Abstract: The paper considers the role of industry-specific input taxes and aggregate production efficiency in economies with imperfect competition. It is first established that differentiated employment taxes can increase efficiency and the determinants of the relative rates of such taxes are investigated. The employment of an industry will be taxed at a lower rate when that industry has low returns to scale and the tax-shifting effect is large. These findings are then combined with previous analysis of the taxation of final commodities and intermediate inputs. The results show that the optimal tax system will not, in general, maintain production efficiency. A characterisation of optimal input taxes based on the elasticity of input demand is derived.

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INTRODUCTION

It has become a widely accepted proposition in the theory of taxation that labour should be taxed at the same rate in all forms of employment.\textsuperscript{1} The basis for this view is the celebrated "Production Efficiency Lemma" of Diamond and Mirrlees (1971), which asserts the optimality of equalising marginal rates of substitution between firms, and the related work of Dasgupta and Stiglitz (1972) and Mirrlees (1972). Common to all these analyses is the assumption of competitive behaviour on the part of firms. In contrast, recent work has considered the consequences of extending the theory of commodity taxation to incorporate imperfectly competitive behaviour (Konishi 1990; Myles 1989a,b). Given these new insights, it seems natural to reassess the use of employment taxes and the desirability of production efficiency in situations with imperfect competition.

Previous analyses of taxation with imperfect competition have studied the taxation of final commodities and the taxation of produced, or intermediate, inputs but have not addressed the taxation of labour inputs. Consequently, the second section of the paper analyses the effects of employment taxes with imperfect competition by way of considering three simple economies. A comparative statics analysis is conducted and it is shown that when imperfect competition is present, differentiated employment taxes can be justified by efficiency considerations. The factors that determine the relative rates of these taxes, in particular returns to scale and industrial conduct, are isolated and interpreted.

These results are then integrated into an analysis of a complete tax system for an imperfectly competitive economy and of whether production efficiency should be maintained by an optimal tax system when imperfect competition is present. The third section approaches this from a tax reform perspective and characterises the optimal direction of reform from a zero-tax initial position. This is followed by the determination of an optimal tax system. In both cases, a single-consumer economy is analysed so that no equity considerations arise and the tax structures characterised are

\textsuperscript{1} In contrast to this theoretical result, differential input taxes have been used in practice. From 1966 until 1973 the U. K. government levied the Selective Employment Tax which took the form of a weekly tax upon employees in service industries. The motivation was to transfer labour from service industries to manufacturing, further discussion can be found in Reddaway (1970).
efficient. The main message of the analysis is that commodity taxation, labour taxation and intermediate good taxation are complementary instruments in an optimal tax system for an imperfectly competitive economy. Conclusions are given in the final section.

DIFFERENTIATED EMPLOYMENT TAXES

The purpose of this section is to demonstrate how differentiated employment taxes can enhance efficiency in economies characterised by imperfect competition. The basic determinants of relative tax rates are also described. These results, as well as being of interest in their own right, provide the basis for the analysis of a comprehensive tax system in the following sections.

The effects of differentiated employment taxes are studied in three economies of approximately increasing generality. In each it is assumed that the revenue requirement is zero, so that only efficiency aspects of taxation are involved, and that utility is additive in labour supply so that income effects can be ignored. The details of the derivations will mostly be omitted since they involve the application of standard comparative statics techniques.

With regard to the incidence of employment taxes, there are two valid representations. The first is to fix the wage rate received by workers at the level $w$. Firm $j$ in industry $i$ with output $x_{ij}$ facing a tax on its labour input of $\tau_i$ per unit then has cost function

$$C_i(x_{ij}, w + \tau_i),$$

where $w + \tau_i$ is the post-tax cost per unit of labour. The alternative representation is to assume the worker pays the tax and maximises utility with respect to a wage rate $w_i - \tau_i$, with a firm $j$ in industry $i$ having costs

$$C_i(x_{ij}, w_i).$$

[2]

It is probably evident that these representations actually describe identical economies. That this is indeed the case can be seen by noting that competition on the labour market must result in the returns across firms being equalised. Hence $w_i - \tau_i = w$ or $w_i = w + \tau_i$. From this equality the two methods of modelling the economy are equivalent and the real equilibrium is independent of the notional incidence of the tax. The class of taxes under consideration can therefore be viewed as either taxes upon
labour income or as taxes on labour inputs. It is the latter interpretation that is adopted below. In addition, the tax will be assumed to be paid by the firms and consequently the wage received by consumers will be normalised at the fixed value $w$. It can also be shown that the real equilibrium is also unaffected by the choice of value for $w$.\(^2\)

In the first economy, let industry 1 be competitive, employing 1 unit of labour to produce each unit of output. Hence its post-tax price, $q_1$, is equal to marginal cost inclusive of taxation and is given by

$$q_1 = w + \tau_1.$$  \[3\]

Industry 2 is composed of a profit maximising monopolist who faces the demand function

$$X_2 = X_2(q_2).$$  \[4\]

and has costs

$$C^2 = C^2(X_2, w+\tau_2).$$  \[5\]

Since [4] is independent of the level of profits, it embodies the assumption that all income effects fall on labour supply. This is a simplifying assumption that does not alter the qualitative properties of the results derived. The analysis of Myles (1989b) shows how it can be relaxed but at the price of increased algebraic complexity. The restriction to the pair of tax instruments focuses attention on the role of input taxes and permits the characterisation of the optimal values. Although a tax on the output of the monopolist would also be of interest in this framework, the inclusion of further tax instruments is postponed until later in the paper.

From the monopolist's first-order condition for profit maximisation it can be calculated that the effect of the employment tax upon price is given by

$$\frac{dq_2}{d\tau_2} = -\frac{C^2_0}{2} \left( \frac{\partial X_2}{\partial q_2} \right)^2 \frac{\partial^2 X_2}{\partial q_2^2} \left[ q_2 - C^2_0 \frac{\partial X_2}{\partial q_2} \right] - C^2_1 \left( \frac{\partial X_2}{\partial q_2} \right)^2 > 0,$$  \[6\]

where $C^2_0$ and $C^2_1$ are the derivatives of [5] with respect to its first and second arguments. The effect of the tax upon profit is

\(^2\) For economies with imperfect competition this result is proved in Cripps and Myles (1989).
\[
\frac{d\pi_2}{d\tau_2} = -c_1^2 < 0.
\]  \[7\]

Writing \( \ell_i \) for the labour demand of industry \( i \), the government budget constraint is

\[
\tau_1\ell_1 + \tau_2\ell_2 = 0.
\]  \[8\]

Finally, social welfare is determined by the indirect utility function

\[
V(q_1,q_2) + \pi_2.
\]  \[9\]

Maximising [9] with respect to \( \tau_1 \) and \( \tau_2 \) subject to [8] and solving the resulting necessary conditions, the optimal value of \( \tau_2 \) can be characterised implicitly by

\[
\tau_2 = \frac{\ell_1 \left[ \frac{\partial V}{\partial q_1} \frac{\partial q_1}{\partial \tau_2} + \frac{\partial \pi_2}{\partial \tau_2} \right] - \ell_2 \left[ \frac{\partial V}{\partial q_2} \frac{\partial q_2}{\partial \tau_2} + \frac{\partial \pi_2}{\partial \tau_2} \right]}{\frac{\partial \ell_2}{\partial \tau_2} \left[ \frac{\partial V}{\partial q_1} \frac{\partial q_1}{\partial \tau_1} + \frac{\partial \ell_1}{\partial \tau_1} \left[ \frac{\partial V}{\partial q_2} \frac{\partial q_2}{\partial \tau_2} + \frac{\partial \ell_2}{\partial \tau_2} \right] \right]}. \]  \[10\]

Since \( \frac{\partial \ell_i}{\partial \tau_i} < 0 \) for \( i = 1, 2 \), the denominator in [10] is positive and the sign of \( \tau_2 \) is determined by that of the numerator. Noting that \( \ell_1 = X_1, \ell_2 = c_1^2 \) and that [9] implies the marginal utility of income is unity, by using Roy's identity and [7] the numerator of [10] can be written

\[
-X_1X_2 \frac{\partial q_2}{\partial \tau_2} < 0,
\]  \[11\]

where the direction of the inequality follows from [6]. Therefore, in this economy, the labour input to the monopolist should be subsidised and that to the competitive firm should be taxed.

The inequality in [11] demonstrates that differentiated employment taxes can be justified on efficiency grounds in an imperfectly competitive economy since if \( \tau_2 < 0 \) then, in order to satisfy the budget constraint, \( \tau_1 > 0 \). The sign of the optimal tax in [11] is in accordance with the standard partial equilibrium conclusion that the output from the monopolist should be increased. However, it must be noted that [11] was derived in a general equilibrium context and that the subsidy to the monopolist is financed by a tax on the labour input of a competitive industry. That the monopolist should always be subsidised is, perhaps, surprising. It should also be stressed that
since the employment taxes are non-uniform they are not equivalent to an income tax nor to a uniform commodity tax structure.

The second economy is designed to provide insight into the determinants of relative rates of tax on labour employed by two distinct imperfectly competitive industries. Hence let both industries 1 and 2 be monopolistic\(^3\) with demands \(X_1 = X_1(q_1)\) and \(X_2 = X_2(q_2)\). Repeating the maximisation of welfare, with indirect utility now equal to \(V(q_1, q_2) + \pi_1 + \pi_2\), the solution for \(\tau_2\) can be written implicitly as

\[
\tau_2 = \frac{\ell_1 \left[ \frac{\partial V}{\partial q_1} \frac{\partial q_1}{\partial q_1} + \frac{\partial \pi_1}{\partial q_1} \frac{\partial q_1}{\partial \tau_1} \right] - \ell_2 \left[ \frac{\partial V}{\partial q_2} \frac{\partial q_2}{\partial q_2} + \frac{\partial \pi_2}{\partial q_2} \frac{\partial q_2}{\partial \tau_2} \right]}{\ell_2 \left[ \frac{\partial V}{\partial q_1} \frac{\partial q_1}{\partial q_1} + \frac{\partial \pi_1}{\partial q_1} \frac{\partial q_1}{\partial \tau_1} \right] + \ell_1 \left[ \frac{\partial V}{\partial q_2} \frac{\partial q_2}{\partial q_2} + \frac{\partial \pi_2}{\partial q_2} \frac{\partial q_2}{\partial \tau_2} \right]}. \tag{12}
\]

The denominator of (12) is positive so the sign of \(\tau_2\) is again determined by that of the numerator. Substituting from Roy's identity and using the fact that \(\frac{\partial \pi_i}{\partial \tau_i} = -C_{i1}\), the numerator can be written

\[
X_1 C_{11} \frac{\partial q_1}{\partial \tau_1} - X_2 C_{12} \frac{\partial q_2}{\partial \tau_2}. \tag{13}
\]

Using the definitions of the tax-shifting terms\(^4\), and dividing across by the first derivatives of the cost functions, the sign of (13) is given by the sign of

\[
X_1 \frac{C_{011}}{C_{11}} - X_2 \frac{C_{012}}{C_{12}}. \tag{14}
\]

To interpret (14), note that \(C_i\) is a measure of returns to scale: \(\frac{\partial q}{\partial \tau} \rightarrow 0\), \(i = 1, 2\) represents decreasing returns to scale, a value of 1 constant returns and \(< 1\) increasing returns. Now assume that the two bracketed terms, which can be said to capture "market conditions", are equal. It then follows that if

\[
X_1 \frac{C_{011}}{C_{11}} > X_2 \frac{C_{012}}{C_{12}}. \tag{15}
\]

\(^3\) The use of "monopolist" is justified here due to the separable demands.

\[
\frac{dq}{d\pi} = \frac{X_1}{q_1} > 0, \ i = 1, 2.
\]

\(^4\) The use of "monopolist" is justified here due to the separable demands.
then $\tau_2 > 0$. That is, the firm with lower returns to scale should have its labour input subsidised when the market conditions are identical. This is a reflection of the fact that this firm will be proportionately further from the competitive output level. In contrast, if the returns to scale are the same for both firms, the bracketed terms describe the rate at which the firms would forward-shift any commodity tax, a response determined by the demand function it faces. Hence the firm that would forward-shift most should have the labour input subsidised. In this case, if $C_{00}^i = 0$, then $\tau_2 > 0$ if the elasticity of the slope of the inverse demand function, given by Seade's "E" (see Seade 1985 and Myles 1995), facing firm 1 is greater than the elasticity of that facing firm 2.

From the analysis above, it can be seen how relative taxes are determined by a composition of market demand and returns to scale properties. The role of these can be understood by viewing the aim of the taxes as the achievement of a more balanced exploitation of returns to scale but the cost of this process is the consequent change in market prices, with the rate of change of prices being dependent upon the demand elasticity facing the firm. It is the resolution of these two effects that is captured in [14].

The final economy comprises two oligopolistic industries and allows for differences in both the size, in terms of the number of firms, and the conduct of the industries. The firms comprising each industry are assumed to compete by choosing quantities of an homogeneous product. To introduce differences of conduct, it is assumed that each firm conjectures how aggregate output will change in response to a change in their output. As the industries are taken to be symmetric, all firms in each industry have the same conjecture. The conjecture is denoted by $\lambda_i$, $i = 1, 2$. A value of $\lambda_i = 0$ represents "Bertrand" competition and $\lambda_i = 1$ "Cournot" competition.

With $n_i$ firms in industry $i$ and $x_i^j$ denoting the output of firm $j$ in industry $i$, the inverse demand functions are defined as

$$q_i = q_i(X_i), \quad X_i = \sum_{j=1}^{n_i} x_i^j, \quad i = 1, 2.$$  \[16\]

From the conditions for profit maximisation, it follows that

$$\frac{dq}{d\tau_i} = \frac{n_i C_{01} \frac{\partial q}{\partial X_i} + \lambda_i \frac{\partial^2 q}{\partial X_i^2}}{[n_i + \lambda_i \frac{\partial q}{\partial X_i} + x_i n_i \lambda_i]} - C_{00} > 0, i = 1, 2,$$  \[17\]
and

\[
\frac{d\pi^i}{d\tau_i} = \frac{C_{i0}^j \left[ q_i + x_i n_i \frac{\partial q_i}{\partial X_i} - C_0^i \right]}{\left[ n_i + \lambda_i \right] \frac{\partial q_i}{\partial X_i} + x_i n_i \lambda_i \frac{\partial^2 q_i}{\partial X_i^2} - C_{i0}^i} - C_{i0}^i, \quad j = 1, \ldots, n_i, i = 1, 2,
\]

[18]

where \( x_i \) is the output of each firm at the symmetric equilibrium and \( C^i(\cdot) \) is the cost function common to all firms in industry \( i \).

The optimal value of \( \tau_2 \) will again be characterised by [12]. Assuming the denominator to be positive, substitution from [17] and [18] allows the numerator to be written as

\[
n_1 C_i^1 \left[ \frac{n_2 C_0^1 \left[ q_2 - C_0^1 \right]}{\left[ n_2 + \lambda_2 \right] \frac{\partial q_2}{\partial X_2} + x_2 n_2 \lambda_2 \frac{\partial^2 q_2}{\partial X_2^2} - C_{i0}^1} - n_2 C_0^1 \right] - n_2 C_i^2 \left[ \frac{n_1 C_{i0}^1 \left[ q_1 - C_{i0}^1 \right]}{\left[ n_1 + \lambda_1 \right] \frac{\partial q_1}{\partial X_1} + x_1 n_1 \lambda_1 \frac{\partial^2 q_1}{\partial X_1^2} - C_{i0}^1} \right].
\]

[19]

From [19] it is possible to describe the result in a number of limiting cases. Firstly, it illustrates that with marginal cost pricing in both industries the employment taxes will both be zero, which is equivalent to the standard result for competitive economies, and that if one industry practices marginal cost pricing, the labour input for the other, assuming price is above cost, should be subsidised. Secondly, provided the derivatives are bounded, as the number of firms in both industries increases the taxes will both tend to zero.

Using the first-order condition for profit maximisation of the individual firms, [19] can be usefully re-written as

\[
n_1 C_i^1 \left[ \frac{-n_2 C_0^1 x_2 \lambda_2 \frac{\partial q_2}{\partial X_2}}{\left[ n_2 + \lambda_2 \right] \frac{\partial q_2}{\partial X_2} + x_2 n_2 \lambda_2 \frac{\partial^2 q_2}{\partial X_2^2} - C_{i0}^1} \right] - n_2 C_i^2 \left[ \frac{n_1 C_{i0}^1 x_1 \lambda_1 \frac{\partial q_1}{\partial X_1}}{\left[ n_1 + \lambda_1 \right] \frac{\partial q_1}{\partial X_1} + x_1 n_1 \lambda_1 \frac{\partial^2 q_1}{\partial X_1^2} - C_{i0}^1} \right].
\]

[20]

If competition is Bertrand in both industries (\( \lambda_i = 0, \ i = 1, 2 \)) the taxes will both be zero; in other cases, with demands and costs equal, the industry with the larger value of \( \lambda_i \) will be subsidised. Finally, as \( n_1 \) tends to infinity, the labour input for industry 2 should be subsidised and vice versa.

This completes the consideration of the example economies. The fact that differential employment taxes can increase welfare in an imperfectly competitive economy has been established and several factors have emerged as relevant to the
determination of relative rates of tax. Amongst these factors are the returns to scale of the industries' production processes, the form of the demand function facing the industry as captured by Seade's $E$, the conduct of each industry and the size of the industry in terms of the number of firms. This latter factor is representative of a notion of the competitiveness of the industry.

**TAX REFORM AND PRODUCTION EFFICIENCY**

A fundamental result in the theory of taxation for competitive economies is that the optimal tax structure should be designed so that production efficiency is attained, which that the marginal rate of substitution between any two inputs is the same for all firms. In the absence of taxation, such a position is achieved by the profit maximising behaviour of firms in competitive markets. When taxation is introduced, profit maximisation will still lead to productive efficiency provided that all firms face identical post-tax prices for inputs so that input taxes should not be differentiated between firms. Furthermore, the observation that any equilibrium set of consumer prices and level of government revenue that can be achieved by combining input taxes and final goods taxes can also be achieved by taxes on final goods alone renders the use of input taxes redundant.

For economies with imperfect competition, the role of production efficiency has only been partially investigated. Myles (1989a) considers optimal commodity taxation but in an economy where labour is the only input so that, as in the analysis of the previous section, production inefficiency can never arise. Myles (1989b) and Konishi (1990) both consider the taxation of intermediate goods but do not treat the taxation of labour inputs. The analysis above has, however, shown that the differentiated taxation of labour is also likely to be desirable.

It is the purpose of this section to demonstrate that since it will, in general, be desirable to differentiate input taxes between firms, production efficiency will be violated. To simplify the presentation and to sharpen the results, the economy studied will be restricted to one with two monopolistic firms each employing labour and an intermediate input. The intermediate input is produced by a competitive firm. This economy is chosen since it is one of the simplest that can combine imperfect competition with intermediate inputs and the possibility of production inefficiency.
There is nothing essential in any of the restrictions and the results that are derived will hold true in more general settings.

This section will employ the approach to characterisation of the tax structure adopted in the literature on tax reform (especially Dixit (1979) and Kanbur and Myles (1992)) which is to maximise the increase in welfare that can be attained whilst restricting the tax changes to be "small". The value of this approach is that it yields simple formula for the direction in which tax reform should proceed. In addition, since it can also be interpreted as being the first step in an iterative procedure to find the optimal taxes, it also indicates the sign pattern that the optimal taxes will take. The derivation of optimal taxes will be presented in the following section.

The structure of the economy is as follows. Firm 0 (or, equivalently, an industry of identical small firms) employs labour to produce an intermediate good and acts competitively on all markets. Units of measurement are chosen so that its production technology requires one unit of labour for each unit of produced good; the price of its output is therefore $q_0 = w + \tau_0$ where $\tau_0$ is the tax paid on each unit of labour input\(^5\). The wage rate is again normalised at $w$. Firms 1 and 2 are monopolistic. Each faces the demand function $X_i(q_i)\), $i=1,2$ and pays taxes $t_i$ on output, $\zeta_i$ on each unit of intermediate good used and $\tau_i$ on employment. The profit function of firm $i$ is therefore

$$\pi_i = q_i X_i(q_i) - t_i X_i - C_i^1(X_i(q_i), q_0 + \zeta_i, w + \tau_i)$$

$$= q_i X_i(q_i) - t_i X_i - C_i^1(X_i(q_i), w + \tau_0 + \zeta_i, w + \tau_i).$$

[21]

From [21] it can be seen immediately that the simplification $\tau_0 = 0$ can be adopted without introducing any restriction. Maximisation by firms 1 and 2 will result in optimal prices and profit levels that are functions of the tax rates. These are denoted by $q_i = q_i(t_i, \tau_i, \zeta_i)$ and $\pi_i = \pi_i(t_i, \tau_i, \zeta_i), i=1,2$. To maintain the focus upon efficiency, the consumption side of the economy consists of a single consumer whose preferences can be represented by the indirect utility function $V(q_1, q_2) + \pi_1 + \pi_2$.

The characterisation of optimal taxes via welfare-maximising reform involves choosing the direction of tax change to maximise the increase in welfare that can be obtained whilst restricting the tax changes to be small. The simplifying aspect of this approach is that the restriction to small changes allows the problem to be linearised by

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\(^5\) Given the structure of the production technology, the competitive intermediate good could also be interpreted as a second form of labour service employed by the monopolistic firms. The results then show that different forms of labour should be taxed at different rates within a firm (as well as across firms) even though they yield the same disutility to the worker.
using the gradient vectors of the functions involved. To proceed, the vector of tax changes is written
\[ dT' = (dt_1, d\tau_1, d\zeta_1, dt_2, d\tau_2, d\zeta_2) = (dT_1, \ldots, dT_6), \]
the gradient vector of the indirect utility function with respect to changes in the tax rates is denoted by
\[ \nabla V' = \begin{bmatrix} \frac{\partial V}{\partial q_1} \partial t_1 & \frac{\partial V}{\partial q_2} \partial t_2 & \ldots & \frac{\partial V}{\partial \zeta_1} \partial t_1 & \frac{\partial V}{\partial \zeta_2} \partial t_2 \\ \frac{\partial \pi_1}{\partial t_1} & \frac{\partial \pi_1}{\partial t_2} & \ldots & \frac{\partial \pi_1}{\partial \zeta_1} & \frac{\partial \pi_1}{\partial \zeta_2} \\ \frac{\partial \pi_2}{\partial t_1} & \frac{\partial \pi_2}{\partial t_2} & \ldots & \frac{\partial \pi_2}{\partial \zeta_1} & \frac{\partial \pi_2}{\partial \zeta_2} \end{bmatrix} = (\nabla V_1, \ldots, \nabla V_6) \]
and the vector of quantities is
\[ Z' = (X_1, \ell_1, y_1, X_2, \ell_2, y_2) = (Z_1, \ldots, Z_6) \]
where \( \ell_i, y_i = C_i \) are the labour demand and intermediate good demand of firm \( i \). Expressed formally, starting from an initial position with no taxation the tax rate changes solve
\[ \max_{(ar)} \nabla V' dT', \]
subject to
\[ \begin{array}{l}
(i) \quad Z dT = 0, \\
(ii) \quad dT' dT = 1,
\end{array} \]
where all derivatives are evaluated at pre-tax prices. Condition (i) ensures that the taxes introduced raise zero revenue and (ii) that the sum of squares of the tax changes is 1. The interpretation of this condition is that units have been chosen such that 1 is small. This choice of restriction is inconsequential and the algebra is only slightly modified by choosing a number other than 1.

Introducing multipliers \( \lambda \) and \( \mu \) for the two constraints, the first-order condition for the choice of \( dT_i \) is
\[ \nabla V_i + \lambda Z_i - 2\mu dT_i = 0. \]
[22]
Multiplying [22] by \( Z_i \), summing over \( i \) and using (i) gives
\[ \lambda = -\frac{\nabla V Z}{Z' Z}. \]
[23]
Similarly, multiplying [22] by \( dT_i \), and summing gives
\[ \mu = \frac{\nabla V' dT}{2}. \]
[24]
It is assumed that welfare can be raised by some combination of the tax instruments chosen so that \( \mu > 0 \). Using [22], [23] and [24], the solution for \( dT_i \) can then be written
\[ dT_i = \frac{[Z' Z \nabla V_i - [\nabla V' Z]Z_i]}{2\mu [Z' Z]} = \frac{[Z' Z \nabla V_i - [\nabla V' Z]Z_i]}{[Z' Z]^T [Z' Z \nabla V' \nabla V - 2 \nabla V' Z]^T}. \]
[25]
Employing Roy’s identity and the envelope theorem

\[
\nabla V' = -X_1 \frac{\partial q_1}{\partial t_1} + \ell_1, X_1 \frac{\partial q_1}{\partial \tau_1} + y_1, X_2 \frac{\partial q_2}{\partial t_2} + \ell_2, X_2 \frac{\partial q_2}{\partial \tau_2} + y_2.\]

[26]

Therefore

\[
\nabla V' Z = -(Z' Z + S),
\]

[27]

where \( S = \sum_{i=1}^{2} X_i \left[ X \frac{\partial q_i}{\partial t_i} + \ell_i, X \frac{\partial q_i}{\partial \tau_i} + y_i, \frac{\partial q_i}{\partial \xi_i} \right] > 0. \) The expression \( S \) is composed of the sum of rates of tax shifting multiplied by the quantity of the taxed good weighted by the levels of final demand. \( S \) will be large if the price of either firm is particularly sensitive to tax increases.

As zero revenue is to be raised, any move from the initial position must make some taxes positive and others negative. Since \( \mu > 0 \), it follows from [25] that

\( \text{sgn}\{dT_i\} = \text{sgn}\left\{ Z' Z \nabla V_i - [\nabla V' Z]Z_i \right\}. \)

For instance, the commodity tax on good 1 will become positive if \( SX_1 - [Z' Z]X_1 \frac{\partial q_1}{\partial t_1} > 0 \) and the labour tax positive if \( S\ell_1 - [Z' Z]X_1 \frac{\partial q_1}{\partial \tau_1} > 0. \) Since \( S \) is effectively an average of shifting terms, it can be seen from these expressions that the tax will tend to become positive when its shifting effect is less than average and will be negative when it is greater than average. Similar conclusions hold for the other tax rates.

The main focus here, though, is upon whether the reform will lead to input taxes that are differentiated between the firms. Returning to [25], the tax reform will move towards differentiated employment taxes if

\[ Z' Z \nabla V_2 - [\nabla V' Z]Z_2 \neq Z' Z \nabla V_5 - [\nabla V' Z]Z_5, \]

[28]

or

\[ Z' Z \left[ -X_1 \frac{\partial q_1}{\partial \tau_1} - \ell_1 \right] - [\nabla V' Z] \ell_1 \neq Z' Z \left[ -X_2 \frac{\partial q_2}{\partial \tau_2} - \ell_2 \right] - [\nabla V' Z] \ell_2. \]

[29]

\[
\frac{dk_i}{dt_i} = \frac{X_i}{q_i^2} > 0, \frac{dk_1}{d\tau_1} = C_{01} \frac{dk_1}{dt_1} > 0, \frac{dk_2}{d\tau_2} = C_{10} \frac{dk_2}{dt_2} > 0, i = 1, 2.
\]
Using [27], inequality [29] reduces to

\[
S[\ell_1 - \ell_2] + Z' Z \left[ X_2 \frac{\partial q_2}{\partial \tau_2} - X_1 \frac{\partial q_1}{\partial \tau_1} \right] \neq 0. \quad [30]
\]

Following the same derivation, the intermediate input taxes will be differentiated if

\[
S[y_1 - y_2] + Z' Z \left[ X_2 \frac{\partial q_2}{\partial \zeta_2} - X_1 \frac{\partial q_1}{\partial \zeta_1} \right] \neq 0. \quad [31]
\]

By inspection it is clear that [30] and [31] will generally be satisfied and that the tax reform will lead to the differentiation of input taxes between firms.

However situations do exist in which [30] and [31] will not be satisfied and production efficiency will be maintained. The most obvious case occurs when \( \ell_1 = \ell_2 \) and \( X_2 \frac{\partial q_2}{\partial \ell_2} = X_1 \frac{\partial q_1}{\partial \ell_1} \), for which \( X_i(q_i) = X_2(q_2) \) and \( C'(X_i) = C'(X_2) \) are sufficient, but not necessary, conditions. To identify a more relevant case of non-differentiation, assume that both firms produce with constant returns to scale so that \( C_i(w + \tau_i, w + \zeta_i) = X_i c_i(w + \tau_i, w + \zeta_i) \). It then follows that \( C'_{00} = 0, \ C'_{01} = \ell_i / X_i \) and \( C'_{02} = y_i / X_i \). Using these identities and the derivatives given in fn. 7, in the case of constant returns [30] reduces to

\[
\left[ \frac{\partial q_2}{\partial \ell_2} - \frac{\partial q_i}{\partial \ell_i} \right] \sum_{i=1,2} \sum_{j=1,2 \atop j \neq i} \left[ X_i^2 \ell_j^2 + y_i^2 \right], \quad [32]
\]

and an analogous expression, with \( y_j \) replacing \( \ell_j \), holds for [31]. Hence non-differentiation occurs with constant returns when the effect of the final good taxes on price are equal. For instance, with linear demand \( \frac{\partial q_i}{\partial \ell_i} = \frac{1}{2} \) so that linear demand and constant returns imply that production efficiency should be maintained. This result also gives an alternative insight into why production efficiency should be maintained in a competitive economy with constant returns since, in such an environment, \( \frac{\partial q_i}{\partial \ell_i} = 1 \).

Despite these special cases, the general conclusion remains that the employment tax and the intermediate goods tax will be differentiated between the firms. To confirm that such differentiation implies that production efficiency will not be achieved, it also needs to be shown that the reform will be such that the ratio of
The post-tax price of labour to post-tax intermediate input price for firm 1 is not equal to that of firm 2 or that \( \frac{w + \tau_1}{w + \zeta_1} \neq \frac{w + \tau_2}{w + \zeta_2} \). The calculation of this expression is straightforward using [25] but, other than leading to the observation that it will generally be satisfied when [30] and [31] are, doing so does not add any further insight.

The finding of production inefficiency above is of a different nature to the temporary production inefficiency that arises along the reform paths in Dixit (1979) and Fogelman, Quinzii and Guesnerie (1978). The analysis here has shown that the tax rates themselves will diverge at the initial stage of the reform. If the reform problem is sufficiently monotonic a continuation of the reform process would witness tax rate changes continuing in the direction they adopted at the start of the reform. This would inevitably lead to production inefficiency at the optimum. In contrast, the temporary production inefficiency arises as the result of the shortest path between points on the frontier of the production set passing through its interior.

**OPTIMAL TAXATION**

Although the results of the analysis of the reform problem are indicative of the structure of optimal taxes, they do not provide a complete characterisation. The contention that production inefficiency will remain at the optimum is now examined by determining the optimal tax rates directly. From the results obtained, it will be argued that employment taxes and intermediate input taxes will in general be differentiated between firms.

The optimisation problem now under consideration is given by

\[
\max_{\{t_i, \tau_i, \zeta_i, j=1,2\}} V(q_1, q_2) + \pi_1 + \pi_2, \quad [33]
\]

subject to

(i) \( q_i = q_i(t_i, \tau_i, \zeta_i), \pi_i = \pi_i(t_i, \tau_i, \zeta_i), i = 1,2, \)

(ii) \( \sum_{i=1,2} t_i X_i + \sum_{i=1,2} \tau_i \ell_i + \sum_{i=1,2} \zeta_i y_i = 0. \)

Condition (i) states the dependence of price and profits levels upon the tax rates and (ii) is the zero-revenue budget constraint. The requirement of zero revenue implies
that divergence of the optimal taxes from zero will occur only in order to increase efficiency.

The first-order conditions for the maximisation can be arranged to give the following (implicit) solution for the tax rates

\[
\begin{bmatrix}
  t_1 \\
  \tau_1 \\
  \xi_1 \\
  t_2 \\
  \tau_2 \\
  \xi_2
\end{bmatrix}
= \frac{1}{\lambda}
\begin{bmatrix}
  A_1^{-1} & 0 \\
  0 & A_2^{-1}
\end{bmatrix}
\begin{bmatrix}
  X_1[1 - \lambda] + \frac{\partial q_1}{\partial t_1} \\
  \ell_1[1 - \lambda] + \frac{\partial q_1}{\partial \tau_1} \\
  X_1[1 - \lambda] + \frac{\partial q_1}{\partial \xi_1} \\
  y_1[1 - \lambda] + \frac{\partial q_2}{\partial t_1} \\
  \ell_2[1 - \lambda] + \frac{\partial q_2}{\partial \tau_2} \\
  y_2[1 - \lambda] + \frac{\partial q_2}{\partial \xi_2}
\end{bmatrix},
\]

where

\[
A_i = \begin{bmatrix}
\frac{\partial X_i}{\partial t_i} & \frac{\partial \ell_i}{\partial t_i} & \frac{\partial \xi_i}{\partial t_i} \\
\frac{\partial X_i}{\partial \tau_i} & \frac{\partial \ell_i}{\partial \tau_i} & \frac{\partial \xi_i}{\partial \tau_i} \\
\frac{\partial X_i}{\partial \xi_i} & \frac{\partial \ell_i}{\partial \xi_i} & \frac{\partial \xi_i}{\partial \xi_i}
\end{bmatrix},
\]

\[i = 1, 2,\]

and \(\lambda\) is the multiplier on the revenue constraint.

The structure of [34] shows immediately why production efficiency does not apply. Since the demand for each good is assumed to depend only upon the price of that good, the two industries are effectively separate. The structure of taxation is then chosen to control each industry individually with the Lagrange multiplier providing the link between the two. As the industries are controlled separately, there is no reason to expect any particular link between the input taxes levied on one and those levied on the other. The total separability is, of course, a reflection of the structure of demand chosen for the example and was selected to emphasise the point to be made. However, production efficiency cannot be a general property of the optimal tax system since it does not hold for this example. Under alternative assumptions, the separability would be lost but there would remain no uniform relation between the input taxes.

To investigate the factors that determine the structure of the taxes it is best to write the first-order conditions for the optimisation in a different form. For the choice of \(\tau_i\) the necessary condition can be written, using Roy’s identity and the envelope property of the profit function, as
\[-X_i \frac{\partial q_i}{\partial \tau_i} - \ell_i + \lambda \left[ \ell_i + t_i \frac{\partial X_i}{\partial q_i} \frac{\partial q_i}{\partial \tau_i} + \tau_i \frac{\partial \ell_i}{\partial \tau_i} + \zeta_i \frac{\partial \tau_i}{\partial \tau_i} \right] = 0. \]  

[36]

The relation \( \frac{\partial q_i}{\partial \tau_i} = \frac{C_{10}^i}{\partial t_i} \) (see fn.7) can then be employed to place [36] in the form

\[
\frac{X_i C_{10}^i \ell_i}{\partial t_i} = \frac{\left[ \frac{1 - \lambda}{\lambda} \right] - \tau_i C_{1i}^i - \zeta_i C_{2i}^i}{\left[ \frac{t_i + \tau_i C_{1n}^i + \zeta_i C_{2n}^i}{X_i} \right] \frac{\partial X_i}{\partial q_i} - \frac{1}{\lambda} \frac{\partial q_i}{\partial t_i}}.
\]  

[37]

Similarly, for the choice of \( \zeta_i \)

\[
\frac{X_i C_{20}^i \gamma_i}{\partial t_i} = \frac{\left[ \frac{1 - \lambda}{\lambda} \right] - \tau_i C_{12}^i - \zeta_i C_{22}^i}{\left[ \frac{t_i + \tau_i C_{1n}^i + \zeta_i C_{2n}^i}{X_i} \right] \frac{\partial X_i}{\partial q_i} - \frac{1}{\lambda} \frac{\partial q_i}{\partial t_i}},
\]  

[38]

and, for \( t_i \)

\[
1 = \frac{\left[ \frac{1 - \lambda}{\lambda} \right]}{\left[ \frac{t_i + \tau_i C_{1n}^i + \zeta_i C_{2n}^i}{X_i} \right] \frac{\partial X_i}{\partial q_i} - \frac{1}{\lambda} \frac{\partial q_i}{\partial t_i}}.
\]  

[39]

These equations can be used in two ways. Firstly, [39] shows that the higher are \( \frac{\partial X_i}{\partial q_i} \) and \( \frac{\partial q_i}{\partial t_i} \) the lower should be the relative taxes on industry \( i \). This observation captures the result (shown in Myles (1987)) that industries that overshift taxes to a greater extent are poor candidates for taxation since a given revenue raised will be met by a higher price. Similarly, the higher is \( C_{j0}^i \) the lower should be the tax on input \( j \). As \( C_{j0}^i \) is marginal cost, this is just the statement that if marginal cost rises rapidly with an increase in the price of an input, that input should not be taxed heavily.

The second use of is to provide a characterisation of the optimum in terms of input use and the cost function. It is first noted that \( \frac{X_i C_{10}^i \ell_i}{\partial t_i} \equiv \epsilon_i^\ell \) and \( \frac{X_i C_{20}^i \gamma_i}{\partial t_i} \equiv \epsilon_i^\gamma \) are the elasticities of factor demand with respect to changes in output. Then, combining [39] with [37] and [38] shows that at the optimum
\[ \frac{\tau_i C_{11} + \zeta_i C_{12}}{\tau_i C_{21} + \zeta_i C_{22}} = \frac{1 - \epsilon_i}{1 - \epsilon_i}, \quad i = 1, 2. \] \[40\]

For small taxes, the numerator of the left-hand side of [40] is approximately the reduction in demand for labour by firm \( i \) due to the imposition of the taxes with output held constant while the denominator is the reduction in intermediate input demand. From [40] the reductions in input demand should be inversely related to the elasticities of input demand - a form of input Ramsey rule. Alternatively, [40] can be solved to give the characterisation of relative tax rates

\[ \frac{\tau_i}{\zeta_i} = \left[ \frac{1 - \epsilon_i C_{22} - (1 - \epsilon_i C_{12})}{1 - \epsilon_i C_{11} - (1 - \epsilon_i C_{21})} \right], \quad i = 1, 2. \] \[41\]

Eq. [41] reinforces how the relative optimal tax rates are determined by the elasticities of input demand and, since the ratio for each firm is determined by the properties of that firm’s cost function, the ratios for the two firms will be independent.

This section has considered the structure of optimal taxation of both outputs and inputs. Characterisations of optimal taxes have been given and it is clear from these that production efficiency will not in general be maintained at the optimum. Although the analysis was conducted for a simplified economy it can be extended but at the expense of cumbersome notation.

**CONCLUSIONS**

The paper has considered the role of industry-specific input taxes in models of optimal taxation. In competitive economies, it follows from the Diamond-Mirrlees Production Efficiency lemma that such taxes are not efficient. With imperfect competition it was established, via the analysis of three simple economies, that differential employment taxes could be justified by efficiency considerations. In addition, the economies that were studied illustrated the determinants of the relative rates of taxation. The labour input into an industry will generally be taxed at a lower rate when the industry has low returns to scale and equilibrium price increases markedly in response to the tax. The second of these factors can be broken down further to distinguish between the consequences of the shape of the demand curve facing the industry (summarised by Seade’s \( E \)), the conduct of the industry in terms of conjectures and its competitiveness measured, approximately, by the number of firms in the industry.
These results were then integrated with the taxation of final commodities and produced inputs. A complete commodity tax system for an imperfectly competitive economy must consist of taxes on all final goods and labour; with the results of this paper suggesting that the input taxes should be industry specific. Labour and commodity taxes are therefore complementary in the optimal tax system. The analysis of the tax reform problem and the characterisation of optimal taxes demonstrated that production efficiency will not be desirable with imperfect competition.

REFERENCES


