( d''' = R''' \neq 0 \), the fact that \( V_a = 0 \) and \( A > 0 \) at the optimum imply from (19) that \( \frac{d w_B}{d \mu} < 0 \).

Finally, \( \frac{d w_A}{d \mu} > \frac{d w_B}{d \mu} \) if \( V_{ab}V_{b\mu} - V_{bb}V_{a\mu} > V_{ab}V_{a\mu} - V_{aa}V_{b\mu} \). Since \( V_{ab} = - V_{bb} \), the inequality is satisfied if \( V_{b\mu}[V_{aa} - V_{bb}] > 0 \). Now \( V_{aa} < V_{bb} < 0 \) and \( V_{b\mu} - B + [w_B^0 - w_B^0] (\partial B / \partial w_B) \). At the optimum for the union \( V_b = 0 \Rightarrow B + [w_B^0 - w_B^0] (\partial B / \partial w_B) = - [w_A - w_A^0] (\partial A / \partial w_B) < 0 \). This proves the result.

Proof of Lemma 1

Let \( w_A(\lambda) = w_A^0, w_B(\lambda) = w_B^0 \) and \( \lambda \in (0, 1] \). Evaluating \( V \) at these values gives \( V = 0 \) and hence \( V^\lambda U^{1-\lambda} = 0 \). From Assumption 2, \( U(w_A^0, w_B^0) > 0 \). By continuity there exists \( \epsilon > 0 \) such that \( U(w_A^0 + \epsilon, w_B^0 + \epsilon) > 0 \). By continuity there exists \( \epsilon > 0 \) such that \( U(w_A^0 + \epsilon, w_B^0 + \epsilon) > 0 \) and so it follows that \( V(w_A^0 + \epsilon, w_B^0 + \epsilon) \) is strictly positive. The pair \( (w_A^0 + \epsilon, w_B^0 + \epsilon) \) therefore give a higher value of the maximand than \( (w_A^0, w_B^0) \) so that \( (w_A^0, w_B^0) \) cannot be the solution to the maximisation. This proves (i).

Now assume that \( w_A(\lambda) = w_A^0, w_B(\lambda) > w_B^0 \). It follows that if this is a corner solution, then

\[
\lambda V_a U + (1 - \lambda) V U_a < 0, \quad \lambda V_b U + (1 - \lambda) V U_b = 0.
\]

Given (i), these equations are equivalent to

\[
U_a V_b < U_b V_a.
\]
Now using (15), (19) and (20), this inequality can be written
\[ B + \frac{d'}{d''} - \frac{(w_A^0 - w_B^0)}{d''} \leq \left[ A + \frac{R'}{R''} - \frac{d'}{d''} + \frac{(w_A^0 - w_B^0)}{d''} - \frac{w_A^0}{R''} \right] \cdot \left[ - A \right]. \]

Since by assumption \( R' = w_A(\lambda) = w_A^0 \), the inequality reduces to
\[ B + \frac{d'}{d''} - \frac{(w_A^0 - w_B^0)}{d''} \leq \left[ A + \frac{(w_A^0 - w_B^0)}{d''} - \frac{d'}{d''} \right] \cdot \left[ - A \right]. \]

Using the fact that \( w_B(\lambda) + d' = w_A = w_A^0 \) and collecting terms, this is equivalent to
\[ \frac{A(w_B - w_B^0)}{d''} + \frac{B(w_B - w_B^0)}{d''} < 0, \]
which is false since by assumption \( w_B(\lambda) > w_B^0 \). Therefore \( w_A(\lambda) = w_A^0 \), \( w_B(\lambda) > w_B^0 \) cannot be a corner solution.

Considering \( w_A(\lambda) > w_A^0 \), \( w_B(\lambda) = w_B^0 \) as a possible corner solution implies that
\[ \lambda V_a U + (1 - \lambda) V U_a = 0, \quad \lambda V_b U + (1 - \lambda) V U_b < 0, \]

or
\[ B + \frac{d'}{d''} - \frac{(w_A^0 - w_B^0)}{d''} \leq \left[ A + \frac{R'}{R''} - \frac{d'}{d''} + \frac{(w_A^0 - w_B^0)}{d''} - \frac{w_A^0}{R''} \right] \cdot \left[ B \right]. \]

which, using the assumption that \( w_B(\lambda) + d' = w_B^0 + d' = w_A(\lambda) \) reduces to
\[ B - \frac{(w_A^0 - w_A(\lambda))}{d''} \leq \left[ A + \frac{R'}{R''} + \frac{(w_A^0 - w_A(\lambda))}{d''} \right] \cdot \left[ B \right]. \]

As \( R' = w_A(\lambda) > w_A^0 \), the left-hand side of the inequality is greater than 1 and the right-hand side less than 1. Therefore the inequality is false and contradicts the assumption that \( w_A(\lambda) > w_A^0, w_B(\lambda) = w_B^0 \) occurs as a corner solution. This proves (ii) of the lemma.

(iii) follows from noting that since the maximum of \( V^{\lambda} U^{1-\lambda} > 0 \), then for \( \lambda > 0, V > 0 \).
Proof of Theorem 1

Substitute into $V_bU_a + V_aU_b$ from (15), (19) and (20) to give

$$\left[ B + \frac{d'}{d''} - \frac{(w_A^0 - w_B^0)}{d''} \right] [-A] = [-B]$$

$$\times \left[ A + \frac{R'}{R''} - \frac{d'}{d''} + \frac{(w_A^0 - w_B^0)}{d''} - \frac{w_A^0}{R''} \right].$$

From this,

$$A \equiv \frac{\left[ \frac{R'}{R''} - \frac{d'}{d''} + \frac{(w_A^0 - w_B^0)}{d''} - \frac{w_A^0}{R''} \right]}{\left[ \frac{d'}{d''} - \frac{(w_A^0 - w_B^0)}{d''} \right]}.$$

(A.1)

Since Lemma 1 proves that $w_A^0, w_B^0$ is not a solution to the maximisation, it follows from the satisfaction of the first-order conditions that both $[R'/R'' - d'/d'' + (w_A^0 - w_B^0)/d'' - w_A^0/R'']$ and $[d'/d'' - (w_A^0 - w_B^0)/d'']$ must be non-zero and of the same sign. Now assume that $[d'/d'' - (w_A^0 - w_B^0)/d''] > 0$. It then follows that, in order to satisfy (A.1), $[R'/R'' - d'/d'' + (w_A^0 - w_B^0)/d'' - w_A^0/R'']$ must be positive. However, since $R' = w_A^0 > w_A^0$ and $R'' < 0$, when $[d'/d'' - (w_A^0 - w_B^0)/d''] > 0$, $[R'/R'' - d'/d'' + (w_A^0 - w_B^0)/R''] < 0$. Therefore, to satisfy (A.1), $[d'/d'' - (w_A^0 - w_B^0)/d''] < 0$ or $(w_A^0 - w_B^0) - d' > 0$ which proves the theorem.

Proof of Lemma 2

Since $|H| = [U^2 V^{1-2}]^2 |J|$, Assumption 4 and Lemma 1 imply that $|J| > 0$. The Jacobian of the mapping defined by (33) and (34) is therefore non-zero at each local maximum. The lemma is then proved by application of the implicit function theorem in the neighbourhood of each local maximum.

Proof of Lemma 3

It is clear that $\lim_{\lambda \to 0} w_A(\lambda) < w_A^0$ and $\lim_{\lambda \to 0} w_B(\lambda) < w_B^0$ can be easily ruled out since, by continuity, this would give a negative value to the union for positive $\lambda$ close to 0. This cannot be a solution, so $\lim_{\lambda \to 0} w_A(\lambda) \geq w_A^0$ and $\lim_{\lambda \to 0} w_B(\lambda) \geq w_B^0$. Now assume that the optimal solution is such that $\lim_{\lambda \to 0} w_A(\lambda) > w_A^0$ and $\lim_{\lambda \to 0} w_B(\lambda) = w_B^0$. Now take any path $w'(\lambda)$ with the properties that for any $\lambda$, $0 < \lambda < \varepsilon$, $w_A^0 < w'(\lambda) < w_A(\lambda)$ and $w_A^0 < \lim_{\lambda \to 0} w'(\lambda) < \lim_{\lambda \to 0} w_A(\lambda)$. Since $U$ is decreasing in $w_A$ and $w_B$
it follows that it is possible to find $\tilde{\lambda}$ sufficiently small that
\[ U'(w'_A(\tilde{\lambda}), w'_B(\tilde{\lambda}))^{1-\frac{1}{\lambda}} V'(w'_A(\tilde{\lambda}), w'_B(\tilde{\lambda}))^{1-\frac{1}{\lambda}} > U'(w_A(\tilde{\lambda}), w_B(\tilde{\lambda}))^{1-\frac{1}{\lambda}} V'(w_A(\tilde{\lambda}), w_B(\tilde{\lambda}))^{1-\frac{1}{\lambda}}. \]
This inequality contradicts the optimality of the proposed solution. Choosing
a path $w_B^0(\lambda)$ with $w_B^0 < w_B(\lambda) < w_B(\lambda)$ for any $\lambda$ such that $0 < \lambda < \varepsilon$, shows by
the same argument that $\lim_{\lambda \to 0} w_A(\lambda) = w_A^0$, $\lim_{\lambda \to 0} w_B(\lambda) > w_B^0$ cannot be a solution. Finally, taking any pair of paths $w_A^0(\lambda), w_B^0(\lambda)$ with the properties that for
any $\lambda$ such that $0 < \lambda < \varepsilon$, $w_A^0 < w_A^0(\lambda) < w_A^0(\lambda)$, $w_B^0 < w_B^0(\lambda) < w_B^0(\lambda)$ and
$w_A^0 < \lim_{\lambda \to 0} w_A^0(\lambda) < \lim_{\lambda \to 0} w_A^0(\lambda)$ and $w_B^0 < \lim_{\lambda \to 0} w_B^0(\lambda) < \lim_{\lambda \to 0} w_B^0(\lambda)$ shows that, $\lim_{\lambda \to 0} w_A^0(\lambda) > w_A^0$, $\lim_{\lambda \to 0} w_B^0(\lambda) > w_B^0$ cannot be a solution. Therefore
$\lim_{\lambda \to 0} w_A^0(\lambda) = w_A^0$ and $\lim_{\lambda \to 0} w_B^0(\lambda) = w_B^0$, as was to be proved.

**Proof of Theorem 2**

The first-order conditions for the maximisation of (30) can be written
\[ \lambda V_a U + (1 - \lambda) U_a V = 0, \]
and
\[ \lambda V_b U + (1 - \lambda) U_b V = 0. \]

Considering variations in $w_A$, $w_B$ and $\lambda$ gives the system
\[
\begin{bmatrix}
\lambda (V_{aa} U + V_a U_a) + (1 - \lambda)(U_{aa} V + V_a U_a) & \lambda (V_{ab} U + V_a U_b) + (1 - \lambda)(U_{ab} V + V_b U_a) \\
\lambda (V_{ab} U + V_b U_a) + (1 - \lambda)(U_{ab} V + V_a U_b) & \lambda (V_{bb} U + V_b U_b) + (1 - \lambda)(U_{bb} V + V_b U_b)
\end{bmatrix}
\]

\[
\begin{bmatrix}
w_A \\
w_B
\end{bmatrix}
= \left[
\begin{bmatrix}
(U_a V - V_a U) d\lambda \\
(U_b V - V_b U) d\lambda
\end{bmatrix}
\right].
\]

The matrix on the left-hand side is the Jacobian $J$. From Assumption 4,
$|J| > 0$ at each local maximum. Solving the system gives
\[
dw_A/d\lambda = \frac{1}{|J|} \left[
(U_b V - V_b U)(\lambda(V_{aa} U + V_a U_a) + (1 - \lambda)(U_{aa} V + V_a U_a))
-(U_a V - V_a U)(\lambda(V_{ab} U + V_a U_b) + (1 - \lambda)(U_{ab} V + V_b U_a))
\right],
\]
\[
dw_B/d\lambda = \frac{1}{|J|} \left[
(U_b V - V_b U)(\lambda(V_{aa} U + V_a U_a) + (1 - \lambda)(U_{aa} V + V_a U_a))
-(U_a V - V_a U)(\lambda(V_{ab} U + V_a U_b) + (1 - \lambda)(U_{ab} V + V_b U_a))
\right].
\]

Therefore $dw_B/d\lambda > dw_A/d\lambda$ if
\[
\begin{bmatrix}
(U_b V - V_b U)(\lambda(V_{aa} U + V_a U_a) + (1 - \lambda)(U_{aa} V + V_a U_a)) \\
-(U_a V - V_a U)(\lambda(V_{ab} U + V_a U_b) + (1 - \lambda)(U_{ab} V + V_b U_a))
\end{bmatrix}
\]
\[
> \left[
(U_b V - V_b U)(\lambda(V_{aa} U + V_a U_a) + (1 - \lambda)(U_{aa} V + V_a U_a))
-(U_a V - V_a U)(\lambda(V_{ab} U + V_a U_b) + (1 - \lambda)(U_{ab} V + V_b U_a))
\right].
\]
Using (35), substitution into the first-order conditions yields the identities

\[ (U_a V - V_a U) = \frac{1}{\lambda} U_a V = -\frac{1}{1 - \lambda} V_a U < 0, \]

and

\[ (U_b V - V_b U) = \frac{1}{\lambda} U_b V = -\frac{1}{1 - \lambda} V_b U < 0. \]

Using these in the previous inequality reduces it to

\[ U_b V(V_{aa} U + V_a U_a) - V_b U(U_a V + V_a U_a) - U_a V(V_{ab} U + V_b U_a) + V_a U(U_{ab} V + V_a U_b) \]

\[ > U_a V(V_{bb} U + V_b U_b) - V_a U(U_{bb} V + V_b U_b) - U_b V(V_{ab} U + V_b U_a) + V_b U(U_{ab} V + V_a U_b). \]

Cancelling terms, and using (35) again, provides the inequality

\[ VU [(V_{aa}U_b - V_{ab} U_a) + (U_{aa} V_a - V_b U_{aa})] > VU [(V_{bb}U_a - V_{ab} U_b) + (U_{aa} V_b - V_a U_{bb})]. \]

Simplifying again and collecting terms,

\[ U_b(V_{aa} + V_{ab}) - U_a(V_{bb} + V_{ab}) + V_a(U_{bb} + U_{ab}) - V_b(U_{aa} + U_{ab}) > 0. \]

Using (15), (16) and (19)–(23), the above inequality can be evaluated as

\[ -\frac{B}{R''} + \frac{B(R' - w_0^A)R''}{(R'')^3} - \frac{(w_A^0 - w_B^0 - d')}{d''R''} > 0. \]  \( \text{(A.2)} \)

It is clear from Assumption 1, Theorem 1 and (A.2) that the inequality is satisfied.

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References


