The Marginal Cost of Public Funds in Growing Economies

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Abstract

The marginal cost of public funds (MCF) measures the cost to the economy of raising government revenue. The MCF can be used to guide reform of the tax system and to determine an efficient level of government expenditure. It can also be used as an input into cost-benefit analysis. Previous applications of the concept have developed a methodology appropriate to single countries. The application of the MCF within the EU context raises several important questions concerning tax and expenditure externalities between member states of an economic union. We extend the concept of the MCF to a setting that combines growth with infrastructural spill-overs. It is intended that this represents a setting in which issues relevant to the EU can be addressed.

JEL Classification: E6, H4, H7

Key Words: Growth, infrastructure, externalities, public funds

1 Introduction

All implementable government tax instruments are distortionary and, as a consequence, impose deadweight losses upon the economy. These distortions are the inevitable cost of collecting the finance required to support public spending. The correct level of spending achieves a compromise between these distortions

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and the benefits of spending. There are large literatures on cost-benefit analysis (Mishan and Quah, 2007) and optimal taxation (Myles, 1995) that describe how this should be done. Results have been derived for a wide variety of economic settings – including all forms of market failure – but some of the unique features of the European Union (EU) have not been modelled. The purpose of this paper is to develop the methodology for determining when additional public spending is justified in a setting that better represents the EU and the policy issues that it faces.

What matters for the spending decision is whether the marginal benefit of spending exceeds the marginal cost of taxation. If it does, then additional public spending is justified. This principle of contrasting marginal benefit and marginal cost underlies the methodology of cost-benefit analysis. The survey of Drèze and Stern (1987) provides a summary of the standard approach to cost-benefit analysis. That approach is based on the Arrow-Debreu representation of the competitive economy and its extensions. The generality of the model permits it to encompass many situations but for specific problems alternative models can be advantageous. The questions we wish to focus upon involve economic growth. Although growth can be handled by dating commodities in the Arrow-Debreu framework it does seem preferable to employ a more specific model of the growth process. The model we analyze in this paper combines endogenous growth and fiscal federalism. In addition, the government provides a productive input (which we view as infrastructure) that has spill-overs between countries. The combination of these features permits us to address policy issues that have been identified in recent EU policy statements.

A government has access to a wide range of different tax instruments. Taxes can be levied on consumption or on income. Different forms of consumption, and different sources of income, can be taxed at different rates. Taxes can also be levied on firms, using profit, turnover, or input use as a base. The cost of collecting revenue will depend upon the tax base that is chosen and the structure of rates that are levied. The marginal cost of public funds (MCF) is a measure of the cost of raising tax revenue from a particular tax instrument. The MCF can be used to identify changes to the tax structure that raise welfare keeping expenditure constant. It can also identify the best tax instruments to use for raising additional revenue. It is a practical tool that permits consistent analysis of taxation choices. Dahlby (2008) provides a detailed summary of the existing methods for calculating the MCF in a wide variety of circumstances and for a range of tax instruments. However, there is very little literature on the derivation of the MCF for a growing economy.

Section 2 of the paper provides a brief review of the MCF from a cost-benefit perspective and a summary of several typical applications. Some relevant features of the operation of EU policy are discussed in Section 3. A general discussion of the MCF for a growing economy is given in Section 4. Section 5 analyses a basic model of endogenous growth and Section 6 extends the model to incorporated infrastructural spill-overs and tax externalities. Section 7 provides concluding comments.

2 Application of the MCF

The MCF provides a numerical summary of the cost of raising additional revenue from each tax instrument. The basic logic can be described as follows. Consider raising the income tax rate so that an extra €1 is collected from a taxpayer. Now offer the taxpayer the alternative of paying some amount directly to the treasury in order to avoid the income tax increase. The maximum they will be willing to pay must be at least €1 since the income tax is distortionary. If the taxpayer is willing to pay €1.20 then the MCF[1] is 1.2. Expressed alternatively, if the MCF is 1.2 an additional euro of government revenue imposes a cost of €1.20 on the taxpayer.

The MCF measures the welfare cost of raising revenue so has a central role in determining the range of projects that will be successful in a cost-benefit calculation. There are two ways in which the MCF interacts with cost-benefit analysis. First, the MCF can be used to judge which projects should be adopted. Continuing the example above, a project that costs the government €1 and produces benefits of €1.15 appears on the surface to be beneficial. However, if the MCF is €1.20 then the marginal benefit of the project falls short of the actual marginal cost faced by the taxpayer. The project should therefore not be adopted. Second, the MCF can be employed to improve the efficiency of the tax system. If the tax structure is inefficient the MCF will be unnecessarily high. This will reduce the range of projects that pass any given cost-benefit criterion. The MCF can be used to improve the efficiency of the tax system and therefore increase the number of beneficial projects. For example, if revision to the structure of income taxation reduced the MCF to €1.10 then the project with benefits of €1.15 should be accepted.

A general construction of the MCF can be provided as follows. Consider an economy that can employ m tax instruments to finance g public goods. Denote the level of tax instrument i by t_i and the quantity of public good j by G_j. Welfare is given by the social welfare function $W(t,G)$ where $t = (t_1,...,t_n)$ and $G = (G_1,...,G_g)$. The level of revenue is $R(t,G)$ and the cost of the public good supply is $C(G)$. Note that there is an interaction in the revenue function between taxes and public good supply in revenue but the cost of public goods is determined by technology alone.

The optimization problem for the government is[2]

$$\max_{\{t,G\}} W(t,G) \quad \text{s.t.} \quad R(t,G) \geq C(G). \tag{1}$$

To analyze the solution to this optimization define λ_i by

$$\lambda_i = -\frac{\partial W/\partial t_i}{\partial R/\partial t_i}, \quad i = 1,...,m.$$

[1] Formally, it is the "normalized MCF" defined in (17) that is described here. The MCF is denominated in units of welfare per dollar whereas the normalized MCF is unit-free.

[2] The solution to this optimization is studied in detail in Atkinson and Stern (1974) .

The term λ_i is the Marginal Cost of Funds (MCF) from tax instrument i. It measures the cost in units of welfare of raising an additional unit of revenue.[3] The optimality condition for the choice of taxes requires that the MCF is equalized across the different tax instruments so that $\lambda_i = \lambda$, for all i. If the value of λ_i differed across tax instruments then welfare could be raised by collecting more revenue from taxes with low λ_i and less revenue from those with high λ_i.

Now assume that the tax instruments have been optimized. The use of the MCF in cost-benefit analysis can be motivated by writing the optimality condition for the level of public good j (or "project" j) as

$$\frac{\partial W}{\partial G_j} = \lambda \left[\frac{\partial C}{\partial G_j} - \frac{\partial R}{\partial G_j} \right], \quad j = 1, ..., g. \tag{2}$$

This conditions says that the quantity of provision of each public good is optimal when the marginal benefit of that extra provision ($\partial W/\partial G_j$) is equal to the net cost of the public good ($\partial C/\partial G_j - \partial R/\partial G_j$) converted into welfare units using the MCF.

This analysis shows that the MCF is denominated in units of welfare/revenue and that it can be used to convert monetary costs into welfare equivalents. The MCF can be used to optimize the tax structure and as a measure of whether a public project is welfare-enhancing (Slemrod and Yitzhaki, 2001). The MCF has received many applications. Several of these applications are now discussed.

Parry (2003) considers the excise taxes on petroleum, alcoholic drinks, and cigarettes in the UK. It is shown that petroleum taxes are the most distortionary of the instruments and the cigarette taxes are the least distortionary. These calculations demonstrate that the current system is only optimal if the goods have external effects or differ in the degree of tax shifting. The role of tax shifting is explored in Delipalla and O'Donnell (2001). Many applications of the MCF are undertaken for linear tax structures but Dahlby (1998) shows how the MCF can be extended to accommodate a progressive income tax. The approach of Dahlby is applied in a study of the Japanese income tax system by Bessho and Hayahi (2005). A alternative extension is made by Kleven and Kreiner (2006) who show how labour force participation can be included in a measure of the MCF. They calculate the MCF for five European countries and show that incorporating the participation margin can significantly increase the MCF in some circumstances.

Poapangsakorn *et al.* (2000) employ the MCF in a cost-benefit analysis of a tax enforcement programme in Thailand. It is concluded that the tax enforcement programme was a high-cost method of raising revenue. Elmendorf and Mankiw (1999) analyze the MCF of public sector debt when interest payments on debt are financed by a distortionary tax on total output. Debt finance can smooth the MCF over time when a lumpy project is financed. This can lead to a significant welfare gain.

[3]In applications revenue is typically denominated in monetary units but in this formal (non-monetary) model it is denominated in units of the numeraire commodity.

This section has shown how the MCF developed in the formal literature into a set of techniques that permit the MCF to be calculated and to be integrated into cost-benefit analysis. Numerous further examples of such applications are described in the comprehensive text of Dahlby (2008). However, despite this extensive literature there are still a number of significant issues that need to be addressed if the MCF is to be used effectively in policy design by the EU.

3 The MCF in an EU Context

The description of the MCF given above is very general and, conceptually, can be applied to any economy. The cost of this generality is an absence of detail about the processes behind the functional representation. This observation is especially important for the application of the MCF to the tax systems and spending decisions of the EU member states. In particular, the case of the EU raises two questions. First, the appropriateness of the use of a single objective function must be questioned especially in an area where subsidiarity permits independence in policy. Secondly, economic integration in the EU has enhanced externalities between member states but the details of these important interactions are hidden by the general formulation.

The issues for the MCF raised by the structure of the EU can be motivated by observing how the EU functions. In very general terms the EU obtains revenue through the mechanism of "Own Resources". In essence, Own Resources are derived from import duties and from the EU taking a share of the VAT revenues of member states. A proportion of these revenues are then spent through the Structural Funds of the EU. This mechanism results in a degree of redistribution between the EU member states since the allocation of Structural Funds across member states is not directly related to the collection of Own Resources. If the MCF were applied to the EU the result would be increased collection of Own Resources from member states with efficient tax structures (low MCF) and reduced collection from member states with inefficient tax structures (high MCF). This is presuming that tax structures were not modified first. The allocation of Structural Funds guided by cost-benefit analysis would direct spending into member states where the marginal benefit was highest. It is possible that these member states would also be those where the MCF was highest. If so, there would be redistribution towards the inefficient member states through the application of the MCF. The political and economic implications of such a process would be significant.

The implication of these comments is that there are benefits to be obtained by refining the analysis of the MCF to explicitly represent the structure and operation of the EU. Such a refinement would need to include the role of individual member states, the externalities that link the member states, and the formal operation of policy in the EU. Dahlby and Wilson (2003) make some progress in this respect by analyzing a model that includes fiscal externalities. Their model has both a central government and a local government. Both governments levy taxes on labour incomes and profit so there are vertical tax externalities linking the two levels of government. The main result is the demonstration that a local government that does not take into account the effect of its choices on the central government may have an MCF that is biased up or down. This study will be the starting point for constructing the methodology that will be employed.

The general version of the model used to derive the MCF can encompass growth but its presentation has the appearance of being static. It is certainly true that the most formal development of both cost-benefit analysis and the MCF has been based upon the Arrow-Debreu model. This model can incorporate time in the form of dated commodities but must have all contracts agreed prior to the commencement of economic activity. Consequently, it is not a compelling representation of the growth process. Similarly, the applications of the MCF reviewed above are typical in being set within a static model. This is a limitation of the current approach since the relationship between fiscal policy and growth has emerged in recent years as a focal point for the attention of EU policy makers. The issue of good governance has been termed the "quality of public finances" (QPF), a phrase that encapsulates all aspects of fiscal policy that increase the rate of sustainable economic growth.

The basic building blocks to achieve enhanced QPF have been identified by the Directorate-General for Economic and Financial Affairs (DG ECFIN) as growth-supportive non-distortionary tax systems, sound fiscal governance, efficient and effective public spending and investment, and the sustainability of public finances. These building blocks are seen as helping to achieve the objectives of the Stability and Growth Pact in two ways: directly by making public spending more efficient, and indirectly by promoting growth potential. The enhancement of growth potential was also a central goal of the Lisbon Strategy for Growth and Jobs. The 2005 EU Report on the Progress of Lisbon Strategy reaffirmed these principles. In Section A1 it is stated that the objective is to "Promote a growth- and employment-orientated and efficient allocation of resources":

> Well-designed tax and expenditure systems that promote an efficient allocation of resources are a necessity for the public sector to make a full contribution towards growth and employment, without jeopardizing the goals of economic stability and sustainability. This can be achieved by redirecting expenditure towards growth-enhancing categories such as Research and Development (R&D), physical infrastructure, environmentally friendly technologies, human capital and knowledge. Member States can also help to control other expenditure categories through the use of expenditure rules and performance budgeting and by putting assessment mechanisms in place to ensure that individual reform measures and overall reform packages are well-designed. A key priority for the EU economy is to ensure that tax structures and their interaction with benefit systems promote higher growth through more employment and investment.

This objective was summarized in Integrated Guideline Number 3:

> Member States should, without prejudice to guidelines on economic stability and sustainability, re-direct the composition of public expenditure towards growth-enhancing categories in line with the Lisbon strategy, adapt tax structures to strengthen growth potential, ensure that mechanisms are in place to assess the relationship between public spending and the achievement of policy objectives, and ensure the overall coherence of reform packages. See also integrated guideline 'To encourage the sustainable use of resources and strengthen the synergies between environmental protection and growth' (No 11).

There are two branches of the economic literature that relate to these issues and provide the basis for extending the analysis of the MCF. First, the literature on endogenous growth has investigated the processes and decisions that explain the growth process. It has identified numerous "engines of growth" and considered how these can be encouraged through appropriate fiscal policy. Second, the literature on fiscal federalism considers the externalities that arise within a federal system and the implications these have for the operation of fiscal policy. Central to this literature are the externalities that link the fiscal policy decisions of independent countries, and the economic inefficiency that can result. An analysis of the MCF that is applicable to the EU requires that these two literatures are linked in order to jointly analyze economic growth and fiscal federalism.

Many of the issues related to EU policy can be interpreted in terms of the financing and provision of public goods. When considering public goods as factors influencing growth, except for some analysis devoted to the role of public infrastructure, most of the questions remain unsettled. But the inter-relationships between public good provision and growth are important and not well studied. For an example of these relationships, there exist public goods generating externalities on a wide scale, such as education and transport infrastructure, but these goods depend on collective financing by a set of agents, be it a group of nations, or a group of regions. A further issue that faces policy making within the EU is the enhanced mobility of capital and labour as a consequence of economic integration. A mobile tax base causes tax externalities between member states and is a cause of tax competition. This gives increased emphasis to the application of the theory of the MCF within a framework of fiscal federalism to ensure that institutional design is consistent with the needs of economic growth.

Endogenous growth occurs when capital and labour are augmented by additional inputs in a production function that otherwise has non-increasing returns to scale. One interesting case for understanding the link between government policy and growth is when the additional input is a public good or public infrastructure financed by taxation. The need for public infrastructure to support private capital in production provides a positive role for public expenditure and a direct mechanism through which policy can affect growth. Introducing infrastructure permits an analysis of the optimal level of public expenditure in an endogenous growth model.

The importance of infrastructure is widely recognized, not least by the EU which pursues an active programme to support the investment activities of member states. The policy problem facing the EU is to ensure that member states undertake an efficient level of infrastructural expenditure that ensures the maximum rate of growth. The determination of the level has to take into account the full consequences of an infrastructure project for the EU, not just the direct benefits for the member state undertaking the investment. The MCF can be used to evaluate public infrastructure provision but its use has to recognize three significant issues. First, infrastructural investment has significant spill-overs across member states. Second, mobility of the tax base results in tax externalities between the member states, and between the member states and the EU. Third, the EU is faced with a decision on how to allocate support for infrastructural expenditure across the different member states. This interacts with the process of revenue-raising, and with the extent to which the projects are financed jointly by the EU and member states.

The economic modelling of the impact of infrastructure on economic growth has focussed on the Barro (1990) model of public expenditure as a public input and its extensions (Chen *et al.* 2005, Turnovsky, 1999). This literature has identified the concept of an optimal level of expenditure, and has highlighted the deleterious effects of both inadequate and excessive expenditure. These are important insights, but do not address the spill-over issues that confront the EU. Infrastructural spill-overs between member states can be positive, which occurs when improvements in infrastructure in one member state raise productivity in another, or they can be negative if they induce relocation of capital between member states. In either case, it is important that the consequences of spill-overs are addressed in order for the role of productive public expenditure to be fully understood. Ignoring either form of spill-over will result in an inefficient level and allocation of expenditure.

4 Dynamic Setting

There has been little investigation of the MCF in growing economies. Two exceptions are Liu (2003) who computes the MCF as a component of a cost-benefit analysis (but taking the intertemporal path of wage rates and interest rates as exogenous) and Dahlby (2006) who uses the MCF to analyze public debt in an AK growth model. Our approach is similar to Dahlby but we employ a more general model of endogenous growth. In principle, it is possible to treat the economy lying behind (1) as intertemporal but the analysis needs to be more specific to generate worthwhile conclusions.

Consider an intertemporal economy set in discrete time. The time path for tax instrument i is a sequence $\{\tau_i^1, \tau_i^2, \ldots\}$. The MCF is computed for a variation in this sequence. A *pulse variation* takes the form of a change in the tax instrument in a single time period, t. The new sequence would then be $\{\tau_i^1, \ldots, \tau_i^{t-1}, \widehat{\tau}_i^t, \tau_i^{t+1}, \ldots\}$. Alternatively, a *sustained variation* in the tax instrument from period t onwards changes the sequence to $\left\{\tau_i^1, \ldots, \tau_i^{t-1}, \widehat{\tau}_i^t, \widehat{\tau}_i^{t+1}, \ldots\right\}$.

We choose to focus on sustained variations. Correspondingly, we extend the definition of the MCF for a static economy to an intertemporal economy by using

$$MCF_i^t \equiv -\frac{\partial W/\partial \tau_i}{\partial R/\partial \tau_i},$$

to denote the MCF of a sustained variation in tax instrument i from period t onwards. In this setting W is the intertemporal social welfare function, and R is discounted value of tax revenue.

In a dynamic, infinite horizon economy

$$R = \sum_{t=0}^{\infty} d(t) R_t,$$

where R_t is tax revenue in period t, and $d(t)$ is the discount factor applied to revenues in period t. There are many well-known issues involved in the choice of the sequence of discount factors $\{d(t)\}$. We choose to remain with the standard convention (see, for example, Nordhaus 2008) of appealing to market equilibrium to endogenously determine the social rate of time preference. In this case

$$d(t) = \frac{\partial W/\partial C_t}{\partial W/\partial C_0},$$

where C_t is consumption at time t. If the welfare function is time separable with exponential discounting, so $W = \sum_{t=0}^{\infty} \beta^t U(C_t)$, then

$$d(t) = \beta^t \frac{U'(C_{t+1})}{U'(C_0)}. \tag{3}$$

The purpose of the analysis is to apply the MCF in settings where economic growth is occurring. This requires us to embed the MCF within a model of endogenous growth. The major difficultly involved in achieving this is that, generally, the entire intertemporal path for the economy must be computed from the present into the indefinite future. To overcome this difficulty we focus upon balanced growth paths. Along a balanced growth path all real variables grow at the same rate, so such a path can be interpreted as describing the pattern of long-run growth. All the commonly used growth models have the property that the economy will converge to a balanced growth path from an arbitrary initial position.

5 Public Infrastructure

This section develops the MCF for a growing economy by building on the Barro (1990) model of productive public expenditure. In particular, the model is used to illustrate the benefits of focussing on the balanced growth path. This analysis provides the developments that we need to combine endogenous growth with fiscal federalism and infrastructural spill-overs in Section 6.

Public infrastructure is introduced by assuming that the production function for the representative firm at time t has the form

$$Y_t = AL_t^{1-\alpha} K_t^{\alpha} G_t^{1-\alpha},$$

where A is a positive constant and G_t is the quantity of public infrastructure. The form of this production function ensures that there are constant returns to scale in labour, L_t, and private capital, K_t, for the firm given a fixed level of public infrastructure. Although returns are decreasing to private capital as the level of capital is increased for fixed levels of labour and public input, there are constant returns to scale in public input and private capital together.

5.1 Output Tax

The first step is to consider the MCF when the productive public input is financed by a tax upon output as in Barro (1990). Denoting the tax upon output by τ the profit level of the firm is

$$\pi_t = (1 - \tau) Y_t - r_t K_t - w_t L_t,$$

where r_t is the interest rate, and w_t the wage rate. Profit maximization requires that the use of capital satisfies the necessary condition

$$\frac{\partial \pi_t}{\partial K_t} = (1 - \tau) \alpha A \left(\frac{G_t}{K_t} L_t \right)^{1-\alpha} - r_t = 0.$$

This can be solved to give

$$r_t = (1 - \tau) \alpha A (g_t L_t)^{1-\alpha}, \tag{4}$$

where $g_t \equiv G_t / K_t$.

The firm belongs to a representative infinitely-lived household whose preferences are described by an instantaneous utility function, $U = \ln(C_t)$. The household maximizes the infinite discounted stream of utility

$$W = \sum_{t=0}^{\infty} \beta^t U(C_t) = \sum_{t=0}^{\infty} \beta^t \ln(C_t), \tag{5}$$

subject to the sequence of intertemporal budget constraints

$$C_t + K_{t+1} = (1 - \delta_K + r_t) K_t + w_t L_t + \pi_t, \tag{6}$$

and with the sequence of taxes and government infrastructure taken as given. Upon substitution of (6) into (5) we can write the first-order conditions for the optimal consumption path as

$$\frac{\partial U / \partial C_t}{\partial U / \partial C_{t+1}} = \beta (1 - \delta_K + r_{t+1}),$$

which for the utility function in (5) become

$$\frac{C_{t+1}}{C_t} = \beta \left(1 - \delta_K + r_{t+1}\right). \tag{7}$$

The public capital input is financed by the tax on output, and there is no government debt so the government budget constraint is

$$G_t = \tau Y_t.$$

This constraint assumes that public capital fully depreciates in one period. We relax this assumption in the later analysis.

On a balanced growth path the real variables (Y_t, C_t, K_t, G_t) grow at the same constant rate, γ. Markets also clear in every period and $r_t = r$ for all t. We assume for this analysis that labour supply is constant, and normalize it to one, so $L_t = 1$. On a balanced growth path $C_{t+1} = (1+\gamma) C_t$ where γ is the rate of growth. Using (4), (6), and (7), the balanced growth path is described by the following set of equations

$$\begin{aligned} c &= (1-\tau) A g^{1-\alpha} - \delta_K - \gamma, & (8)\\ \gamma &= \beta \left(1 - \delta_K + (1-\tau) \alpha A g^{1-\alpha}\right) - 1, & (9)\\ g &= (A\tau)^{1/\alpha}, & (10) \end{aligned}$$

where $c \equiv C_t/K_t$. Along the balanced growth path when utility is logarithmic, the discount factor defined in (3) is

$$d = \left[\frac{\beta}{1+\gamma}\right]^t, \tag{11}$$

which implies

$$R = \sum_{t=0}^{\infty} \left(\frac{\beta}{1+\gamma}\right)^t \tau Y_t = \frac{\tau K_0}{1-\beta} A g^{1-\alpha}. \tag{12}$$

The MCF is calculated with g constant so assuming the sustained variation in tax rate is from period 0 onwards:

$$\frac{\partial R}{\partial \tau} = \frac{K_0}{1-\beta} A g^{1-\alpha}. \tag{13}$$

For the welfare function with logarithmic utility we have

$$W = \sum_{t=0}^{\infty} \beta^t \ln\left(C_t\right) = \frac{\ln\left(c K_0\right)}{1-\beta} + \frac{\beta}{(1-\beta)^2} \ln\left(1+\gamma\right). \tag{14}$$

This implies

$$\frac{\partial W}{\partial \tau} = -\frac{A g^{1-\alpha}}{1-\beta} \left[\frac{1-\alpha\beta}{c} + \frac{\alpha\beta^2}{1-\beta} \frac{1}{1+\gamma}\right]. \tag{15}$$

Combining (13) and (15) allows the MCF to be computed as

$$MCF = \frac{1}{K_0}\left[\frac{1-\alpha\beta}{c} + \frac{\alpha\beta^2}{1-\beta}\frac{1}{1+\gamma}\right]. \tag{16}$$

Following Dahlby (2006) we define the MCF^N (the "normalized MCF") by dividing the MCF in by the marginal utility of income at time 0,

$$MCF^N = -\frac{1}{\partial W/\partial I_0}\frac{\partial W/\partial\tau}{\partial R/\partial\tau} \tag{17}$$

This provides a unit-free measure of the cost of public funds. For the specification of utility in this model the marginal utility of income at time 0 is given by $1/C_0 = 1/cK_0$. Hence, using (16),

$$MCF^N = 1 - \alpha\beta + \frac{\alpha\beta^2}{1-\beta}\frac{c}{1+\gamma}. \tag{18}$$

Figure 1 plots the values of the key endogenous variables against the tax rate. In this specification of the economy the level of welfare is an increasing function of the growth rate, so is at a maximum when the growth rate is highest. For low levels of the tax rate growth is negative because of insufficient provision of public infrastructure. It can be seen that MCF^N is everywhere above 1, and reaches a maximum at the value where welfare is maximized.

We can now use the expression for MCF^N in (18) to demonstrate the link with the optimality condition (2) in this model. Let $C(g)$ denote the cost of providing the public input. The optimal amount of public input is determined by equating the marginal cost of the public input to its marginal benefit, which is the increase in productivity. In the balanced growth path equilibrium $Y_t = AK_tg^{1-\alpha}$, and in the dynamic setting the marginal benefit is calculated as the infinite discounted sum of the marginal productivities in every time period, with the discount factor (11). Hence,

$$\frac{\partial C}{\partial g} = \sum_{t=0}^{\infty}\left(\frac{\beta}{1+\gamma}\right)^t\frac{dY_t}{dg} = \sum_{t=0}^{\infty}\left(\frac{\beta}{1+\gamma}\right)^t(1-\alpha)AK_tg^{-\alpha} = \frac{1-\alpha}{1-\beta}AK_0g^{-\alpha}. \tag{19}$$

With the tax on output the marginal benefit is is the increase in welfare (normalized by the marginal utility of income at time 0):

$$MB_g = \frac{\partial W/\partial g}{\partial W/\partial I}.$$

From (14),

$$\frac{\partial W}{\partial g} = \frac{1}{1-\beta}\frac{1}{c}\frac{\partial c}{\partial g} + \frac{\beta}{(1-\beta)^2}\frac{1}{1+\gamma}\frac{\partial\gamma}{\partial g}$$

and, using (8)-(9),

$$\frac{\partial W}{\partial g} = \frac{(1-\tau)(1-\alpha)}{1-\beta}Ag^{-\alpha}\left[\frac{1-\alpha\beta}{c} + \frac{\alpha\beta^2}{1-\beta}\frac{1}{1+\gamma}\right],$$

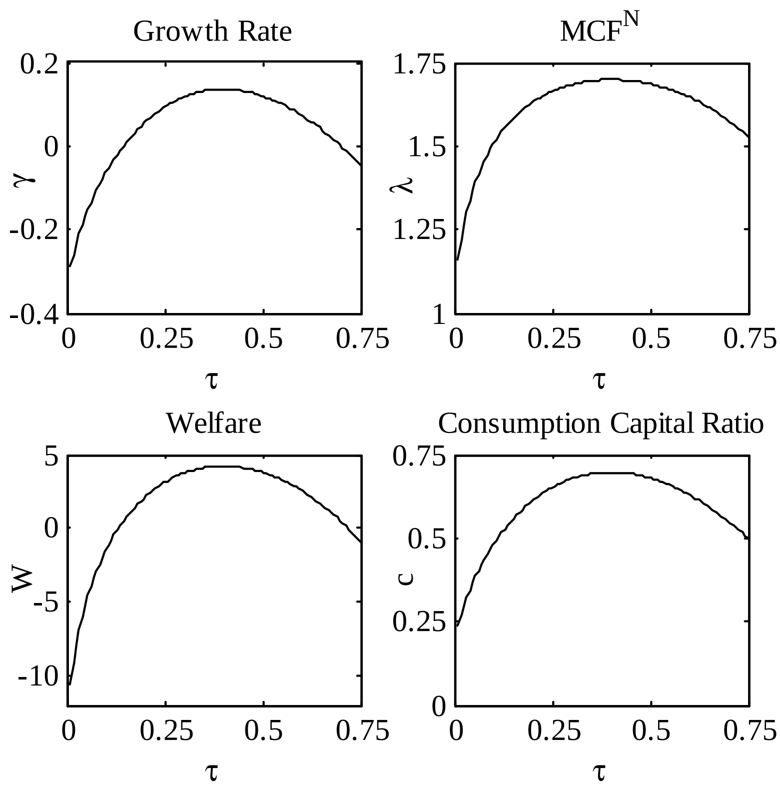

Figure 1: $\alpha = 0.6$, $\beta = 0.8$, $A = 2$, $\delta_K = 0.2$, $K_0 = 2$.

so that

$$MB_g = \frac{(1-\tau)(1-\alpha)}{1-\beta} AK_0 g^{-\alpha} \left[1 - \alpha\beta + \frac{\alpha\beta^2}{1-\beta}\frac{c}{1+\gamma}\right]. \tag{20}$$

The optimality condition requires

$$MB_g = MCF^N \left(\frac{\partial C}{\partial g} - \frac{\partial R}{\partial g}\right),$$

or, using (12), (18), and (20),

$$\begin{aligned}
\frac{\partial C}{\partial g} &= \frac{\partial R}{\partial g} + \frac{MB_g}{MCF^N} \\
&= \frac{\tau K_0}{1-\beta}(1-\alpha) AK_0 g^{-\alpha} + \frac{(1-\tau)(1-\alpha)}{1-\beta} AK_0 g^{-\alpha} \\
&= \frac{1-\alpha}{1-\beta} AK_0 g^{-\alpha},
\end{aligned}$$

which is exactly the same as (19) when the direct productivity benefit of the public input is determined. This is a reflection the Diamond and Mirrlees (1971) result that optimal tax system should not distort production decisions.

5.2 Capital Tax

We now assume that government spending is funded from a tax levied on the private capital input. This tax distorts the choice of inputs so the MCF will reflect this. We also assume that public infrastructure can be built up as a stock. This begins the development of a model of public infrastructure as a capital good.

The consumers' optimization problem remains the same as described in the previous section. The private capital input is taxed at rate τ_K. Net of tax profit is

$$\pi_t = Y_t - (r_t + \tau_K) K_t - w_t L_t.$$

The profit maximization condition implies that

$$r_t = \alpha A (g_t L_t)^{1-\alpha} - \tau_K.$$

We now assume that the public capital stock accumulates over time by assuming that the depreciation rate is $\delta_G < 1$. Again, there is no government debt so the government budget constraint at time t is

$$G_{t+1} = (1 - \delta_G) G_t + \tau_K K_{t+1}.$$

On the balanced growth path the real variables $(Y_t, C_t, K_t, G_t, w_t)$ grow at the same constant rate, γ. Markets also clear in every period and $r_t = r$ for all

t. Normalizing available labour to one, we obtain the following set of equations describing the balanced growth path

$$r = \alpha A g^{1-\alpha} - \tau_K, \tag{21}$$

$$c = A g^{1-\alpha} - \delta_K - \gamma - \tau_K, \tag{22}$$

$$\gamma = \beta\left(1 - \delta_K - \tau_K + \alpha A g^{1-\alpha}\right) - 1, \tag{23}$$

$$g = \frac{1+\gamma}{\gamma+\delta_G}\tau_K. \tag{24}$$

In this case there is no closed form solution for γ in terms of τ_K when the dependence of g upon τ_K is taken into account.

For the present value of tax revenues we have

$$R = \sum_{t=0}^{\infty}\left(\frac{\beta}{1+\gamma}\right)^t \tau_K K_t = \frac{\tau K_0}{1-\beta},$$

from which it follows that

$$\frac{\partial R}{\partial \tau_K} = \frac{K_0}{1-\beta}.$$

The welfare function remains as described by (14) so

$$MCF = \frac{1}{K_0}\left[\frac{1-\alpha\beta}{c} + \frac{\alpha\beta^2}{1-\beta}\frac{1}{1+\gamma}\right].$$

This MCF can be used directly or, dividing by the marginal utility of income at time 0, converted into the normalized version

$$MCF^N = 1 - \alpha\beta + \frac{\alpha\beta^2}{1-\beta}\frac{c}{1+\gamma}. \tag{25}$$

The relationships between the endogenous variables and the tax rate are plotted in Figure 2 for three different values of α. The MCF^N and the consumption-capital ratio are decreasing in α, whereas the growth rate is increasing. These competing effects produce a single-crossing property in welfare: it increases with α for low τ_K but decreases for high τ_K. The MCF^N is monotonically increasing in the tax rate – and reaches high values for modest levels of the tax rate – so the capital tax rapidly becomes increasingly distortionary. This is not surprising given the important role that capital plays in sustaining growth in this economy.It is straightforward to verify that the results are consistent with the optimality condition. Indeed, from (22)-(23),

$$MB_g = cK_0\frac{\partial W}{\partial g} = \frac{1-\alpha}{1-\beta}AK_0 g^{-\alpha}\left[1 - \alpha\beta + \frac{\alpha\beta^2}{1-\beta}\frac{c}{1+\gamma}\right].$$

and $\dfrac{\partial R}{\partial g} = 0$. Therefore, using (25),

$$\frac{\partial C}{\partial g} = \frac{MB_g}{MCF^N} = \frac{1-\alpha}{1-\beta}AK_0 g^{-\alpha},$$

which, again, is the marginal benefit of the public input.

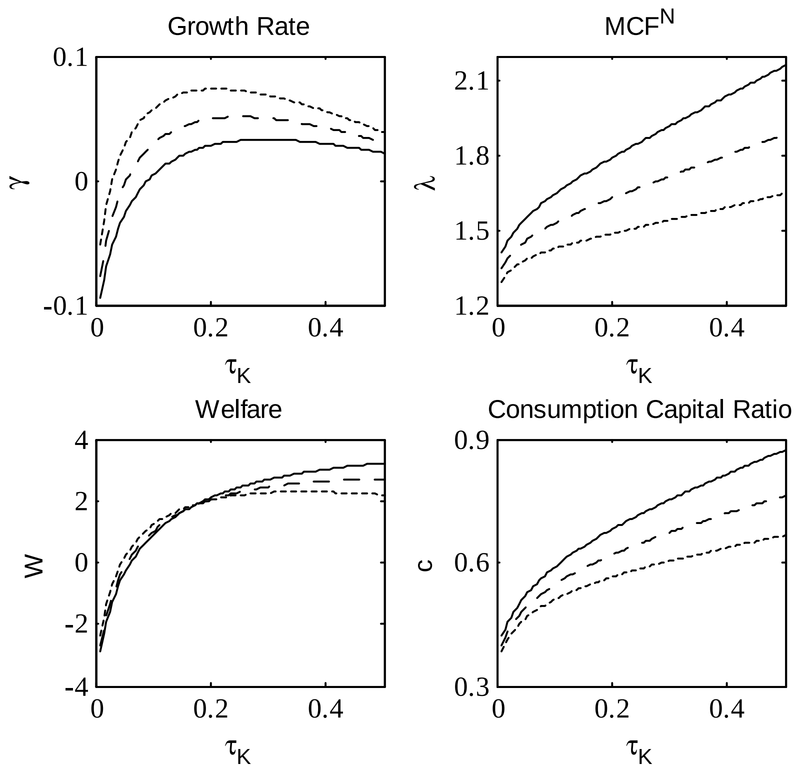

Figure 2: $\alpha = 0.6$ (solid), 0.65 (dash), 0.7 (dot), $\beta = 0.8$, $\delta_K = \delta_G = 0.15$, $A = 1$, $K_0 = 2$.

5.3 Tax on capital and labour

The model is now extended to make the quantity of labour a choice variable. This is achieved by modifying the utility function to include labour as an argument. This makes it interesting to analyze a capital tax and a labour tax since both instruments are distortionary.

The private capital input is taxed at rate τ_K. Net of tax profit is

$$\pi_t = Y_t - (r_t + \tau_K) K_t - w_t L_t.$$

From the necessary conditions for the choice of capital and labour inputs we obtain

$$r_t = \alpha A (g_t L_t)^{1-\alpha} - \tau_K,$$

and

$$w_t = (1-\alpha) A (g_t L_t)^{1-\alpha} \frac{K_t}{L_t},$$

where, as before, $g_t \equiv G_t / K_t$.

The representative consumer has intertemporal preferences

$$W = \sum_{t=0}^{\infty} \beta^t U(C_t, L_t), \tag{26}$$

where the within-period utility has the Cobb-Douglas form

$$U(C_t, 1 - L_t) = \theta \ln(C_t) + (1-\theta) \ln(1 - L_t). \tag{27}$$

Labour income is taxed at rate τ_L so the consumer's budget constraint is

$$C_t + K_{t+1} = (1 - \delta_K + r_t) K_t + (1 - \tau_L) w_t L_t + \pi_t. \tag{28}$$

Upon substitution of (27) and (28) into (26) we can write the first-order conditions for the intertemporal paths of consumption and labour supply as

$$\begin{aligned} \frac{C_{t+1}}{C_t} &= \beta (1 - \delta_K + r_{t+1}), \\ \frac{C_t}{1 - L_t} &= \frac{\theta}{1-\theta} (1 - \tau_L) w_t. \end{aligned}$$

The public capital input is financed by the tax on capital input and on the labour income. We continue to assume that the government does not issue debt. The government budget constraint in period t is therefore

$$G_t = (1 - \delta_G) G_{t-1} + \tau_K K_t + \tau_L w_t L_t.$$

The achievement of the balanced growth path when public capital is modelled as a stock variable has been analyzed in Gómez (2004) and Turnovsky (1997). Turnovsky assumes that investments in public capital and private capital are reversible. This allows immediate adjustment to the balanced growth path via a downward jump in one of the capital stock variables. Without reversibility it is shown by Gómez that the optimal transition path requires investment in one of the two capital variables to be zero until the balanced growth path is reached.

Employing the conditions developed above the balanced growth path is described by the following set of equations

$$
\begin{aligned}
r &= \alpha A\left(gL\right)^{1-\alpha} - \tau_K, && (29)\\
\omega &= \left(1-\alpha\right) A\left(gL\right)^{1-\alpha}, && (30)\\
c &= \left(r-\delta_K-\gamma\right) + \left(1-\tau_L\right)\omega, && (31)\\
\gamma &= \beta\left(1-\delta_K+r\right) - 1, && (32)\\
\frac{1}{L} &= \frac{1-\theta}{\theta}\frac{c}{\left(1-\tau_L\right)\omega} + 1, && (33)\\
g &= \frac{1+\gamma}{\gamma+\delta_G}\left(\tau_K+\omega\tau_L\right). && (34)
\end{aligned}
$$

where $c \equiv C_t/K_t$ and $\omega \equiv w_t L_t/K_t$.

In the balanced growth path equilibrium the present value of tax revenues are given by

$$R = \sum_{t=0}^{\infty}\left(\frac{\beta}{1+\gamma}\right)^t\left(\tau_K K_t + \tau_L w_t L_t\right) = \frac{K_0}{1-\beta}\left(\tau_K+\omega\tau_L\right),$$

so that

$$\frac{\partial R}{\partial \tau_K} = \frac{K_0}{1-\beta},\ \frac{\partial R}{\partial \tau_L} = \frac{K_0\omega}{1-\beta}\left(1+\varepsilon_\omega^L\right), \tag{35}$$

where $\varepsilon_\omega^L \equiv \frac{\tau_L}{\omega}\frac{\partial \omega}{\partial \tau_L}$. The welfare function can be written as

$$
\begin{aligned}
W &= \sum_{t=0}^{\infty}\beta^t U\left(C_t, L_t\right) = \sum_{t=0}^{\infty}\beta^t\left[\theta\ln C_t + \left(1-\theta\right)\ln\left(1-L_t\right)\right]\\
&= \sum_{t=0}^{\infty}\beta^t\left[\theta\ln\left[cK_0\left(1+\gamma\right)^t\right] + \left(1-\theta\right)\ln\left(1-L\right)\right]\\
&= \frac{1}{1-\beta}\left[\theta\ln\left(cK_0\right)+\left(1-\theta\right)\ln\left(1-L\right)\right] + \frac{\beta}{\left(1-\beta\right)^2}\theta\ln\left(1+\gamma\right). && (36)
\end{aligned}
$$

Differentiation with respect to the tax rates gives

$$
\begin{aligned}
\frac{\partial W}{\partial \tau_i} &= \frac{1}{1-\beta}\left(\frac{\theta}{c}\frac{\partial c}{\partial \tau_i} - \frac{1-\theta}{1-L}\frac{\partial L}{\partial \tau_i}\right) + \frac{\beta}{\left(1-\beta\right)^2}\frac{\theta}{1+\gamma}\frac{\partial \gamma}{\partial \tau_i}\\
&= \frac{1}{1-\beta}\frac{\theta}{\tau_i}\left[\varepsilon_c^i - \left(1-\tau_L\right)\frac{\omega}{c}\varepsilon_L^i\right] + \frac{\beta}{\left(1-\beta\right)^2}\frac{\theta}{1+\gamma}\frac{\gamma}{\tau_i}\varepsilon_\gamma^i,
\end{aligned}
$$

where

$$\varepsilon_c^i \equiv \frac{\tau_i}{c}\frac{\partial c}{\partial \tau_i},\ \varepsilon_L^i \equiv \frac{\tau_i}{L}\frac{\partial L}{\partial \tau_i},\ \varepsilon_\gamma^i \equiv \frac{\tau_i}{\gamma}\frac{\partial \gamma}{\partial \tau_i},\ i = K, L.$$

Using (35) and (36) the MCF for the two tax instruments can be expressed in terms of elasticities by

$$\begin{aligned} MCF_K &= \frac{\theta}{\tau_K K_0}\left[\varepsilon_c^K - (1-\tau_L)\frac{\omega}{c}\varepsilon_L^K + \frac{\beta}{1-\beta}\frac{\gamma}{1+\gamma}\varepsilon_\gamma^K\right], \\ MCF_L &= \frac{\theta}{\omega\tau_L(1+\varepsilon_\omega^L)K_0}\left[\varepsilon_c^L - (1-\tau_L)\frac{\omega}{c}\varepsilon_L^L + \frac{\beta}{1-\beta}\frac{\gamma}{1+\gamma}\varepsilon_\gamma^L\right]. \end{aligned}$$

The marginal utility of income is now $\theta/(cK_0)$. Thus, the normalized MCFs for the two instruments are

$$\begin{aligned} MCF_K^N &= \frac{c}{\tau_K}\left[\varepsilon_c^K - (1-\tau_L)\frac{\omega}{c}\varepsilon_L^K + \frac{\beta}{1-\beta}\frac{\gamma}{1+\gamma}\varepsilon_\gamma^K\right], \\ MCF_L^N &= \frac{c}{\omega\tau_L(1+\varepsilon_\omega^L)}\left[\varepsilon_c^L - (1-\tau_L)\frac{\omega}{c}\varepsilon_L^L + \frac{\beta}{1-\beta}\frac{\gamma}{1+\gamma}\varepsilon_\gamma^L\right], \end{aligned}$$

The expressions we have derived for two tax instruments are now numerically analyzed for a calibrated version of the model. For the model's parameters we employ values that are broadly consistent with the calibration of business cycle and growth models; see, for example, Cooley and Prescott (1995). The first step is to evaluate the elasticities that appear in the MCF formulae to provide an insight into the relative magnitude of effects in the model. The second step is to present an evaluation of the MCF.

To calculate the elasticities we use (29) to eliminate r from (30) to (33) and take the total differential of each resulting equation holding g constant. This process produces the matrix equation

$$\mathbf{A}\cdot\left[\begin{array}{cccc} dc & d\gamma & d\omega & dL \end{array}\right]^T = \mathbf{B}\left[\begin{array}{cc} d\tau_K & d\tau_L \end{array}\right]^T,$$

where

$$\mathbf{A}=\left[\begin{array}{cccc} 0 & \frac{1}{\beta} & -\frac{\alpha}{1-\alpha} & 0 \\ -1 & \frac{1-\beta}{\beta} & 1-\tau_L & 0 \\ \frac{1}{c} & 0 & -\frac{1}{\omega} & \frac{1}{L(1-L)} \\ 0 & 0 & -1 & (1-\alpha)\frac{\omega}{L} \end{array}\right],\ \mathbf{B}=\left[\begin{array}{cc} -1 & 0 \\ 0 & \omega \\ 0 & -\frac{1}{1-\tau_L} \\ 0 & 0 \end{array}\right].$$

This equation can be solved to yield the derivatives and, hence, the elasticities with respect to the tax rates. It should be noted that these are not behavioral elasticities but are instead the elasticities of equilibrium values. The elasticities are also not constant but depend on the tax rates at which they are evaluated. The results reported in Table 1 show that the elasticity of the consumption-capital ratio, c, and the elasticity of labor have the expected signs but are small in value. In contrast, the elasticity of the growth rate (which reflects changes in capital accumulation) is negative and large for both taxes.

ε_c^K	ε_L^K	ε_γ^K
-0.0225	0.0347	-15.91
ε_c^L	ε_L^L	ε_γ^L
-0.3878	-0.0629	-1.648

Table 1: Elasticities ($\tau_K = \tau_L = 0.25$)

Figure 3 shows how the growth rate, level of welfare, consumption-capital ratio, and MCF_K^N for the capital tax change with the rate of tax on private capital input, for three different levels of labour income tax. In Figure 4 the roles of the capital tax and the labour tax are reversed and the MCF_L^N is plotted for three values of the tax on labour. These figures show that there is no longer a direct link between the growth rate and the level of welfare. In fact, the maximum rate of growth is achieved before welfare is maximized. Over the range plotted in Figure 3 one can see that the decrease in consumption with a higher rate of labour income tax in this economy is more than offset by the increase in the growth rate, so that for a given level of the capital tax the welfare level is higher with a higher labour tax rate. A similar pattern is observed for the capital tax. MCF_K^N is reduced by an increase in labour tax but, conversely, MCF_L^N is raised by an increase in capital tax. Both MCF_K^N and MCF_L^N increase rapidly as the tax rates are raised and MCF_L^N is clearly convex in the tax rate. Both are less than one for low tax rates since the growth-enhancing effect is dominant and exceed two for moderate values of the tax rates.

The MCF^N for capital reaches a value of 5 as the capital tax rate approaches 0.5. This very high value reveals how distortionary a tax upon capital is in a growing economy. The tax on capital distorts the choice of capital stock and, because capital is accumulated as a stock, the distortion is compounded throughout the life of the economy. This is the reason why the optimal tax analyses of Chamley (1986) and Judd (1985) find that the optimal tax rate on capital should be zero.

6 Infrastructural Spill-Overs

This section analyzes a model that incorporates infrastructural spill-overs between countries. The model is designed to capture the important feature of the EU that productive investments by one member state have benefits for other neighboring states. This implies that the calculation of the MCF must take into account the linkage between the growth rates of countries implied by the infrastrucutral externality.

We first introduce externalities in productive public input between two countries. Let G_t and G_t^* denote productive public input in the home and in the foreign country, respectively, and let $\Gamma_t = G_t + G_t^*$ be the total public input at home and abroad. The level of output in the home country is given by

$$Y_t = AL_t^{1-\alpha} K_t^{\alpha} \left(G_t^{1-\rho} \Gamma_t^{\rho} \right)^{1-\alpha}.$$

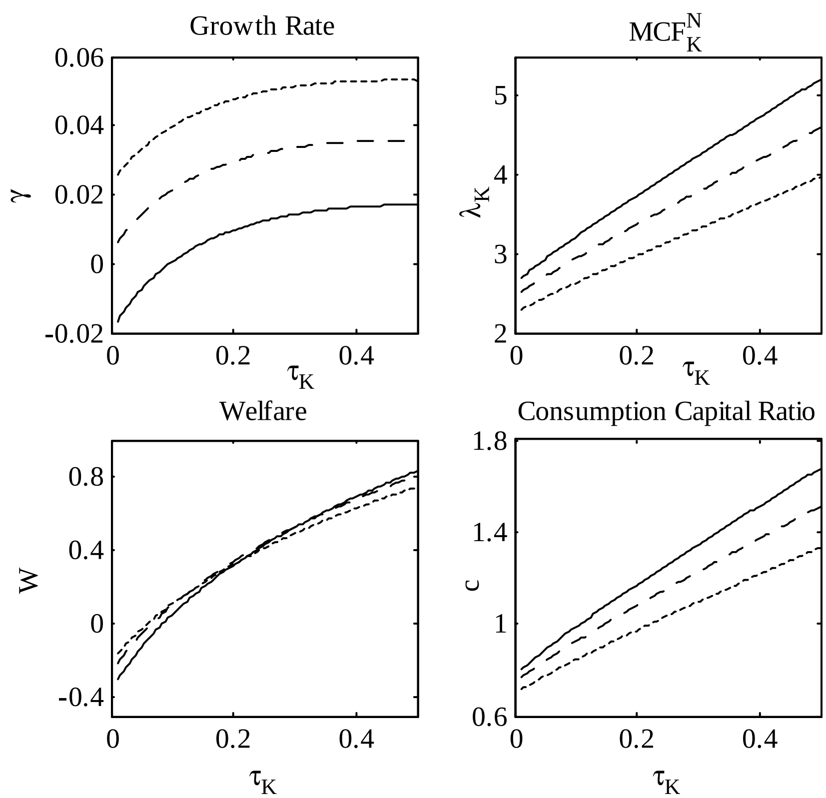

Figure 3: $\alpha = 0.3$, $\beta = 0.8$, $\delta_K = \delta_G = 0.1$, $A = 1.5$, $K_0 = 2$, $\theta = 0.3$, $\tau_L = 0.3$ (solid), 0.4 (dash), 0.5 (dot)

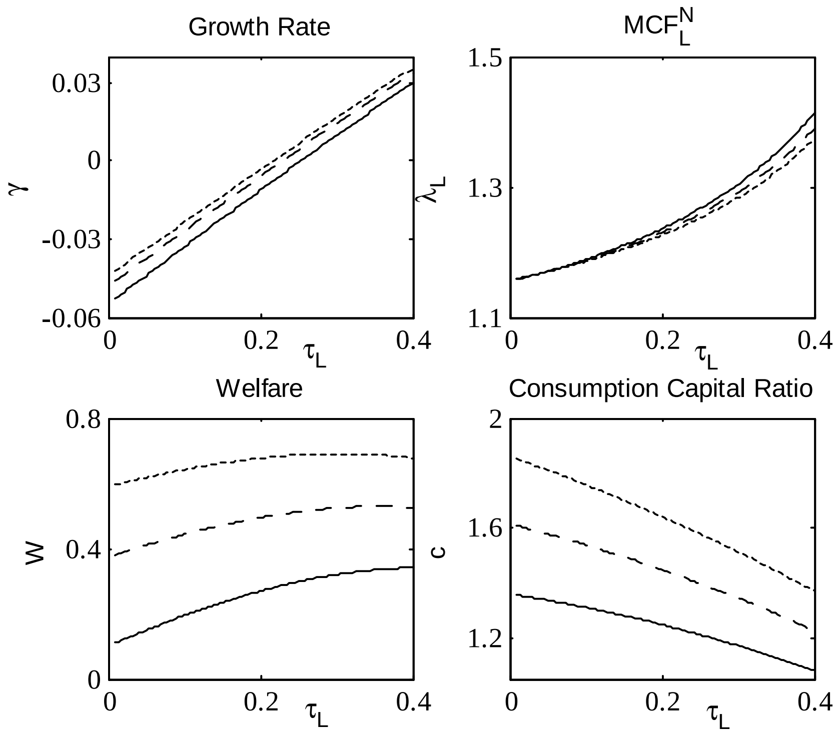

Figure 4: $\alpha = 0.3$, $\beta = 0.8$, $\delta_K = \delta_G = 0.1, A = 1.5$, $K_0 = 2$, $\theta = 0.3$, $\tau_K = 0.3$ (solid), 0.4 (dash), 0.5 (dot)

There is no externality when $\rho = 0$. To simplify the analysis we assume labour is inelastic and normalize the quantity to one. The production function can be rewritten as

$$\begin{aligned} Y_t &= AK_t \left[\frac{G_t}{K_t} \left(\frac{\Gamma_t}{G_t} \right)^{\rho} \right]^{1-\alpha} = AK_t \left[\frac{G_t}{K_t} \left(\frac{G_t + G_t^*}{G_t} \right)^{\rho} \right]^{1-\alpha} \\ &= AK_t \left[\frac{G_t}{K_t} \left(1 + \frac{G_t^*}{G_t} \right)^{\rho} \right]^{1-\alpha} = \widetilde{A} K_t \left(\frac{G_t}{K_t} \right)^{1-\alpha}, \end{aligned}$$

where

$$\widetilde{A} \equiv A \left(1 + \frac{G_t^*}{G_t} \right)^{\rho(1-\alpha)}.$$

The optimization problem of the home consumer is to maximize intertemporal utility taking as given the levels of capital and public good plus the rate of growth in the foreign country.

We assume there is no redistribution of tax revenues across countries and that the home and the foreign countries set their tax rates independently. The home (foreign) country finances their public spending by taxing the private capital input at rate τ_K (τ_K^*), and there is no government debt. Thus, for the home country

$$G_t = (1 - \delta_G) G_{t-1} + \tau_K K_t.$$

We focus on balanced growth paths along which all real variables in all countries grow at the same rate and the tax rates are constant over time. The equality of the growth rates across countries here is imposed, since the law of motion of the public capital in one country only ensures that the growth rates of the stock of public and private capital are equal in that country, but there is no reason of why the growth rates should be equal across countries. If we did not impose this assumption then the output of one country would eventually become arbitrarily small relative to the output of the other. An extension to the model that ensures the endogenous equalization of the growth rates is currently under development.

The solution of the home consumer's optimization problem is described by the following equation:

$$\frac{1+\gamma}{\beta} = \alpha A \left[1 + \left(\frac{1+\gamma^*}{1+\gamma} \right)^t \frac{G_0^*}{G_0} \right]^{\rho(1-\alpha)} + 1 - \delta_K - \tau_K, \tag{37}$$

with a similar equation for the foreign consumer,

$$\frac{1+\gamma^*}{\beta} = \alpha A^* \left[1 + \left(\frac{1+\gamma}{1+\gamma^*} \right)^t \frac{G_0}{G_0^*} \right]^{\rho(1-\alpha)} + 1 - \delta_K - \tau_K^*, \tag{38}$$

so the growth rate in each country depends not only on this country's own tax rate but also on the tax rate in the other country, as long as $\rho \neq 0$. One can see that along the balanced growth path it must be the case that $\gamma = \gamma^*$. When the externality from the infrastructural spill-over is present, W depends on the growth rate in both home and foreign country. Thus, the welfare of the home country is given by

$$W = \frac{\ln K_0}{1-\beta} + \sum_{t=0}^{\infty} t\beta^t \ln(1+\gamma) + \sum_{t=0}^{\infty} \beta^t \ln c(\gamma, \gamma^*) \tag{39}$$

where

$$c(\gamma, \gamma^*) = Ag^{1-\alpha} \left[1 + \left(\frac{1+\gamma^*}{1+\gamma}\right)^{t+1} \frac{G_0^*}{G_0}\right]^{\rho(1-\alpha)} - \gamma - \delta_K - \tau_K. \tag{40}$$

Note that in (37)-(40) dependence on time disappears for $\gamma = \gamma^*$. We need, however, to keep the term with t explicit, in order to take into account interdependence between γ, γ^* and τ_K, τ_K^* in the presence of externalities.

The set of equations describing the balanced growth path equilibrium is analogous to (21)-(24)

$$\begin{aligned}
c &= \frac{1-\alpha\beta}{\alpha\beta}(1+\gamma) - \frac{1-\alpha}{\alpha}(1-\delta_K-\tau_K), && (41)\\
\frac{1+\gamma}{\beta} &= 1-\delta_K-\tau_K+\alpha\widetilde{A}g^{1-\alpha}, && (42)\\
g &= \frac{1+\gamma}{\gamma+\delta_G}\tau_K, && (43)\\
\widetilde{A} &= A\left(1+\frac{G_0^*}{gK_0}\right)^{\rho(1-\alpha)}, && (44)
\end{aligned}$$

with a similar set of equations for the foreign country plus the requirement that $\gamma = \gamma^*$.

As in the one country case, we have

$$\frac{\partial R}{\partial \tau_K} = \frac{K_0}{1-\beta}.$$

The expression for $\dfrac{\partial W}{\partial \tau_K}$ is now more complicated, since it now depends on both γ and γ^*, which, in turn, depend on both τ_K and τ_K^*, as it can be seen from (37)-(38). Thus, in the presence of externalities, the welfare of the home consumer depends on the home tax rate through its own growth rate as well as the growth rate in the foreign country

$$\frac{\partial W}{\partial \tau_K} = \left.\frac{\partial W}{\partial \tau_K}\right|_{\gamma,\gamma^*} + \frac{\partial W}{\partial \gamma}\frac{\partial \gamma}{\partial \tau_K} + \frac{\partial W}{\partial \gamma^*}\frac{\partial \gamma^*}{\partial \tau_K}.$$

The expressions for $\frac{\partial \gamma}{\partial \tau_K}$ and $\frac{\partial \gamma}{\partial \tau_K}$ are obtained by taking the total differential of (37)-(38) and solving the resulting system of linear equations. The details of the calculations are provided in Hashimzade and Myles (2009).

The expressions for the MCF and MCF^N are as the following:

$$MCF = \frac{1-\beta}{K_0} \left[\left. \frac{\partial W}{\partial \tau_K} \right|_{\gamma,\gamma^*} + \frac{\partial W}{\partial \gamma} \frac{\partial \gamma}{\partial \tau_K} + \frac{\partial W}{\partial \gamma^*} \frac{\partial \gamma^*}{\partial \tau_K} \right]_{\gamma=\gamma^*},$$

and

$$MCF^N = (1-\beta)\, c \left[\left. \frac{\partial W}{\partial \tau_K} \right|_{\gamma,\gamma^*} + \frac{\partial W}{\partial \gamma} \frac{\partial \gamma}{\partial \tau_K} + \frac{\partial W}{\partial \gamma^*} \frac{\partial \gamma^*}{\partial \tau_K} \right]_{\gamma=\gamma^*}.$$

Figure 5 depicts the solution for symmetric equilibrium in the model with two identical countries. It can be seen that an increase in the extent of the spill-over (measured by ρ) reduces MCF^N, so that greater spill-overs decrease the cost of funding projects. One explanation for an increased spill-over could be economic integration, which suggests that the single-market programme may have consequences for the cost of financing public projects. The MCF^N increases with the tax rate but over the range displayed so do the growth rate and welfare.

7 Conclusions

The marginal cost of public funds has a central role in the assessment of tax policy and in cost-benefit analysis. The MCF provides a measure of the cost of raising revenue through distortionary taxation that can be set against the benefits of a public sector project. Despite the importance of the concept the current literature has focussed upon the MCF in static settings. Only a very small literature has so far considered it within a growth setting.

In this paper we have computed the MCF in a variety of endogenous growth models with public infrastructure. To do this we have built upon the definition of the MCF in an intertemporal setting provided by Dahlby (2006). The models that have been analyzed are extensions of the Barro model of productive public expenditure, but with the public input represented as a stock rather than a flow. In addition, we have also introduced externalities between countries which are a consequence of spill-overs from public infrastructure. To evaluate the MCF we assume that the economy is on a balanced growth path which permits the evaluation of welfare in terms of a balanced growth rate. This technique provides a basis for determining the MCF for a variety of tax instruments in a form that can be empirically evaluated.

We have employed the standard parameter values for calibration used in real business cycle and growth models to simulate the models. Our results demonstrate that there is a link between the MCF and the growth rate, and that the MCF is sensitive to the tax rate. In the calibrated simulation the

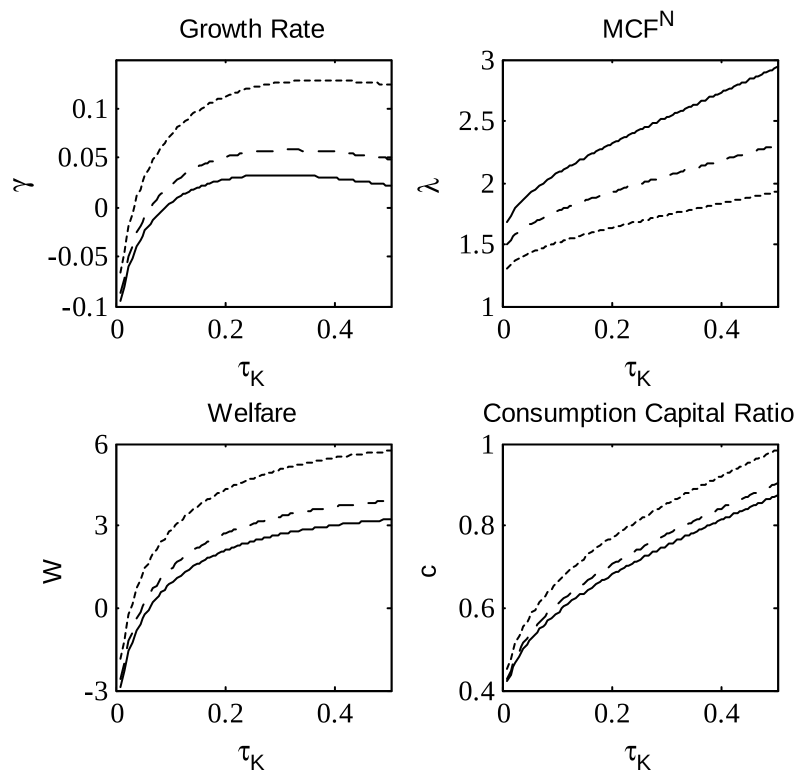

Figure 5: $\alpha = 0.6$ $\beta = 0.8$, $\delta_K = \delta_G = 0.15$, $A = 1$, $K_0 = 2$,
$\rho = 0$ (solid), 0.3 (dash), 1 (dot).

normalized MCF can exceed a value of two for quite reasonable values of tax rates. This indicates that the effect upon the growth rate can exacerbate the static distortions caused by taxation. In every case the MCF is increasing and monotonic over a range of capital and income tax rates similar to those seen in practice. In the model with an infrastructural spill-over it is interesting to observe that an increase in the spill-over effect reduced the MCF.

The analysis has been restricted here by the focus on balanced growth paths. It might be thought necessary to consider the transition path but there is limited evidence on the length of such transition. We have considered only sustained variations in tax rates and have implicitly assumed credibility of government announcements and commitment to announced policies. This removed any need to consider the formation of expectations or games played between the public and private sectors. If there is any strategic interaction this would change the value of the MCF.[4] The benefit of these restrictions is the simplification they provide to the analysis and the fact that they can be applied in a similar manner to more complex models.

The MCF is an important concept in tax policy and cost-benefit analysis. Although it generally appears in a static setting it can be extended to growth models. Our approach to the MCF is suitable for numerical evaluation in more complex economic environments. We aim to explore the practical value of this methodology in further work in this area. In particular, we aim to explore the effect of capital mobility upon the MCF and then construct a more general model that can incorporate several countries and a broader range of tax instruments. This latter model will form the basis of empirical implementation using a combination of calibration and estimation.

References

Atkinson, A.B. and N.H. Stern (1974) "Pigou, taxation and public goods", *Review of Economic Studies*, **41**, 119 – 128.

Barro, R.J. (1990) "Government spending in a simple model of endogenous growth", *Journal of Political Economy*, **98**, S103 – S125.

Bessho, S. and Hayahi, M. (2005) "The social cost of public funds: the case of Japanese progressive income taxation", Ministry of Finance, Japan, *Policy Research Institute Paper* 05A-16.

Chamley, C. (1986) "Optimal taxation of capital income in general equilibrium with infinite lives", *Econometrica*, **54**, 607 - 622,

[4] An example can illustrate the potential effects of game-playing. A tax on the profit from developing land has been introduced twice in the past 100 years in the UK. On both occasions developers held back from undertaking development in the belief that the next government would repeal the tax. This belief was proved correct. Hence, a significant short-term welfare cost was incurred for the generation of very little revenue.

Cooley, T.F. and E.C. Prescott (1995) "Economic growth and business cycles", in: Cooley, T.F. (Ed.) *Frontiers of Business Cycle Research*, Princeton: Princeton University Press.

Dahlby, B. (1998) "Progressive taxation and the social marginal cost of public funds", *Journal of Public Economics*, **67**, 105 – 112.

Dahlby, B. (2006) "The marginal cost of funds from public sector borrowing", *Topics in Economic Analysis and Policy*, **6**, 1 – 28.

Dahlby, B. (2008) *The Marginal Cost of Public Funds: Theory and Applications*, Cambridge: MIT Press.

Dahlby, B. and Wilson, L.S. (2003) "Vertical fiscal externalities in a federation", *Journal of Public Economics*, **87**, 917 – 930.

Delipalla, S. and O'Donnell, O. (2001) "Estimating tax incidence, market power and market conduct", *International Journal of Industrial Organization*, **19**, 885 – 908.

Diamond, P.A. and Mirrlees, J.A. (1971) "Optimal taxation and public production 1: Production efficiency and 2: Tax rules", *American Economic Review*, **61**, 8 - 27 and 261 - 278.

Drèze, J., and Stern, N. (1987) "The theory of cost-benefit analysis", in Auerbach A.J., and M. Feldstein (eds.), *Handbook of Public Economics Volume* 2, Amsterdam: North-Holland.

Elmendorf, D. and Mankiw, G. (1999) "Government debt", in J. Taylor and M. Woodford (eds.) *Handbook of Macroeconomics*, Amsterdam: Elsevier.

Gómez, M.A. (2004) "Optimal fiscal policy in a growing economy with public capital", *Macroeconomic Dynamics*, **8**, 419 – 435.

Hashimzade, N. and Myles, G.D. (2009) "Growth and public infrastructure", mimeo.

Judd, K. (1985) "Redistributive taxation in a simple perfect foresight model", *Journal of Public Economics*, **28**, 59 - 83.

Kleven, H. and Kreiner, C. (2006) "The marginal cost of public funds: hours of work versus labor force participation", *Journal of Public Economics*, **90**, 1955 – 1973.

Liu, L. (2003) "A marginal cost of funds approach to multi-period public project evaluation: implications for the social discount rate", *Journal of Public Economics*, **87**, 1707 – 1718.

Mishan, E.J. and Quah, E. (2007) *Cost Benefit Analysis*, New York: Routledge.

Myles, G.D. (1995) *Public Economics*, Cambridge: Cambridge University Press.

Nordhaus, W. (2008) *A Question of Balance*, Yale: Yale University Press.

Parry, I. (2003) "On the costs of excise and income taxes in the UK", *International Tax and Public Finance*, **10**, 281 – 304.

Poapangsakorn, N., Charnvitayapong, K., Laovakul, D, Suksiriserekul, S. and Dahlby, B. (2000) "A cost-benefit analysis of the Thailand taxpayer survey", *International Tax and Public Finance*, **7**, 63 – 82.

Slemrod, J. and Yitzhaki, S. (2001) "Integrating expenditure and tax decisions: the marginal cost of funds and the marginal benefit of projects", *National Tax Journal*, **54**, 189 – 201.

Turnovsky, S.J. (1997) "Fiscal policy in a growing economy with public capital", *Macroeconomic Dynamics*, **1**, 615 – 639.