ECONOMIC Mismeasurement and the Bias in Policy Choice

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Abstract

The level of economic activity is never measured perfectly because of problems of definition, inaccuracies in data collection, and the existence of the hidden economy. Such mismeasurement implies that government policies based on official statistics can be optimal only by chance. The analysis formalizes this observation in a two-sector economy and attempts to quantify the direction and extent of the bias introduced into policy by the failure to account for the true size of the economy. It is shown that short-term reform (which need not balance the government budget) can be detrimental. When a budget constraint is imposed, this ensures that reforms will be beneficial no matter how bad is the mismeasurement.

1. Introduction

Government statistics on economic activity can never be perfect. There are issues of definition that remain unresolved, such as whether criminal acts\(^1\) or work in the home\(^2\) should be included in national income, and, for any given definition, there will be errors in collecting and compiling statistics. For instance, rural production that is consumed at home or traded on informal markets will not be measured. Also not included, and

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\(^1\)In principle, the UN System of National Accounts includes both legal and illegal activities. It has been suggested that criminal activity should be made explicit when the system is revised (see Blade 1985; Carson 1984). The Italian national accounts include estimates for smuggling and domestic distribution of cigarettes. In addition, when the proceeds of crime are spent they are very likely to then appear in the accounts.

\(^2\)The benefits of including this and other such items have been discussed by Nordhaus and Tobin (1973), among others.

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arguably of great significance, is the hidden economy, which comprises activity that is deliberately concealed from measurement. Despite these shortcomings in the measurement of economic activity, governments are generally seen to base policy decisions upon their official published statistics. Since such statistics can disregard a significant fraction of total economic activity, there is a strong possibility that incorrect and potentially damaging policies will be adopted.

Although the potential magnitudes involved may be large, the consequences of such mismeasurement and false selection of policy have not previously been explored. What needs to be known is the extent of the bias that can be introduced into policy and whether there is anything systematic in this. If the bias is systematic then appropriate corrections could be made. As noted, mismeasurement can arise for a variety of reasons, but, in terms of analysis, they all have the same common structure. Essentially, an activity that is generating economic welfare is not recorded, so the effects of policy upon this are not taken into account. Consequently, a policy may be chosen that is harmful for the activity. If this happens, the actual level of economic welfare may fall even though the measured level rises.

To provide a framework in which these issues can be investigated, the paper considers a simple two-sector model that is intended to capture in a very stylized way the economy of a small developing country. The problem of mismeasurement is introduced into this economy by assuming that the output of the urban sector is traded on a formal market and so is subject to accurate measurement, but in contrast cultivation in the rural sector is not correctly measured. The failure to measure correctly arises from the fact that some rural production is consumed at home, so is never traded, and the output that is sold may be traded on informal markets where recording of sales information is limited. This fits naturally with the explanations for incorrect measurement of national income. The hidden economy is also particularly prevalent in some developing countries.

Having said this, the general principles that emerge from the analysis apply equally in the context of a developed economy.

Within this two-sector economy, the paper considers the issue of policy reform. This is undertaken for three different objective functions: measured national income, true national income, and economic welfare. The aim is to contrast the reforms that are optimal for each measure and from this to identify the direction of bias in policy caused by the mismeasurement and to determine its consequences. The reform problems are studied in two circumstances. In the first, which is termed short-term reform, it is assumed permissible for a surplus or deficit to be developed in trade balance (and by Walras’s law a corresponding deficit or surplus in govern-

\[^3\] Estimates of the extent of the hidden economy range up to about 30% of measured GDP. See Smith (1981) for a summary presentation of estimates.

\[^4\] See the evidence reported in Chugh and Uppal (1986) and Tanzi (1982).
ment revenue) on the reform path. This is meant to represent a reform that can only be undertaken over the short period in which such an imbalance can be sustained. Naturally, it has to be interpreted as the first step in a longer-term process that eventually must lead back to balance. The second formulation imposes budget balance from the outset and is termed balanced-budget reform. As the feasible directions of reform are more restricted in this case than for short-term reform, the initial welfare gain can never be larger.

The results obtained catalogue the possible outcomes that can arise and details the contrasts between them under the various objectives. When short-term reforms are undertaken, it is shown that true income will generally be reduced by a policy that is optimal given measured income. Balanced-budget reforms are more sensitive to misspecification of the objective function than short-term reform, and, as certain boundaries are passed, the direction of change in the policy instruments can switch signs. This arises from the interaction of the budget constraint and the direction of optimal reform. However, this does not imply that the reform will reduce true income or welfare. In fact the converse is true, and the balanced-budget reform will usually raise true income and welfare no matter how badly measured is income.

These findings illustrate the two different aspects of the bias problem: that of the direction of change in policy variables and that of the change in income or welfare. Even if the use of measured income results in a change in policy that moves in the wrong direction, the change may still increase true income or welfare. Contrasting short-term and balanced-budget reforms, the budget constraint can be seen to have two distinctly different roles. On one hand it acts to enforce differences in directions of change. On the other, it has the effect of ensuring that these differences are not too great. In this second role the budget constraint is a form of discipline device that keeps the direction of reform within acceptable bounds.

In respect of the general consequence of mismeasurement, these findings have a number of implications. First, they show that the application of standard techniques in a novel setting can provide insight into the problem, whereas at the outset it might have been felt that the “consequences of mismeasurement” would be too vague an inquiry. Second, although the results do identify cases where the policy based on measured income can be damaging, these are not especially widespread. There is something reassuring in the observation that even if policy is chosen essentially “in the dark,” the outcome may not be too disastrous. The final implication is the role of the budget constraint in giving support to the previous remark. What can be called the “discipline of the budget constraint” ensures that chosen policy will not deviate too far from what should have been chosen.

Section 2 describes the two-sector economy that is analyzed and characterizes its equilibrium. The general forms of the policy-reform problems
are described in Section 3. Short-term reforms are then analyzed in Section 4 and balanced-budget reforms in Section 5. Conclusions are given in Section 6. All proofs are collected in the Appendix.

2. The Two-Sector Economy

The economy consists of two sectors: one rural and one urban. The rural sector uses labor and a fixed amount of land to produce a single output. Part of this output is consumed within the rural sector and the remainder sold on the urban market. The urban sector uses labor and a fixed stock of capital to produce an output that is exported. Both the urban and rural sectors consume an imported good. The economy is small, so it trades at given world prices. The policy instruments at the government’s disposal consist of a tax (or subsidy) on the rural good and a tariff (or subsidy) on the imported good. The profits generated by the urban sector are remitted abroad\(^5\) so the urban firm can be given the interpretation, for instance, of being a subsidiary of a foreign corporation.

Mismeasurement of economic activity is introduced by assuming that only a fraction of actual income is included in the government’s measure of national income. This fraction is taken to remain constant as policy changes since this provides some helpful simplification of the analysis. In the cases of incorrectly defined national income and difficulties in obtaining data, this is a reasonable assumption. With a hidden economy interpretation, it is a little less appealing since the extent of tax evasion can be expected to be dependent upon the tax rates. However, some support for it in the present context follows from the fact that the taxes considered are not levied on income so do not have a direct bearing on the evasion decision. If income taxes were considered, the assumption would be less tenable.

A formal description of the model is now given. The individual optimization problems are described, and from these the economy’s equilibrium conditions are constructed. Some results are then established that are used in later sections of the paper.

2.1 Rural Sector

Rural production requires the input of land and labor. Since the quantity of land is fixed, the level of output, \(y_r\), is determined by the labor input alone via the production function

\[
y_r = f'(\ell),
\]

\(^5\)An equivalent assumption is that profit in the urban sector is taxed at a rate of 100% and revenue spent on the imported good. The only change is to the interpretation. Alternatively, the urban sector could be treated as a nationalized industry. Again, this only changes the interpretation.
where \( \ell_r \) is the labor supply of the representative rural consumer. Some of this production is consumed by the rural household, and the remainder is traded for the imported good. Denoting the consumer price of the imported good by \( q_i \) and the producer price of the rural good by \( p_r \), the resulting budget constraint is

\[
q_i x'_i = p_r [y_r - x'_r],
\]

where \( x'_r \) is rural consumption of the import and \( x'_r \) consumption of the rural good. With preferences described by the strictly quasiconcave utility function \( U'^*(x'_r, x'_r, \ell_r) \), the rural household solves the optimization

\[
\max_{(x'_r, x'_r, \ell_r)} U'^*(x'_r, x'_r, \ell_r), \quad \text{subject to: } y_r = f'*(\ell_r), \quad q_i x'_i = p_r [y_r - x'_r].
\]

The solution to (3) generates demand functions

\[
x'_j = x'_j \left( \frac{p_r}{q_i} \right), \quad j = r, i,
\]

a labor supply function

\[
\ell_r = \ell_r \left( \frac{p_r}{q_i} \right),
\]

and an output supply function

\[
y_r = f'^* \left( \ell_r \left( \frac{p_r}{q_i} \right) \right) = y_r \left( \frac{p_r}{q_i} \right).
\]

Substituting (4) and (5) into the utility function determines the rural household’s indirect utility function

\[
V'^* = V'^* (p_r, q_i).
\]

The structure of the decision problem in (3) requires some further comment. When purchases of the rural good are taxed, it makes obvious sense to assume that the rural household consumes it directly rather than purchases it through the market. In contrast, when sales are subsidized, the specification chosen is not so obviously appealing since the household would then benefit from selling its entire production to the market and then purchasing what it needs at the subsidized price. What eliminates this behavior is the implicit assumption that the rural households are separated by geographical distance from the urban market place and cannot directly trade upon it. To support this assumption necessitates the existence of intermediaries who, acting in a competitive manner, purchase from the rural household, transport the good to the urban market, and then sell to urban households. Since these intermediaries act competitively, they do not need explicit modeling. The subsidy or tax is intro-
duced at the point where the intermediaries trade with the urban households and therefore does not directly affect the behavior of the rural household.\footnote{Allowing the rural household to purchase at subsidized prices also introduces a nonconvexity into the budget set. An alternative resolution to this problem (e.g., applied in Shoven 1974) is to rule out subsidies. This second approach seems less attractive than that adopted here.}

### 2.2 Urban Production

The representative urban firm combines labor with a fixed stock of capital to produce output subject to the production function \( y_u = f(u(\ell_u)) \). Denoting the fixed world price of their output by \( p_u \) and the urban wage by \( w_u \), the production plan, \( \{ y_u, \ell_u \} \), solves the optimization

\[
\max_{\{y_u, \ell_u\}} p_u y_u - w_u \ell_u \quad \text{subject to: } y_u = f(u(\ell_u)).
\]

(8)

From (8), profit maximization by the urban firm determines its labor demand as

\[
\ell_u^d = \ell_u^d \left( \frac{w_u}{p_u} \right),
\]

(9)

and its output

\[
y_u = y_u \left( \frac{w_u}{p_u} \right).
\]

(10)

As already noted, profit earned in the urban sector is remitted abroad.

### 2.3 Urban Consumption

The urban household supplies labor to the urban firm and consumes the rural good and the imported good. With preferences represented by the strictly quasiconcave utility function \( U^u(x^u_r, x^u_i, \ell_u) \), the optimization problem of the representative urban household is

\[
\max_{\{x^u_r, x^u_i, \ell_u\}} U^u(x^u_r, x^u_i, \ell_u) \quad \text{subject to: } q_r x^u_r + q_i x^u_i = w_u \ell_u.
\]

(11)

The optimization in (11) generates demand functions

\[
x^u_j = x^u_j(q_r, q_i, w_u), j = r, i,
\]

(12)

and labor supply function

\[
\ell_u = \ell_u(q_r, q_i, w_u).
\]

(13)
2.4 Equilibrium

The equilibrium of the economy can now be characterized. There are four prices to be determined: \( p_u, p_i, p_r, w_u \). The small-country assumption implies that the producer prices of the urban good, \( p_u \), and the imported good, \( p_i \), are fixed at the world level. Either of these can be adopted as the numeraire. This leaves only the price of the rural good, \( p_r \), and the urban wage, \( w_u \), to determine.

Before doing this, note that in the absence of taxation if the markets for urban labor and the rural good are in equilibrium then, by Walras’s law, the country is in trade-balance.\(^7\) When taxes are introduced, trade may be in surplus or deficit dependent on whether the government is in surplus or deficit. In the analysis of Section 4 trade-balance is not enforced, whereas it is in Section 5. Even when trade-balance is not imposed, it is assumed that equilibrium is attained on the internal markets for the rural good and urban labor.

Equilibrium on the internal markets is determined by an urban wage rate and a rural commodity price that satisfy

\[
\ell^u(q_r, q_i, w_u) = \ell^u\left(\frac{w_u}{p_u}\right) \tag{14}
\]

and

\[
x^r_\tau\left(\frac{p_r}{q_r}\right) + x^u(q_r, q_i, w_u) = y_i\left(\frac{p_i}{q_i}\right) \tag{15}
\]

for given \( p_u \) and \( p_i \), where

\[
q_r = p_i + t \tag{16}
\]

and

\[
q_i = p_i + \tau, \tag{17}
\]

with \( t \) and \( \tau \) being the tax and tariff, respectively. When taxes are zero, standard theorems on the existence of equilibrium guarantee that there is a solution to (14) and (15).

Substituting from (16) and (17) into the equilibrium conditions (14) and (15) and applying the implicit function theorem, the equilibrium producer price of the rural good and the urban wage are determined as functions of the policy instruments

\[
p_r = p_r(t, \tau) \tag{18}
\]

\(^7\)This refers to trade balance on the current account. There may be a deficit on the capital account depending on the treatment on profits (see footnote 5).
2.5 Tax and Price Effects

The purpose of this section is to determine the effect of changes in the tax and tariff upon the economy's equilibrium prices. These are derived from an analysis of market clearing conditions. The section is completed by an analysis of the welfare effects of price changes.

In order to generate results, it is necessary to place some restrictions upon the utility and production functions. The restrictions contained in Assumption 1 are adopted from this point onward.

**Assumption 1:**

(i) The production and utility functions satisfy

1. Rural production function: \( f' > 0, f'' < 0 \);
2. Rural utility function: \( U_1' > 0, U_2' > 0, U_3' < 0 \);
3. Urban production function: \( f'' > 0 \) and \( f''' < 0 \);
4. Urban utility function: \( U_1'' > 0, U_2'' > 0, U_3'' < 0 \).

(ii) The demand for the imported good from the rural household satisfies

\[
\left( q_i/x_i \right) \left( \partial x_i'/\partial q_i \right) < -1. 
\]

The conditions in (i) are standard. The condition in (ii) requires that the demand elasticity for the imported good from the rural household is greater than 1. This is consistent with the view that the imported good is some form of luxury rather than a necessity.

The demand responses of the urban household are easily dealt with. The optimization described in (11) is entirely standard in form, so the responses will be the usual resolution of income and substitution effects. It is assumed from this point that all goods are normal for the urban household and that the two consumption goods are gross substitutes. Additional restrictions that are adopted will be introduced where necessary.

Turning to the rural household, the situation is somewhat different. Since the household is engaging in production, its response to price changes is not entirely that of the standard household. The following result describing the effect upon the household is employed extensively below. The proof of this result, and of all those that follow, is contained in the Appendix.

**Lemma 1:** The optimal choices of the rural household satisfy

\[
\frac{dx_i'}{dx} - \frac{df'}{d\ell} \frac{d\ell}{dx} \frac{bp_i}{q_i} = x_i' - y_i' < 0. 
\]
The interpretation of Lemma 1 is that elastic demand for the imported good implies the surplus of production over consumption of the produced good for the rural household increases as the selling price rises. It would not have been unreasonable to impose this directly.

The next result shows how equilibrium prices are affected by changes in the policy instruments.

**LEMMA 2:** If (a) the urban labor supply function (13) satisfies \( \ell_u \geq 0 \) and \( \ell_u \geq 0 \), where \( \ell_{uj} \) is the partial derivative with respect to the \( j \)th argument, (b) the urban demand for the rural good (12) satisfies \( x_r^1 < 0 \), \( x_r^2 > 0 \), and \( x_r^3 > 0 \), and (c) \( \ell_u - \ell_u^{dr}/p_u > 0 \), then

(i) \( \partial p_r/\partial t < 0 \)

(ii) \( \partial w_u/\partial t < 0 \), \( \partial w_u/\partial \tau < 0 \).

Furthermore, if \( \ell_u^2 = 0 \) then

(iii) \( \partial p_r/\partial \tau > 0 \).

Conditions (a) and (b) are standard. Condition (c) can be written as \( (\partial/\partial w)[\ell_u - \ell_u^{dr}] > 0 \), which shows that it is the mild restriction that the excess supply of labor is increasing in the nominal wage. The condition \( \ell_u^2 = 0 \) used for (iii) is sufficient; the necessary condition is much weaker.

The final result of this section prepares for the welfare analysis by determining the effects of price changes upon the utility levels of the rural and urban households.

**LEMMA 3:**

(i) The rural indirect utility function has the following properties:

\[
\frac{\partial V_r}{\partial q_r} = -\alpha^r x_r^i
\]

and

\[
\frac{\partial V_r}{\partial p_r} = \alpha^r \frac{q_i}{p_r} x_r^i,
\]

where \( \alpha^r \) is the rural marginal utility of income.

(ii) The urban indirect utility function satisfies

\[
\frac{\partial V_u}{\partial q_r} = -\alpha^u x_r^u, \quad \frac{\partial V_u}{\partial q_i} = -\alpha^u x_i^u, \quad \frac{\partial V_u}{\partial w_u} = \alpha^u \ell_u.
\]
3. Income, Welfare, and Reforms

The focus of the paper is the contrast between the policy reforms that are optimal under alternative measures of income and welfare. The analysis is designed to capture the observation that the government formulates policy given its observation of income and that this level of income differs from the true level. This section introduces the measures of income and welfare and describes the modeling of the policy reform process.

The alternative measures of income and welfare considered below are

(i) measured national income;
(ii) true national income;

and

(iii) social welfare.

Each of these is now described formally in the context of the two-sector economy of Section 2.

Denoting the government’s measure of the level of national income by $Y^M$, the proportion of rural income observed by the government by $\theta_r$, and the proportion of urban income observed by $\theta_u$, it follows that

$$Y^M = \theta_r p_r y_r + \theta_u w_u \ell_u,$$

where $0 \leq \theta_r \leq 1$, $0 \leq \theta_u \leq 1$. Urban profit does not appear in this expression since it is remitted abroad. Corresponding to (24), the true level of national income, $Y^T$, is determined as

$$Y^T = p_r y_r + w_u \ell_u.$$

Both (24) and (25) can be seen as proxies for the true level of economic welfare, $W$, which, with a utilitarian objective, is

$$W = \phi^r V^r(p_r, q_r) + \phi^u V^u(q_u, q_r, w_u),$$

where $\phi^r \geq 0$ and $\phi^u \geq 0$.

The policy reforms that are studied here consist of small changes in the government’s policy instruments. It is assumed that prior to reform, the government’s budget is in balance. After an arbitrary reform, this need no longer be the case. Two distinct scenarios are therefore considered. In the first, called short-term reform, the government is permitted to run a deficit or surplus after the reform. Since such a reform cannot be sustained in the long run, this motivates the study of balanced-budget reform. This is defined as a reform that must ensure that budget-balance is preserved. Naturally, this additional condition restricts the feasible set of reforms.

The basic methodology for studying reform problems introduced by Guesnerie (1977) characterized the set of directions of policy change that
lead to an increase in the government’s objective function.\textsuperscript{8} This was modified by Dixit (1979) to that of finding the optimal reform that maximized the increase in the objective function for a permissible size of change. Restricting the feasible changes in policy to be “small” captures the idea of a reform rather than a full optimization. It is this latter approach that is adopted here. Consequently, only changes $dt$ in the tax and $d\tau$ in the tariff that satisfy the condition\textsuperscript{9}

$$|dt|^2 + |d\tau|^2 = 1$$

are permitted. The constraint in (27) implies the tax changes must lie on the unit circle.

\section*{4. Short-Term Reforms}

This section begins by defining the short-term reform problems for the different measures and describes the geometry of their solutions. The optimal reform based on measured income is then analyzed, and the central role played by the ratio of the proportion of rural income measured to the proportion of urban income measured is made clear. The reforms based on measured income are then contrasted to those based on true income and then to those based on welfare.

\subsection*{4.1 The Reform Problems}

Using the observed measure of national income, (24), the short-term reform that would appear optimal to the government based on measured income solves the maximization

\begin{equation}
M: \max_{\{dt, d\tau\}} dY^M = \frac{\partial Y^M}{\partial t} dt + \frac{\partial Y^M}{\partial \tau} d\tau \quad \text{subject to (27),}
\end{equation}

The reform that would maximize the increase in correctly measured national income actually solves

\begin{equation}
T: \max_{\{dt, d\tau\}} dY^T = \frac{\partial Y^T}{\partial t} dt + \frac{\partial Y^T}{\partial \tau} d\tau \quad \text{subject to (27),}
\end{equation}

while the reform that maximizes the increase in social welfare is

\begin{equation}
W: \max_{\{dt, d\tau\}} dW = \frac{\partial W}{\partial t} dt + \frac{\partial W}{\partial \tau} d\tau \quad \text{subject to (27).}
\end{equation}

\textsuperscript{8}Detailed coverage of this and later developments can be found in Myles (1995).

\textsuperscript{9}This form of constraint was introduced by Dixit (1979) and also employed in Kanbur and Myles (1992). The interpretation is that units are chosen so that “1” is small.
The optimizing values for these reform problems are denoted $(dt^M, d\tau^M)$, $(dt^T, d\tau^T)$, and $(dt^W, d\tau^W)$, respectively.

Geometrically, the reform problems $M$, $T$, and $W$ have a common structure, and much use will be made of this fact later. To understand this structure, consider problem $M$. Defining $\nabla Y^M = (\partial Y^M/\partial t, \partial Y^M/\partial \tau)$ and $dT = (dt, d\tau)$, the objective function for $M$ becomes $dY^M = \nabla Y^M \cdot dT$ and the constraint $dT \cdot dT = 1$. The optimal reform is the vector $dT^M = (dt^M, d\tau^M)$ on the unit circle that points in the same direction as $\nabla Y^M$, since this is the vector that maximizes the product $\nabla Y^M \cdot dT$. See Figure 1. An alternative expression of this solution is obtained by defining the hyperplane $dY^M = 0$ as the set $\{dt, d\tau: \nabla Y^M \cdot dT = 0\}$. This hyperplane passes through the origin and is orthogonal to $\nabla Y^M$. Furthermore, the solution vector $dT^M$ is also orthogonal to this hyperplane.

The basis for comparing the solutions to the reform problems can now be explained. From Figure 1 it can be seen that for problem $M$ it is the direction of the vector $\nabla Y^M$ that determines the direction of reform. Similarly, for the reform problems $T$ and $W$, whose objectives can be written $dY^T = \nabla Y^T \cdot dT$ and $dW = \nabla W \cdot dT$, respectively, the solutions will depend upon the directions of the vectors $\nabla Y^T$ and $\nabla W$. Given these observations, the bias in policy arising from using measured income can be found by determining the extent to which the mismeasurement of the economy changes the direction of the vector $\nabla Y^M$ relative to $\nabla Y^T$ and $\nabla W$. This, in turn, can be traced back through the structure of economy described in Section 2.

![Figure 1: The solution to $M$.](image_url)
The analysis of reform problem $M$ begins by determining how the solution is related to the relative degree of mismeasurement of the two sectors. Since relative mismeasurement is captured by the value of $\theta_u/\theta_u$, and the solution vector $dT^M$ points in the same direction as $\nabla Y^M$, this is achieved by relating $\theta_u/\theta_u$ to the direction of the vector $\nabla Y^M$. This is the content of Lemma 4, which shows that the effect of a change in the relative degree of mismeasurement is determined by the technology of urban production.

One further piece of notation is required. The slope of the vector $\nabla Y^M$, which is given by $(\partial Y^M/\partial \tau)/(\partial Y^M/\partial t)$, is denoted $S(\nabla Y^M)$. (This is also the tangent of the angle between the vector and the horizontal axis.)

**Lemma 4:** If the urban production function satisfies $\ell_u f'' u f''' < -1$, then an increase in $\theta_u/\theta_u$ reduces $S(\nabla Y^M)$. The converse is true if $\ell_u f'' u f''' > -1$.

The condition that $\ell_u f'' u f''' < -1$ is essentially a requirement that the urban production function must exhibit a sufficient degree of decreasing returns to scale. As constant returns to scale are approached, $\ell_u f'' u f''' \to 0$ from below. The analysis below will consider only the case of $\ell_u f'' u f''' < -1$, so that a degree of decreasing returns is assumed for urban production. This agrees with the interpretation of the urban sector remitting profit to a foreign owner. Similar arguments to those presented here can easily be constructed for the other case.

It is now possible for the optimal reform to be related to $\theta_u/\theta_u$. The proof of Lemma 4 shows that $S(\nabla Y^M) = [(\theta_u/\theta_u) b_1 + b_2]/[(\theta_u/\theta_u) c_1 + c_2]$, where $b_1 > 0$, $b_2 < 0$, $c_1 < 0$, and $c_2 < 0$. From these facts it follows that $0 > \partial Y^M/\partial t = (\theta_u/\theta_u) c_1 + c_2$. $\nabla Y^M$ must therefore lie in the halfspace to the left of the $dt$ axis as shown in Figure 2. Recalling that the solution to $M$ lies along $\nabla Y^M$, this implies that $dt < 0$ at the solution. Furthermore,

$$\lim_{\theta_u \to 0} S(\nabla Y^M) = \frac{b_2}{c_2} > 0,$$

and

$$\lim_{\theta_u \to \infty} S(\nabla Y^M) = \frac{b_1}{c_1} < 0.$$  

Equation (31) implies that $\nabla Y^M$ is downward sloping for $\theta_u/\theta_u$ close to zero and (32) that it is upward sloping for large $\theta_u/\theta_u$. Combining these

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10These inequalities can be deduced from the proof of Lemma 4.
observations with Lemma 4 shows that the effect of increasing $\theta_r/\theta_u$ from a value close to zero is to rotate the $\nabla Y^M$ vector clockwise.

Referring to Figure 2 it can be seen that for $\theta_r/\theta_u$ close to zero the optimal reform is to reduce both the rural tax and the tariff. Increasing $\theta_r/\theta_u$ increases the optimal reduction in the rural tax until the change in the tariff is zero. Further increases in the value of $\theta_r/\theta_u$ make the reduction in the rural tax smaller and are accompanied by an increase in the tariff.

Now define $\theta^*_r/\theta_u$ by $S(\nabla Y^M)|_{(\theta_r/\theta_u)_{=(\theta^*_r/\theta_u)}} = 0$. This is the value at which the optimal policy is to leave the tariff unchanged. This construction is also shown in Figure 2. An interpretation of this value can be obtained by returning to the definition of $S(\nabla Y^M)$ in the proof of Lemma 4. From this it can be seen that $\theta^*_r/\theta_u$ satisfies

$$\theta_r \frac{\partial p_r y_r}{\partial \tau} + \theta_u \frac{\partial w_u \ell_u}{\partial \tau} = 0. \quad (33)$$

Hence $\theta^*_r/\theta_u$ is given by the relative weights that make the weighted aggregate income change equal to zero following a differential change in the tariff. This interpretation provides a simple implementable method of evaluating $\theta^*_r/\theta_u$.

The discussion above is summarized as Theorem 1.

THEOREM 1: The optimal reform for problem M defined in (28) is characterized as follows:
4.3 Contrasting $M$ and $T$

Theorem 1 has determined the dependence of the solution to $M$ on the value of $\theta^*_u/\theta_u$. These results can now be contrasted with the solution to $T$. Theorem 2 describes the differences in the direction of reform, while Theorem 3 determines when it is possible for a reform based on measured income to reduce true income.

Define the vector $\nabla Y^T = (\partial Y^T/\partial t, \partial Y^T/\partial \tau)$ and its slope $S(\nabla Y^T) = (\partial Y^T/\partial \tau)/(\partial Y^T/\partial t)$. From (24) and (25) it follows that

$$S(\nabla Y^T) = S(\nabla Y^M)|_{(\theta^*_u, \theta_u)} = 1.$$  

(34)

Hence $\nabla Y^T$ must lie between the two limiting cases of $\nabla Y^M$ determined by $\theta^*/\theta_u = 0$ and $\theta^*/\theta_u = \infty$, respectively. It is this observation that permits the contrast between the reform based on the measured economy and the optimal reform.

To undertake this contrast it is assumed that $\theta^*/\theta_u \equiv 0$. This is the limiting case in which the rural sector is entirely unmeasured. This assumption is made to simplify the analysis and allows the most precise results to be derived. There are also situations in which the justification for this is clear. The argument can easily be extended to consider other values.11

Denoting the optimal reforms arising from (29) by $dt^T$ and $d\tau^T$, Theorem 2 contrasts the solutions to (28) and (29).

THEOREM 2: The contrast between the optimal reforms for $M$ and $T$ is given by

- (i) If $\theta^*/\theta_u < \theta^*_u/\theta_u$ then $dt^M < 0$ and $d\tau^M < 0$;
- (ii) If $\theta^*/\theta_u = \theta^*_u/\theta_u$, then $dt^M < 0$ and $d\tau^M = 0$;
- (iii) If $\theta^*/\theta_u > \theta^*_u/\theta_u$, then $dt^M < 0$ and $d\tau^M > 0$.

All three of the cases identified in Theorem 2 have $0 > dt^M > dt^T$ and either $d\tau^M < d\tau^T \leq 0$ (cases (i) and (ii)) or $d\tau^M < 0 < d\tau^T$ (case (iii)). In consequence, the failure to measure income correctly results in an insufficient reduction in the commodity tax compared to what would follow from knowledge of the true level of income. In contrast, the tariff is reduced too far; in case (ii) the tariff is actually reduced when it should be

11 The details of this can be found in the working paper (available from the author), which gives the complete characterization for all possible values.
increased. The mechanism behind this finding is that measured income does not take full account of the benefit to the rural household from the increase in demand they face after the cut in the tax rate. Consequently, too much emphasis is placed upon the benefit of a tariff reduction.

To provide a perspective on which of the outcomes identified in Theorem 2 will actually arise, it is necessary to identify a value for \( u_r \geq 0 \). To do this, suppose that an increase in the tariff raises the income of rural households more than it reduces the income of urban households, so that \( \theta_r > \theta_u \). This is not unreasonable since if the import is a substitute for the rural good then demand for the rural good, and hence its price, will rise. From (33) it can then be seen that \( \theta^*_r/\theta_u < 1 \). This reasoning places the focus of interest on outcome (iii). Here the policy based on measured income reduces the commodity tax, though not sufficiently, but reduces the tariff when it should in fact be increased. Consequently, one policy instrument is changed in entirely the wrong direction.

Theorem 2 has isolated the differences in direction of the reforms based on measured and true income. What remains is to determine whether a reform based on the measured economy can actually reduce true income. If this is the case, it would be better for the government not to intervene until it has better information. Theorem 3 evaluates the true income change, \( \Delta Y^T \cdot dT^M \), for the three cases identified in Theorem 2.

To do this, define \( \theta_{r^*}^{\cdot}/\theta_u \) by

\[
\nabla Y^M|_{\theta_r^{\cdot}/\theta_u} \cdot \Delta Y^T = 0.
\]

(35)

Hence if the proportions of income measured equal \( \theta_r^{\cdot}/\theta_u \), then \( \nabla Y^M \) is orthogonal to \( \nabla Y^T \). It should be noted that, unlike \( \theta_r/\theta_u \), there is no necessity that \( \theta_r^{\cdot}/\theta_u \) should be non-negative.
THEOREM 3:

(i) If \( \theta^*/\theta_u \geq 1 \) then \( \nabla Y^T.dT^M > 0 \);

(ii) If \( \theta^*/\theta_u < 1 \) then

- \( \nabla Y^T.dT^M < 0 \) if \( \theta^{**}/\theta_u > 0 \),
- \( \nabla Y^T.dT^M = 0 \) if \( \theta^{**}/\theta_u = 0 \),
- \( \nabla Y^T.dT^M > 0 \) if \( \theta^{**}/\theta_u < 0 \).

The discussion following Theorem 2 has already given reasons for focusing on the case of \( \theta^*/\theta_u < 1 \). From Theorem 3 this can be seen to open the possibility that a reform based on measured income can actually reduce true income. The critical issue then becomes the value of \( \theta^{**}/\theta_u \), since the reform will reduce measured income if this is positive. The value of \( \theta^{**}/\theta_u \) is determined by the orthogonality of \( \nabla Y^M \) and \( \nabla Y^T \). Employing this fact gives

\[
\frac{\theta^{**}}{\theta_u} = \frac{-b_2[b_1 + b_2] + c_2[c_1 + c_2]}{b_1[b_1 + b_2] + c_1[c_1 + c_2]}.
\] (36)

If \( \theta^*/\theta_u < 1 \) (hence \( \partial/\partial \tau(p(y_e + w_u \epsilon_u) > 0) \) then \( b_1 + b_2 > 0 \), and the denominator of (36) is positive. Accepting this, \( \theta^{**}/\theta_u \) is positive when the numerator is negative. This requires \( |b_2[b_1 + b_2]| > |c_2[c_1 + c_2]| \), which is the condition that the marginal effect of the tariff on income is greater than that of the tax.

Collecting these observations allows the statement of Corollary 1.

COROLLARY 1: If an increase in the tariff raises total income and the tariff has a stronger effect on income than the tax, then \( \nabla Y^T.dT^M < 0 \) and the reform based on measured income reduces true income.

The content of Theorem 2 was to show the bias in the direction of reform due to the incorrect measurement of income. However, even though the policy is biased, it need not lead to a reduction in true income. In fact, Theorem 3 demonstrates that there are circumstances where the reform will raise true income despite being based on the wrong objective. Despite this, in the circumstances described in Corollary 1 the reform will reduce income. In this case the bias in policy is so significant that it becomes better not to reform at all.

4.4 Contrasting \( M \) and \( W \)

The contrast between the reform arising from \( M \) and that arising from \( W \) can be approached in the same way. To proceed with this requires the calculation of the direction of the vector that determines the solution to \( W \). From the definition of social welfare given in (26), it follows that
\[
\frac{\partial W}{\partial t} = \phi' \frac{\partial V'}{\partial p_r} \frac{\partial p_r}{\partial t} + \phi'' \frac{\partial V''}{\partial q_i} \left[ 1 + \frac{\partial p_r}{\partial t} \right] + \phi'' \frac{\partial V''}{\partial w_u} \frac{\partial w_u}{\partial t}
\]

\[
= \phi' \alpha \frac{q_i}{p_r} x_i' \frac{\partial p_r}{\partial t} - \phi'' \alpha x_i' \left[ 1 + \frac{\partial p_r}{\partial t} \right] + \phi'' \alpha \epsilon_x \frac{\partial w_u}{\partial t} < 0, \quad (37)
\]

where the inequality follows from Lemma 2 and the fact that \(1 + (\partial p_r/\partial t)\) > 0. To demonstrate the latter inequality, note that

\[
\frac{\partial p_r}{\partial t} = \frac{-x_i' \left[ \frac{x_i' - y_i'}{p_i + \tau} + x_i'' \right] + \epsilon_x x_i''}{\left[ \frac{x_i' - y_i'}{p_i + \tau} + x_i'' \right] \left[ \epsilon_x - \frac{\epsilon_x'}{p_u} \right] - \epsilon_x x_i''}, \quad (38)
\]

which, since the denominator of (38) is negative, implies \(1 + (\partial p_r/\partial t)\) > 0 if

\[
x_i' \left[ \frac{x_i' - y_i'}{p_i + \tau} + x_i'' \right] > \left[ \epsilon_x - \frac{\epsilon_x'}{p_u} \right] \left[ \frac{x_i' - y_i'}{p_i + \tau} + x_i'' \right], \quad (39)
\]

a condition that is always satisfied due to Lemma 1 and the assumptions placed on the model.

The effect of changing \(\tau\) upon welfare is given by

\[
\frac{\partial W}{\partial \tau} = \phi' \left[ \frac{\partial V'}{\partial p_r} \frac{\partial p_r}{\partial \tau} + \frac{\partial V'}{\partial q_i} \right] + \phi'' \left[ \frac{\partial V''}{\partial p_r} \frac{\partial p_r}{\partial \tau} + \frac{\partial V''}{\partial q_i} \right] + \phi'' \frac{\partial V''}{\partial w_u} \frac{\partial w_u}{\partial \tau}
\]

\[
= \phi' \alpha \left[ \frac{q_i}{p_r} x_i' \frac{\partial p_r}{\partial \tau} - x_i' \right] + \phi'' \alpha \epsilon_x \left[ \frac{\partial w_u}{\partial \tau} - x_i'' \frac{\partial p_r}{\partial \tau} - x_i'' \right]. \quad (40)
\]

From the definition of \(\partial p_r/\partial \tau\) given in the proof of Lemma 2, it follows that \((q_i/p_r)x_i'(\partial p_r/\partial \tau) - x_i' > 0\) when \((p_i/q_i)x_i'' + x_i'' < 0\). In addition, \(\epsilon_x(\partial w_u/\partial \tau) - x_i''(\partial p_r/\partial \tau) - x_i'' < 0\).

Defining the social welfare weights \(\beta^i\) by \(\beta^i = \phi^i \alpha^i, i = r, u\), these calculations give

\[
\nabla W = \left( \frac{\partial W}{\partial t}, \frac{\partial W}{\partial \tau} \right) = (\beta^r d_1 + \beta^u d_2, \beta^r e_1 + \beta^u e_2), \quad (41)
\]
with \(d_1, d_2, e_2 < 0\) and \(e_1 > 0\). Therefore,

\[
S(\nabla W) = \frac{\partial W}{\partial \tau} = \frac{\beta' e_1 + e_2}{\beta' d_1 + d_2},
\]

with \(\lim_{\beta' \rightarrow 0} S(\nabla W) > 0\) and \(\lim_{\beta' \rightarrow \infty} S(\nabla W) < 0\).

A first insight into the contrast between the reform based on welfare with that based on measured income is given in the following theorem. Denoting the solution to \(W\) by \(\{dtW, dr^W\}\), the solutions at the two limiting values of \(\beta' / \beta''\) are described in Theorem 4.

**THEOREM 4:** The solution to \(W\) satisfies

(i) \(dtW < 0, dr^W < 0\) as \(\beta' / \beta'' \rightarrow 0\);

(ii) \(dtW < 0, dr^W > 0\) as \(\beta' / \beta'' \rightarrow \infty\).

Recalling that the solution to \(M\) for \(\theta_1 / \theta_u \equiv 0\) has \(dt^M < 0, dr^M < 0\), Theorem 4 shows that the solutions to \(M\) and \(W\) are in agreement with respect to the direction of change when the rural (unmeasured) sector is given no weight in the social welfare function. It is an obvious corollary of this theorem that if, for example, the rural sector is given a high weight in social welfare but its income is grossly underestimated, then a reform based upon measured income will result in the tariff being changed in the wrong direction. Given sufficient divergence in the direction of the two vectors, this will lead to a reduction in social welfare.

Although Theorem 4 shows clearly what happens at the limiting values of \(\beta' / \beta''\), it says little about the intermediate values except that there must be some critical value at which \(S(\nabla W) = 0\). Theorem 5 partially answers this by putting a lower bound on the values of \(\beta' / \beta''\) for which \(S(\nabla W) > 0\).

**THEOREM 5:** If \(\beta' / \beta'' \leq \theta^*_r / \theta_u\) and \(\partial e_u / \partial \tau > 0\), then \(S(\nabla W) > 0\) and \(dt^W < 0, dr^W > 0\).

Returning to the baseline case of \(\theta_1 / \theta_u \equiv 0\), Theorem 5 can be interpreted as saying that when the government places a significant weight upon the urban households in the social welfare function, then the direction of change in the tariff based on measured income will be in the wrong direction. This follows since the solution to \(M\) with \(\theta_1 / \theta_u \equiv 0\) has \(dr^M < 0\).

**5. Balanced-Budget Reform**

When the requirement that the budget must be balanced is imposed upon the reform problem, the degrees of freedom in the choice of the policy
variables are much reduced. It will become apparent below that this can have the effect of enforcing increased divergence between the solutions to the various reform problems but, rather more surprisingly, reduces the likelihood that a reduction in true income or welfare will occur.

5.1 The Revenue Constraint

As a consequence of the mismeasurement of the economy, two alternative measures of revenue can be employed. The first incorporates the mismeasurement and defines revenue as what the government expects to receive. Formally, this is given by

$$R^M = \theta_t \tau x_t^t + \theta_{u_r} \left[ \frac{\partial x_t^u}{\partial t} + \tau x_t^u \right]. \quad (43)$$

The alternative measure is the level of revenue that the government actually receives, which equals

$$R^T = \tau x_t^t + t x_t^u + \tau x_t^u. \quad (44)$$

It follows from the definitions that $R^T > R^M$. Both of these measures have arguments in their favor. $R^M$ will be consistent with the government’s measure of economic activity, but it will be contradicted by the level of revenue actually raised. Conversely, $R^T$ will match the level of revenue raised but will not be rationalizable against the measured level of activity. The focus below will be placed upon $R^T$ because this will be the revenue level observed in practice by the government. The analytical arguments if $R^M$ were used are very similar and can easily be worked out.

Given this choice the restriction that the reform must not reduce revenue requires that the tax changes satisfy

$$dR^T = \frac{\partial R^T}{\partial t} dt + \frac{\partial R^T}{\partial \tau} d\tau = \nabla R^T.dT \geq 0. \quad (45)$$

The reform problems $MR$, $TR$, and $WR$ then constitute optimizations (28), (29), and (30), respectively, plus the constraint in (45).

From (44)

$$\frac{\partial R^T}{\partial t} = x_t^u + t \frac{\partial x_t^u}{\partial t} + \tau \left[ \frac{\partial x_t^t}{\partial t} + \frac{\partial x_t^u}{\partial t} \right]. \quad (46)$$

and

$$\frac{\partial R^T}{\partial \tau} = x_t^u + x_t^t + t \frac{\partial x_t^u}{\partial \tau} + \tau \left[ \frac{\partial x_t^t}{\partial \tau} + \frac{\partial x_t^u}{\partial \tau} \right]. \quad (47)$$
Hence

\[
S(\nabla R^T) = \frac{\partial R^T}{\partial \tau} = \frac{x_t^u + x_t + t \frac{\partial x_t^u}{\partial t} + \tau \left[ \frac{\partial x_t^u}{\partial \tau} + \frac{\partial x_t^u}{\partial t} \right]}{x_t^u + t \frac{\partial x_t^u}{\partial t} + \tau \left[ \frac{\partial x_t^u}{\partial \tau} + \frac{\partial x_t^u}{\partial t} \right]}. \tag{48}
\]

It is assumed that the standard Laffer condition is satisfied so that the economy is on the correct side of the Laffer curve with \(S(\nabla R^T) > 0\). This will always be the case if the reform is beginning from an initial position with \(t = \tau = 0\), as can be readily seen from (48).

### 5.2 Contrasting \(MR\) and \(TR\)

The solutions to the reform problems \(MR\) and \(TR\) depend upon the gradients of the vectors \(\nabla Y^M\), \(\nabla Y^T\), and \(\nabla R^T\). Without the revenue constraint, the optimal reform is chosen to point along the gradient vector of the objective. In general, this is not possible when there is a budget constraint. The tax changes satisfying (45) are those that lie above the hyperplane orthogonal to \(\nabla R^T\), while those that increase measured income lie below the hyperplane orthogonal to \(\nabla Y^M\). Similarly, reforms raising true income lie below the hyperplane orthogonal to \(\nabla Y^T\). The optimal reform will be given by the vector that makes the narrowest angle with the gradient of the objective while still satisfying the revenue constraint.

To see the role played by the slopes of the vectors, consider Figure 4. This depicts the contrast between the optimal reforms for the case that

![Figure 4: Contrast between \(MR\) and \(TR\).](image)
$S(\nabla Y^T) < S(\nabla R^T) < S(\nabla Y^M)$. The reasoning that lies behind this drives the results reported in Theorem 6.

In Figure 4 the vector on the unit circle closest to $\nabla Y^M$ but also orthogonal to $\nabla R^T$ is shown by point $a$. This is the solution to the reform problem $MR$. The existence of the revenue constraint can then be seen to force a very different solution to the problem $TR$. This solution, denoted by $b$, is also on the hyperplane orthogonal to $\nabla R^T$ but is the vector closest to $\nabla Y^T$. Although $\nabla Y^M$ and $\nabla Y^T$ are quite similar, the solutions at $a$ and $b$ are diametrically opposed.

For the configuration depicted in Figure 4 the solutions to problems $M$ and $T$ without the revenue constraint would not differ greatly since they would each lie along the relevant vector. However in the case of $MR$ and $TR$ they are sufficiently different that the optimal reform for $MR$ reduces true income because it lies below the hyperplane orthogonal to $\nabla Y^T$. These observations support the claim that the revenue constraint exacerbates the bias problem.

The complete characterization of outcomes is now stated in Theorem 6.

**THEOREM 6:**

(i) If $S(\nabla Y^M) > S(\nabla R^T)$ then $dt^M > 0$, $d\tau^M < 0$ and

(a) if $S(\nabla Y^T) > S(\nabla R^T)$ then $dt^T = dt^M$, $d\tau^T = d\tau^M$, and $\nabla Y^T.dT^M > 0$;

(b) if $S(\nabla Y^T) = S(\nabla R^T)$ then $dt^T = dt^M$, $d\tau^T = d\tau^M$, and $\nabla Y^T.dT^M = 0$;

(c) if $S(\nabla Y^T) < S(\nabla R^T)$ then $dt^T < 0$, $d\tau^T > 0$, and $\nabla Y^T.dT^M < 0$.

(ii) If $S(\nabla Y^M) = S(\nabla R^T)$ then there is no improving reform.

(iii) If $S(\nabla Y^M) < S(\nabla R^T)$ then $dt^M < 0$, $d\tau^M > 0$ and

(a) if $\nabla Y^T.\nabla R^T \leq 0$ then $dt^T = dt^M$, $d\tau^T = d\tau^M$, and $\nabla Y^T.dT^M > 0$;

(b) if $\nabla Y^T.\nabla R^T > 0$ then $0 > dt^T > dt^M$, $d\tau^T > d\tau^M$, and $\nabla Y^T.dT^M > 0$.

The points already made about increased bias in policy are emphasized by observing in Theorem 6 how the existence of the revenue constraint can cause the optimal policy to switch from the portion of the hyperplane orthogonal to the revenue constraint lying in one quadrant to the portion lying in the diagonally opposite quadrant. This is illustrated by the contrast between the outcomes in (i) (a) and (b) as $S(\nabla Y^T)$ crosses $S(\nabla R^T)$: the direction of change of the tax rates switch signs as does the change in true income. The solutions can therefore be sensitive to the precise directions of the vectors involved.

Having shown that the revenue constraint can exaggerate the bias in the direction of policy reform, the question that now arises is whether this is reflected in an increased likelihood of the reform based on measured income reducing true income. To address this, it should first be noted that when $\theta_1/\theta_2 \equiv 0$ it follows that $S(\nabla Y^M) = b_2/c_2 > (b_1 + b_2)/(c_1 + c_2) = S(\nabla Y^T)$. Now assume the initial position is one with no policy interven-
tion, so \( t = \tau = 0 \), and that \( x^u_{u2} = \ell_{u2} = 0 \). Under these conditions, \( S(\nabla Y^M) > S(\nabla R^T) \) if \( p_1 x^u_1 > p_1 [x^u_1 + x^T_1] \). But from (4), this last inequality cannot hold. Hence \( S(\nabla Y^M) < S(\nabla R^T) \). This locates the outcome in (iii) of Theorem 6. Consequently, starting from an initial position with no intervention, the balanced-budget reform must raise welfare even if the policy is based on a very badly defined measure of income. In this case the discipline of the budget constraint prevents policy diverging too far from what should be adopted.

5.3 Contrasting \( MR \) and \( WR \)

The budget-constrained welfare reform problem, \( WR \), is now easily dealt with. Theorem 6 remains true with \( \nabla Y^T \) replaced everywhere by \( \nabla W \), so the discussion of the direction of policy following the theorem also applies. Furthermore, under the conditions of the previous paragraph, the fact that \( S(\nabla Y^M) < S(\nabla R^T) \) again locates the solution in (iii). Hence, starting from the initial position with no intervention, the balanced-budget reform must raise welfare.

This last claim merits some further discussion. Contrasting (24) and (25) it can be seen that there is little significant connection between the two. Yet the results show there are grounds for believing that a reform founded on (24) will also raise (25). Looked at in this way, the role of the budget constraint in mediating between the two becomes apparent. It enforces a discipline on the reform problem that ensures the choice is not wholly damaging. This is underlined further by recalling the discussion of the short-term reform. Given the importance of the budget constraint, there may be some questions raised here about using the true budget constraint when income is measured incorrectly. Support for doing so comes from the fact that the budget constraint can easily be identified by some experimentation with tax rates or by reviewing historical data. The same claim cannot be made for discovering true income (or welfare).

6. Conclusions

The objective of the paper was to explore the possible biases that can arise in policy choice due to the mismeasurement of income. This was undertaken for a two-sector economy, which was interpreted as a simple representation of the economy of a developing country. Reforms were interpreted as either short term, in which case a budget deficit or surplus could be incurred, or as being conducted under the imposition of a balanced budget. In all cases, the alternative outcomes under the three measures of income and welfare were contrasted.

The results demonstrate that even at the level of generality adopted, clear predictions can be obtained about the direction of the policy biases that arise. When no budget constraint is imposed, in a reform based on measured income there is a good chance that a policy will be chosen that reduces the level of true income and welfare. The effect of the budget
constraint is to make the direction of policy more sensitive to the specification but to remove the possibility that the reform will reduce true income or welfare. The basic reason for this is that the budget constraint is roughly proportional to quantities as are welfare and income. This observation about the role of the budget constraint is independent of the particular model that is used.

Overall, these findings suggest that the consequences of economics mismeasurement are something that can be addressed using a standard framework of analysis. The present paper has provided an insight into what can be expected to arise in a more general framework. The adoption of the two-sector economy with the precise interpretation of one sector being rural production is clearly very special, but it does provide the simplest possible framework for analysis and generates easily interpretable results.

Appendix

Proof of Lemma 1: The budget constraint of the rural household is given by

\[ q_i' \left( \frac{p_i}{q_i} \right) = y_i' \left( \frac{p_i}{q_i} \right) - x_i' \left( \frac{p_i}{q_i} \right), \tag{A.1} \]

which, when differentiated with respect to \( q_i \), gives

\[ \frac{q_i}{x_i'} \frac{\partial x_i'}{\partial q_i} + \frac{p_i^2}{q_i^2 x_i'} \left[ y_i' - x_i'' \right] = -1. \tag{A.2} \]

Under Assumption 1(ii) this equality proves the lemma. \( \blacksquare \)

Proof of Lemma 2: From (14) and (15), it follows that the effects of the policy instruments upon the equilibrium price and wage rate are given by the solution to

\[
\frac{dp_i}{dw_u} = \left[ \begin{array}{cc}
\frac{x_i'' - y_i'}{p_i + \tau} + x_i' u & x_i'' u \\
\ell_{u1} & \ell_{u3} - \ell_u^{dt}
\end{array} \right]^{-1} \left[ \begin{array}{c}
\frac{[x_i'' - y_i'] p_i}{[p_i + \tau]^2} - x_i'' u \\
-\ell_{u1} dt - \ell_{u2} d\tau
\end{array} \right]. \tag{A.3}
\]

Under the restrictions placed on urban consumption demand and labor supply, it is apparent that the determinant of the inverted matrix, denoted \( |A| \), is negative since \( \ell_u^{dt} < 0 \). This implies immediately that
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\( \partial p_i/\partial t < 0 \) and \( \partial w_u/\partial \tau < 0 \). Solving the system and canceling terms gives

\[
\frac{\partial w_u}{\partial t} = -\frac{1}{|A|} \frac{\ell_u [x_u'' - y_r]}{p_i + \tau}.
\] (A.4)

Since \(|A|\) is negative, it follows from Lemma 1 that \( \partial w_u/\partial t < 0 \). This proves (i) and (ii).

To prove (iii), note that the derivative is given by

\[
\frac{dp_i}{d\tau} = \frac{1}{|A|} \left[ \ell_{u3} - \frac{\ell_u [x_u'' - y_r]}{p_u} \right] \left[ \frac{p_r [x_r'' - y_r]}{(p_r + \tau)^2} - x_r'' \right] + x_u'u_{u2} \ell_{u2}.
\] (A.5)

When \( \ell_{u2} = 0 \) this is positive, proving (iii).

Proof of Lemma 3: The optimization in (3) has necessary conditions

\[
\frac{p_r}{q_i} U_i' f'' + U_i' = 0
\] (A.6)

and

\[
\frac{p_r}{q_i} U_i' - U_i'' = 0.
\] (A.7)

Using (A.1) and (A.2), the maximum value function for the optimization is given by

\[
V' = V'(p_r, q_i)
\]

\[
= U' \left( f' \left( \frac{p_r}{q_i} \ell_i \left( \frac{p_r}{q_i} \right) - \frac{p_r}{q_i} x_i' \left( \frac{p_r}{q_i} \right), \frac{p_r}{q_i} x_i' \left( \frac{p_r}{q_i} \right) \right) \right).
\] (A.8)

Hence

\[
\frac{\partial V'}{\partial q_i} = -\ell_i' \left[ \frac{p_r}{q_i} \left( \frac{p_r}{q_i} U_i' f'' + U_i' \right) + x_i'' \left( \frac{p_r}{q_i} U_i' - U_i'' \right) \right]
\]

\[
= -\alpha' x_i' - u_i',
\] (A.9)

using (A.6) and (A.7) and noting that \( U_i'/q_i \) is the marginal utility of income for the rural household.
Similarly,

\[
\frac{\partial V'}{\partial p_r} = \frac{\ell_i'}{q_i} p_r \left[ \frac{p_r}{q_i} U''_1 f'' + U''_3 \right] + \frac{x''_r}{q_i} \left[ U'_2 - \frac{p_r}{q_i} U'_i \right] + \frac{U'_i}{q_i} \left[ f' - x'_i \right]
\]

\[= \alpha' \frac{q_i}{p_r} x'_r. \quad (A.10)\]

This proves (i). The proof of (ii) is standard. ■

Proof of Lemma 4: By definition,

\[
S(\nabla Y^M) = \frac{\theta_r}{\theta_u} \frac{b_1 + b_2}{c_1 + c_2}, \quad (A.11)
\]

where \(b_1 = y_r(\partial p_r/\partial \tau) + p_r(\partial y_r/\partial \tau), b_2 = \ell_u(\partial w_u/\partial \tau) + w_u(\partial \ell_u/\partial \tau), c_1 = y_r(\partial p_r/\partial t) + p_r(\partial y_r/\partial t), \) and \(c_2 = \ell_u(\partial w_u/\partial t) + w_u(\partial \ell_u/\partial t). \) Differentiating (A.11)

\[
\frac{\partial S(\nabla Y^M)}{\partial \theta_r} = \frac{b_1 c_2 - c_1 b_2}{\left[ \theta_r \theta_u c_1 + c_2 \right]^2}, \quad (A.12)
\]

so that

\[
\text{sgn}\left\{ \frac{\partial S(\nabla Y^M)}{\partial \theta_r} \right\} = \text{sgn}\{b_1 c_2 - c_1 b_2\}. \quad (A.13)
\]

Noting that

\[
\frac{\partial y_r}{\partial t} = \frac{\partial y_r}{\partial p_r} \frac{\partial p_r}{\partial t} + \frac{\partial y_r}{\partial \tau} \frac{\partial \tau}{\partial t} = \frac{\partial y_r}{\partial p_r} \frac{\partial p_r}{\partial \tau} \quad (A.14)
\]

it can be seen that

\[
b_1 c_2 - c_1 b_2 = \frac{\partial p_r}{\partial \tau} \left[ \ell_u \frac{\partial w_u}{\partial t} + w_u \frac{\partial \ell_u}{\partial t} \right] - \frac{\partial p_r}{\partial \tau} \left[ \ell_u \frac{\partial w_u}{\partial \tau} + w_u \frac{\partial \ell_u}{\partial \tau} \right]. \quad (A.15)
\]

From the market clearing conditions (20)

\[
\ell_u \frac{\partial w_u}{\partial t} + w_u \frac{\partial \ell_u}{\partial t} = \frac{\partial w_u}{\partial t} \left[ \ell_u + \frac{w_u}{\ell_u} \frac{\partial \ell_u}{\partial t} \right]. \quad (A.16)
\]
The optimization of the firm implies that

\[ f^u' = \frac{w_u}{p_u}, \quad \ell^d' = \frac{1}{f^u'}. \tag{A.17} \]

Substituting (A.17) into (A.16), the assumption that \( \ell_u f^u' f^u' < -1 \) implies \( \ell_u + (w_u/p_u)\ell^d' > 0 \). The same argument can be applied to \( \ell_u (\partial w_u/\partial \tau) + w_u (\partial \ell_u/\partial \tau) \). Hence, using the results of Lemma 2, it follows that \( b_1 e_2 - c_1 b_2 < 0 \). When \( \ell_u f^u' f^u' > -1, \ell_u + (w_u/p_u)\ell^d' < 0 \) and \( b_1 e_2 - c_1 b_2 > 0 \). This completes the proof.

Proof of Theorem 2: This is straightforward from considering the direction of the vectors in Figure 3 in the main text.

Proof of Theorem 3: If \( \theta_r^*/\theta_r > 1 \) then \( \nabla Y^T \) must lie below the horizontal axis. The converse holds if \( \theta_r^*/\theta_r < 1 \). Since vectors making an angle of less than 90° have a positive inner product, the theorem then follows.

Proof of Theorem 4: This follows directly from observation of the directions of the vectors.

Proof of Theorem 5: The theorem can be proved by returning to the definitions of \( \nabla W \) and \( \nabla Y^M \). The terms entering the numerators of these expressions can be ranked as given in Lemma A1.

**Lemma A1:**

(i) \( 0 < e_1 < b_1 \)

and, if \( \partial \ell_u/\partial \tau > 0 \),

(ii) \( e_2 < b_2 < 0 \).

Proof of Lemma A1: Differentiating the budget constraint of the rural household, (4), with respect to \( \tau \) shows that

\[ b_1 = p_r \frac{\partial y_r}{\partial \tau} + y_r \frac{\partial p_r}{\partial \tau} = q_r \frac{\partial x'_r}{\partial \tau} + x'_r \frac{\partial p_r}{\partial \tau} + p_r \frac{\partial x'_r}{\partial \tau}, \tag{A.18} \]

which allows \( e_1 \) to be written as

\[ e_1 = b_1 - p_r \frac{\partial y_r}{\partial \tau} - x_r \frac{\partial p_r}{\partial \tau} - x'_r. \tag{A.19} \]

The results of Lemma 3 then prove (i).
To prove (ii), differentiate the urban budget constraint and use this to write
\[ e_2 = b_2 - w_u \frac{\partial \ell_u}{\partial \tau} - x_i \frac{\partial p_i}{\partial \tau} - x_i u. \] (A.20)

This proves (ii) given the assumption that \( \frac{\partial \ell_u}{\partial \tau} > 0 \). ■

The proof of Theorem 5 is then completed by recalling that \((\theta_1^*/\theta_u)b_1 + b_2 = 0\). The results of Lemma A1 show that \( \beta' e_1 + \beta'' e_2 < 0 \) when \( \beta' / \beta'' < \theta_1' / \theta_u \). Since \( d_1 < 0 \) and \( d_2 < 0 \), it follows from (41) that \( S(\nabla W) > 0 \). This immediately implies the optimal direction of change. ■

References


