Growth and Public Infrastructure

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Abstract

The paper analyzes a multi-country extension of the Barro model of productive public expenditure. In the presence of infrastructural externalities between countries the provision of infrastructure will be inefficiently low if countries do not coordinate. This provides a role for a supra-national body, such as the EU, to coordinate the policies of the individual governments. It is shown how intervention by a supra-national body can raise welfare by internalizing the infrastructural externality. Infrastructural externalities increase the importance of tax policy in the growth process and distribute the benefits of taxation across countries.

Keywords: Public infrastructure, growth, externality

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1 Introduction

One factor promoting endogenous growth is the supply of public infrastructure that complements the capital investments of the private sector. The importance of infrastructure is widely recognized, not least by the EU which pursues an active programme to support the investment activities of member states. The policy problem facing the EU is to ensure that member states undertake an efficient level of infrastructural expenditure that ensures the maximum rate of growth. The determination of the level has to take into account the full consequences of an infrastructure project for the EU, not just the direct benefits for the member state undertaking the investment. There are three significant issues that confront this policy programme. First, infrastructural investment has significant spillovers across member states. Second, mobility of the tax base results in tax externalities between the member states, and between the member states and the EU. Third, the EU is faced with a decision on how to allocate support for infrastructural expenditure across the different member states. This interacts with the process of revenue-raising, and with the extent to which the projects are financed jointly by the EU and member states.

The economic modelling of the impact of infrastructure on economic growth has focussed on the Barro (1990) model of public expenditure as a public input and its extensions (Chen et al. 2005, Turnovsky, 1999). This literature has identified the concept of an optimal level of expenditure, and has highlighted the deleterious effects of both inadequate and excessive expenditure. These are important insights, but they do not address the spillover issues that confront the EU. Infrastructural spillovers between member states can be positive, which occurs when improvements in infrastructure in one member state raise productivity in another, or they can be negative if they induce relocation of capital between member states. In either case, it is important that the consequences of spillovers are addressed in order that the role of productive public expenditure can be fully understood. Ignoring either form of spillover will result in an inefficient level and allocation of expenditure.

The financing of infrastructure in the Barro model is through a simple tax on output levied at the national level. The position in the EU is much more complex. Each member state levies national taxes. Part of the revenue from these taxes is retained by the member states, the remainder is remitted to, and redistributed by, the EU. In economic terms, if there is mobility of the tax base then there are horizontal tax externalities between member states, and a vertical tax externality between member states and the EU. These tax externalities have a key role in determining the growth-maximizing level of expenditure.

In this paper we construct a multi-country extension of the Barro model of productive public infrastructure in which the benefits of infrastructure spillover between countries. The spillover between countries is a form of positive externality which results in inefficient investment in infrastructure if countries act independently. If there are positive infrastructural externalities between countries then the provision of infrastructure will be inefficiently low when countries do not coordinate policies. This gives a role to a supra-national body, such as
the EU, to act as a coordinator of the policies of individual governments. The supra-national body can intervene by revenue-matching to counter the externality and obtain an increase in welfare. The infrastructural externality raises the importance of tax relative to a world without spillovers since additional public infrastructure in one country can raise the growth rate in all. This holds if all countries are operating with less than the optimum level of infrastructure, as they will be in an equilibrium without policy intervention.

Section 2 analyzes a basic version of the endogenous growth model with a productive public input. Section 3 studies the role of a supra-national body in coordinating the choices of individual countries when there is an infrastructural externality. Simulation results are presented in Section 4. Conclusions are given in Section 5.

2 Public Infrastructure

Endogenous growth can occur when capital and labour are augmented by additional inputs in a production function that otherwise has non-increasing returns to scale. One interesting case for understanding the link between government policy and growth is when the additional input is a public good or public infrastructure financed by taxation. The need for public infrastructure to support private capital in production provides a positive role for public expenditure and a direct mechanism through which policy can affect growth. The Barro (1990) model of productive public expenditure was the first to investigate the role of public infrastructure and permitted an analysis of the optimal level of public expenditure in an endogenous growth model.

The analysis of the growth path as undertaken in Barro (1990) works successfully for that particular specification of the model. However, it is difficult to generalize the approach to more complex settings in a way that permits explicit results to be derived. We adopt a different approach in the modelling that follows. The basis of this approach is that instead of looking at the growth path from an arbitrary starting point we focus on balanced growth paths. Along a balanced growth path all real variables grow at the same rate, so it can be interpreted as describing the process of growth in the long-run. We model the consumer as choosing a balanced growth path given the path of tax rates announced by the government. The government chooses the path of tax rates to maximize consumer welfare. If the tax is distortionary the resulting growth rate will not be first-best optimal. We see this analysis as the dynamic equivalent of maximizing welfare in a standard static Diamond-Mirrlees type framework. In characterizing the equilibrium we exploit two equivalences. The first equivalence is that between the market equilibrium and the outcome when the consumer chooses the path of capital directly. This is a standard result that has been widely exploited to simplify the derivation of the path of capital accumulation in growth models. The second is that, in the long-run, the outcome with the consumer choosing the path for capital is equivalent to the consumer directly choosing the rate of growth of the capital stock on the balanced growth path.
The equivalence holds provided the economy always tends to a balanced growth path, a property that we assume applies to the economies we study.

We now demonstrate our approach by applying it to a version of the Barro model. With public infrastructure the production function for the representative firm at time $t$ takes the form

$$Y_t = AL_t^{1-\alpha}K_t^{\alpha}G_t^{1-\alpha}$$

(1)

where $A$ is a positive constant and $G_t$ is the quantity of public infrastructure. The form of this production function ensures that there are constant returns to scale in labour, $L_t$, and private capital, $K_t$, for the firm given a fixed level of public infrastructure. Although returns are decreasing to private capital as the level of capital is increased for fixed levels of labour and public input, there are constant returns to scale in public input and private capital together. For a fixed level of $L_t$, this property of constant returns to scale in the other two inputs permits endogenous growth to occur.

We assume that government spending is funded from a tax levied on the private capital input. Public infrastructure and private capital depreciate in use at rates $\delta_G \geq 0$ and $\delta_K \geq 0$ respectively. The budget constraint of the government, or the law of motion for public infrastructure, at time $t$ is

$$G_t = (1 - \delta_G) G_{t-1} + \tau_t K_t.$$ (2)

The firm belongs to a representative infinitely lived household whose preferences are from this point described by an instantaneous utility function, $U_t = \ln C_t$. The household maximizes the infinite discounted stream of utility

$$\max \sum_{t=0}^{\infty} \beta^t \ln C_t,$$ (3)

subject to the sequence of intertemporal budget constraints,

$$Y_t = C_t + K_{t+1} - (1 - \delta_K) K_t + \tau_t K_t,$$ (4)

and with the sequence of taxes and government infrastructure taken as given.

We focus on balanced growth path equilibria along which the tax rate is constant and all real variables grow at the same constant rate. The first step is to show that when $K_t$ grows at a constant rate $\gamma$ and taxes are constant the law of motion for $G_t$ also converges to growth at the same constant rate. Assume that the private capital stock grows at rate $\gamma$ from time 0 with capital stock $K_0$. Recursive substitution into (2) gives

$$G_{t+1} = (1 - \delta_G) G_t + \tau K_{t+1}$$

$$= (1 - \delta_G)^{t+1} G_0 + \sum_{i=0}^{t} (1 - \delta_G)^i \tau K_{t+1-i}.$$ (5)

From the relation

$$K_{t+1-i} = (1 + \gamma)^{t+1-i} K_0,$$ (6)

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it follows that
\[
\sum_{i=0}^{t} (1 - \delta G)^i \tau K_{t+1-i} = \tau K_0 (1 + \gamma)^{t+1} \sum_{i=0}^{t} \left( \frac{1 - \delta G}{1 + \gamma} \right)^i \\
= \tau K_0 \frac{1 + \gamma}{\gamma + \delta G} \left( (1 + \gamma)^{t+1} - (1 - \delta G)^{t+1} \right).
\]

(7)

Hence,
\[
G_{t+1} = (1 - \delta G)^{t+1} G_0 + \tau K_0 \frac{1 + \gamma}{\gamma + \delta G} \left[ (1 + \gamma)^{t+1} - (1 - \delta G)^{t+1} \right] \\
= (1 - \delta G)^{t+1} \left( G_0 - \tau K_0 \frac{1 + \gamma}{\gamma + \delta G} \right) + \frac{1 + \gamma}{\gamma + \delta G} \tau K_{t+1}.
\]

(8)

From (8) it can be seen that the effect of the initial levels disappears with time, and for \( t \) large enough
\[
\frac{G_t}{K_t} \approx \frac{1 + \gamma}{\gamma + \delta G} \tau.
\]

(9)

In particular, this result is consistent with the static balanced budget constraint \( G_t = \tau K_t \) if \( \delta G = 1 \). Consequently, when the economy is on the balanced growth path at time 0 it must be the case that
\[
G_0 = \tau K_0 \frac{1 + \gamma}{\gamma + \delta G}.
\]

(10)

Assuming that \( L_t \) is constant and setting \( L_t = 1 \), the level of consumption at time \( t \) if the balanced growth path is achieved at time 0 with capital stock \( K_0 \) is
\[
C_t = (1 + \gamma)^t \left[ A (K_0)^{\alpha} (G_0)^{1-\alpha} - K_0 (\gamma + \delta K + \tau) \right].
\]

(11)

This gives the objective of the household as
\[
\max_{\{\gamma\}} \sum_{t=0}^{\infty} \beta^t \ln \left( (1 + \gamma)^t \left[ A (K_0)^{\alpha} (G_0)^{1-\alpha} - K_0 (\gamma + \delta K + \tau) \right] \right).
\]

(12)

The household takes government actions as given when optimizing so the values of \( \tau \) and \( G_0 \) are treated as fixed in the choice of the balanced growth path. Assuming an interior solution exists, the necessary condition for the choice of \( \gamma \) can be solved to give
\[
1 + \gamma = \beta \left[ A \left( \frac{G_0}{K_0} \right)^{1-\alpha} + 1 - \delta K - \tau \right].
\]

(13)

The expression in (13) determines the rate of growth chosen by the household in response to the tax rate and level of public infrastructure selected by the government. This equation summarizes the behavior of the private sector in the model.
The government chooses the tax rate, $\tau$, to maximize $U$ taking into account the effect of $\tau$ on $\gamma$ determined by (13). The value of $\tau$ and the level of government infrastructure are related by (10). Substitution from (13) into the utility function shows that

$$U = \frac{1}{(1-\beta)^2} \ln (1+\gamma) + \frac{1}{1-\beta} \left[ \ln K_0 + \ln \left( \frac{1-\beta}{\beta} \right) \right].$$

Since only the first term depends upon policy instruments and is itself an increasing function of $\gamma$, there is an equivalence between maximizing $\gamma$ and maximizing $U$. Hence, in this single-country model the government objective is to achieve the fastest possible rate of growth.

The necessary condition for the optimal tax rate is

$$\tau - A (1-\alpha) \left( \frac{1+\gamma}{\gamma + \delta_G} \right)^{1-\alpha} = 0. \quad (15)$$

The simultaneous solution to (13) and (15) determines the tax rate and maximized growth rate. It is helpful to illustrate the nature of this solution for comparison with later results. It can be seen in Figure 1 that (15) intersects (13) at the maximum achievable growth rate given the behavior of the household. It should be noted that this is not the first-best outcome for the economy since the tax on capital is distortionary. The first-best is obtained by maximizing the welfare of the household with respect to $\tau$ and $\gamma$ subject only to the government budget constraint. The first-best values of $\tau$ and $\gamma$ are found by simultaneously solving (15) and

$$1 + \gamma = \beta \left[ A \left( \frac{G_0}{K_0} \right)^{1-\alpha} + 1 - \delta_K - \tau \right] - (1-\beta)(1-\delta_G) \tau. \quad (16)$$

Figure 1 shows that the first-best outcome has a higher tax rate than the decentralized outcome but a lower growth rate. Hence, the decentralized outcome is inefficient with an excessive rate of growth.
This analysis has shown how a study of the optimal tax rate to finance public infrastructure can be undertaken by considering choice over different balanced growth paths. We now develop this technique in the context of a world economy with infrastructural externalities between countries.

3 Infrastructural Spillovers

This section extends the model to a two-country economy in which production benefits from positive spillovers created by global infrastructure. The central observation is that independent optimization by countries does not internalize the externality resulting from the infrastructural spillover. This provides a role for a supra-national body to coordinate the decisions of individual countries so as to secure an increase in the growth rate. We interpret the role of this central body as performing the function of the European Commission: it claims a share of the tax revenue of each country and then redistributes funds among countries. The results are developed by retaining the focus upon the comparison of balanced growth paths.

We assume there are two countries; one is called the “home” country and the other the “foreign” country. At time $t$ the level of output in the home country is given by

$$Y_t = AK_t^\alpha \left(G_t^{1-\rho} \Gamma_t^\rho\right)^{1-\alpha}.$$  \hspace{1cm} (17)

The measure of global infrastructure at time $t$, $\Gamma_t$, is defined as the total public investment in infrastructure, $\Gamma_t = G_t + \overline{G}_t$, where $\overline{G}_t$ is the public investment in infrastructure in the foreign country. The infrastructural externality
is generated by the inclusion of the term $G_t$. The interpretation is that both infrastructure within a country (the term involving $G_t$ in (17)) and the total level of infrastructure (the term involving $\Gamma_t$) are relevant. The production function in the foreign country is defined in the same way, and we assume that the parameters $\alpha$ and $\rho$ are the same for both countries.

We focus on balanced growth paths along which all real variables in both countries grow at the same rate and the tax rates are constant over time. We assume the discount rates and depreciation rates are equal in the two countries.

Extending (11) the level of consumption in the home country at time $t$ if the balanced growth path is reached with capital stock $K_0$ and levels of infrastructure $G_0$ and $\bar{G}_0$ is

$$C_t = (1 + \gamma)^t \left[ A(K_0)^\alpha \left( G_0 \left( 1 + \left( \frac{1 + \tau}{1 + \gamma} \right)^t \frac{\bar{G}_0}{G_0} \right) \right)^{1-\alpha} - K_0 (\gamma + \delta_K + \tau) \right],$$

where $\tau$ is the growth rate of $\bar{G}$. This gives the objective function of the household in the home country as

$$U = \ln (1 + \gamma) \sum_{t=0}^\infty t \beta^t + \ln K_0 \sum_{t=0}^\infty \beta^t$$

$$+ \sum_{t=0}^\infty \beta^t \ln \left[ A \left( \frac{G_0}{K_0} \right)^{1-\alpha} \left( 1 + \left( \frac{1 + \tau}{1 + \gamma} \right)^t \frac{\bar{G}_0}{G_0} \right)^{\rho(1-\alpha)} - \delta_K - \tau - \gamma \right]$$

$$= \frac{\beta}{(1 - \beta)^2} \ln (1 + \gamma) + \frac{1}{1 - \beta} \ln K_0 + \sum_{t=0}^\infty \beta^t \ln \phi_t (\gamma; \tau),$$

where

$$\phi_t (\gamma; \tau) \equiv A \left( \frac{G_0}{K_0} \right)^{1-\alpha} \left( 1 + \left( \frac{1 + \tau}{1 + \gamma} \right)^t \frac{\bar{G}_0}{G_0} \right)^{\rho(1-\alpha)} - \delta_K - \tau - \gamma. \quad (20)$$

The household takes the values of $K_0$, $G_0$, $\bar{G}_0$, and $\tau$ as given, so the first-order condition for the choice of growth rate by the household is

$$\frac{\beta}{(1 - \beta)^2} \frac{1}{1 + \gamma} + \sum_{t=0}^\infty \frac{\beta^t}{\phi_t (\gamma; \tau)} \frac{\partial \phi_t (\gamma; \tau)}{\partial \gamma} = 0. \quad (21)$$

A similar construction for the optimization for the foreign country household results in the first-order condition

$$\frac{\beta}{(1 - \beta)^2} \frac{1}{1 + \tau} + \sum_{t=0}^\infty \frac{\beta^t}{\phi_t (\tau; \tau)} \frac{\partial \phi_t (\tau; \tau)}{\partial \tau} = 0. \quad (22)$$

This pair of equations determine simultaneously the chosen growth rates conditioned on the tax rates (and the other variables taken as parametric by the households). We denote these functions $\gamma = \gamma(\tau, \tau)$ and $\tau = \tau(\tau, \tau).$
When the governments do not coordinate their choices each maximizes the welfare of their representative household taking the policy choice of the other government as given. Hence, the home country chooses \( \tau \) to maximize \( U \), taking \( K_0, \gamma, \) and \( G_0 \) as given, but recognizing that along the balanced growth path the level of public good and the level of private capital are related by (9), and the growth rates are determined by \( \gamma = \gamma (\cdot) \) and \( \overline{\gamma} = \overline{\gamma} (\cdot) \). The optimization facing the home government is

\[
\max_{\tau} U = \frac{\beta}{1 - \beta} \ln (1 + \gamma (\tau, \overline{\tau})) + \sum_{t=0}^{\infty} \beta^t \ln (K_0 \Phi_t (\gamma (\tau, \overline{\tau}), \overline{\tau} (\tau, \tau); \tau)), \quad (23)
\]

where

\[
\Phi_t (\gamma, \overline{\gamma}, \tau, \overline{\tau}) = A \left( \frac{1 + \gamma}{\gamma + \delta_G} \right)^{1-\alpha} \left( 1 + \left( \frac{1 + \overline{\gamma}}{1 + \gamma} \right)^{t} \frac{\overline{\tau} \frac{1 + \gamma}{\gamma + \delta_G} K_0}{\tau \frac{1 + \gamma}{\gamma + \delta_G} K_0} \right)^{1-\alpha} - \delta_K - \gamma - \overline{\gamma}.
\]

(24)

It should be observed that the equivalence between the maximization of \( U \) and the maximization of \( \gamma \) that held in the single-country case no longer applies since the objective function in (23) is not monotonically increasing in \( \gamma \). The non-monotonicity arises from the fact that the benefit derived through the infrastructure externality is reduced as \( \gamma \) increases relative to \( \overline{\gamma} \). This point is important for interpreting the simulation results in the following section. Differentiating with respect to \( \tau \) and utilizing (21) produces the necessary condition for the choice of the home government

\[
\sum_{t=0}^{\infty} \beta^t \frac{1}{\Phi_t} \left( \frac{\partial \Phi_t}{\partial \gamma} - \frac{\partial \Phi_t}{\partial \tau} \right) \frac{\partial \gamma}{\partial \tau} + \frac{\partial \Phi_t}{\partial \overline{\gamma}} \frac{\partial \gamma}{\partial \tau} + \frac{\partial \Phi_t}{\partial \overline{\tau}} \frac{\partial \tau}{\partial \tau} = 0. \quad (25)
\]

The equivalent condition for the foreign government is

\[
\sum_{t=0}^{\infty} \beta^t \frac{1}{\Phi_t} \left( \frac{\partial \Phi_t}{\partial \overline{\gamma}} - \frac{\partial \Phi_t}{\partial \overline{\tau}} \right) \frac{\partial \overline{\gamma}}{\partial \tau} + \frac{\partial \Phi_t}{\partial \gamma} \frac{\partial \gamma}{\partial \tau} + \frac{\partial \Phi_t}{\partial \overline{\gamma}} \frac{\partial \overline{\gamma}}{\partial \tau} = 0. \quad (26)
\]

To obtain the expressions for the terms \( \frac{\partial \gamma_t}{\partial \tau}, \frac{\partial \overline{\tau}}{\partial \tau} \) etc. one needs to take the total differential of conditions (21) and (22). We provide a sketch of the details of this construction in the appendix. The general solution is used in the simulations but is too long to report here. The tax rates and growth rates without coordination are found as the simultaneous solution to (21), (22), (25) and (26).

When the two government coordinate their choice of policies the tax rates are chosen simultaneously to maximize the sum of the welfare levels of their representative households. The coordinated optimization still takes into account the fact the households choose the rates of growth. The conditions determining the choice of growth rate for the home country and the foreign country are the same as in the equilibrium without coordination, so \( \gamma \) and \( \overline{\gamma} \) remain determined by the functions \( \gamma (\tau, \overline{\tau}) \) and \( \overline{\gamma} (\tau, \tau) \).
The objective function for the coordinated choice of policy is defined as

\[ U(\gamma, \tau) + U(\gamma; \tau) = \frac{\beta}{(1 - \beta)^2} \ln (1 + \gamma) + \sum_{i=0}^{\infty} \beta^i \ln (K_0 \Phi_i (\gamma; \tau)) \]

\[ + \frac{\beta}{(1 - \beta)^2} \ln (1 + \gamma) + \sum_{i=0}^{\infty} \beta^i \ln (K_0 \Phi_i (\tau; \gamma)) . \]  

(27)

Using the envelope theorem to take account of household optimization, the necessary condition for \( \tau \) is

\[ \sum_{i=0}^{\infty} \beta^i \frac{1}{\Phi_i} \left[ \left( \frac{\partial \Phi_i}{\partial \gamma} - \frac{\partial \phi_i}{\partial \gamma} \right) \frac{\partial \gamma}{\partial \tau} + \frac{\partial \Phi_i}{\partial \tau} \frac{\partial \tau}{\partial \tau} + \frac{\partial \Phi_i}{\partial \tau} \right] 
\]

\[ + \sum_{i=0}^{\infty} \beta^i \frac{1}{\Phi_i} \left[ \left( \frac{\partial \Phi_i}{\partial \tau} - \frac{\partial \phi_i}{\partial \tau} \right) \frac{\partial \tau}{\partial \tau} + \frac{\partial \Phi_i}{\partial \gamma} \frac{\partial \gamma}{\partial \tau} + \frac{\partial \Phi_i}{\partial \tau} \right] \] = 0.  

(28)

An equivalent condition holds for \( \gamma \). The tax rates and growth rates are determined by the simultaneous solution of the conditions for household choice and the necessary conditions for government optimization.

A comparison of (28) with (25) illustrates the effect of the externalities in the model. With uncoordinated optimization each government ignores the effect that its choice of tax rate has upon welfare in the other country. These effects operate through the interaction of the households and directly through the infrastructural spillover. These effects are internalized in the coordinated case so the two outcomes will differ. The simulation of the next section explores the manner in which they differ.

Now consider the possibility of intervention of a supranational central body with redistribution of tax revenues that are used as public input in production. The interaction between the central body and the national government is modelled as the following multi-stage game. At the first stage the central body announces what share of the tax revenues will be collected from each national government for the centralized fund. At the second stage the governments choose optimal tax rates. At the third stage the central body announces how the centralized fund will be divided between the two countries. Finally, the investments are made and the production takes place. There is no coordination between the two national governments at any stage.

The central government takes a fraction \( \theta \) of the home government revenue and a fraction \( \bar{\theta} \) of the foreign government revenue and after the tax (public investment) decisions are made in each country it returns fraction \( \mu \) of the total collected revenues to the home country and fraction \( 1 - \mu \) to the foreign country.

With this system the law of motion for public capital in the home country is

\[ G_{t+1} = (1 - \delta_G) G_t + (1 - \theta) \tau_{t+1} K_{t+1} + \mu \Omega_{t+1}, \]  

where \( \Omega_{t+1} = \theta \tau_{t+1} K_{t+1} + \bar{\theta} \tau_{t+1} K_{t+1} \). Thus, if the balanced growth path is achieved from \( t = 0 \)

\[ G_0 = \frac{1 + \gamma}{\gamma + \delta_G} \left[ (1 - \theta + \theta \mu) \tau K_0 + \bar{\theta} \mu \tau K_0 \right] . \]  

(30)
Hence, we can write
\[ \frac{G_0}{K_0} = \frac{1 + \gamma}{\gamma + \delta_G}, \] (31)
where
\[ \tilde{\tau} = (1 - \theta + \theta \mu) \tau + \theta \mu \frac{K_0}{K_0}. \] (32)

Similarly,
\[ \frac{G_0}{K_0} = \frac{1 + \gamma}{\gamma + \delta_G}, \] (33)
with
\[ \hat{\tau} = (1 - \bar{\theta} + \bar{\theta} (1 - \mu)) \tau + \theta (1 - \mu) \frac{K_0}{K_0}. \] (34)

The optimization problems for the households in the home and foreign countries do not change. The optimization problem of the home country government becomes
\[ \max_{\{r\}} U = \frac{\beta}{(1 - \beta)^2} \ln (1 + \gamma (\tau, \bar{\tau})) + \sum_{t=0}^{\infty} \beta^t \ln \left( K_0 \hat{\Phi}_t (\gamma (\tau, \bar{\tau}), \bar{\tau} (\tau, \bar{\tau}); \tau, \bar{\tau}) \right), \] (35)
where
\[ \hat{\Phi}_t (\gamma, \bar{\tau}; \tau, \bar{\tau}) = A \left( \frac{1 + \gamma}{\gamma + \delta_G} \right)^{1-\alpha} \left( 1 + \frac{1 + \gamma}{1 + \gamma} \right)^t \left( \frac{\tilde{\tau}}{\gamma + \delta_G} \frac{1 + \gamma}{\tilde{\tau} + \delta_G} \right)^{\rho(1-\alpha)} - \delta K - \tau - \gamma. \] (36)

This leads to the necessary condition
\[ \sum_{t=0}^{\infty} \beta^t \frac{1}{\hat{\Phi}_t} \left[ \left( \frac{\partial \Phi_t}{\partial \gamma} - \frac{\partial \phi_t}{\partial \gamma} \right) \frac{\partial \gamma}{\partial \tau} + \frac{\partial \hat{\phi}_t}{\partial \tau} \frac{\partial \gamma}{\partial \tau} + \frac{\partial \hat{\phi}_t}{\partial \tau} \frac{\partial \bar{\tau}}{\partial \tau} + \frac{\partial \hat{\phi}_t}{\partial \bar{\tau}} \frac{\partial \bar{\tau}}{\partial \tau} \right] = 0. \] (37)

An equivalent condition holds for the foreign country.

We now employ a simulation analysis to compare the solutions that emerge for the three equilibrium concepts. The aim of the simulation is to contrast the uncoordinated and the coordinated equilibria, and to investigate what can be achieved by intervention.

4 Simulation

In interpreting the results it should be observed that there are three potential sources of inefficiency. First, the household in each country chooses a growth rate given the tax policy. Both households ignore any externalities arising from their choices. Second, the governments can only indirectly influence the choices of the households through the choice of a tax rate. The tax on capital distorts the intertemporal consumption choice. Third, when the governments do not coordinate, they ignore the positive externality from infrastructural spillovers.
Coordinating the tax choices of the governments addresses only the third source of inefficiency, so coordination alone will not achieve a first-best outcome. This opens the possibility that intervention can improve upon coordination since it can go part way to resolving the first two sources of inefficiency.

In Figure 2 a symmetric equilibrium without coordination between countries is illustrated for the following values of the model parameters: $\beta = 0.9$, $\rho = 0.5$, $\alpha = 0.6$, $\delta_K = \delta_G = 0.2$, $A = \bar{A} = 0.6$, $K_0 = \bar{K}_0 = 2$. The equilibrium occurs at the intersection of the two curves determined by (21) and (25). At the equilibrium $\tau = 0.186$, $\gamma = 0.055$, and $U = -5.41$. Unlike the single-country case the chosen tax rate does not maximize the growth rate because the spillover is not internalized. We now proceed to show that this outcome does not maximize welfare either. This is a consequence of the fact that in this equilibrium all three sources of inefficiency are present.

Figure 3 displays the equilibrium with coordinated policy choice for the same values of the model parameters. It can be seen that the both the tax rate and the growth rate are higher than in the case without coordination. The values in this case are $\tau = 0.299$, $\gamma = 0.057$, and $U = -4.550$. The coordination ensures that the infrastructural externality between the governments is internalized so the third source of inefficiency is removed. The internalization of the externality provides the incentive to set a higher tax rate. The growth rate and the welfare level are increased by coordination, but it does not achieve either the maximum growth rate that is possible given the choice behavior of the consumer or the maximum level of welfare.
Figure 3: Equilibrium with coordination

Table 1 details the effect of intervention for a range of values of $\theta$. Since the countries are symmetric the optimal value of $\mu = 0.5$. The maximum growth rate is achieved by $\theta = -0.6$, but this value does not coincide with the value of $\theta = -1.06$ that delivers maximum welfare. When $\theta = -1.06$ the outcome is given by $\tau = 0.307, \gamma = 0.057$, and $U = -4.547$. Coordination only addresses the third source of inefficiency, so intervention is able to achieve a higher level of welfare. Observe, though, that the value of $\theta$ is negative which represents the central body matching the tax revenues of the individual countries. This is not surprising. We have shown that the equilibrium tax rates are too low in the absence of coordination since the infrastructural spillover causes a positive externality. Intervention by the central body is required to raise the tax rates and this is achieved by a process of revenue-matching. The central body finances this revenue matching by claiming back revenues (through $\mu$) once the tax rates have been determined.

<table>
<thead>
<tr>
<th>$\theta$</th>
<th>-1.5</th>
<th>-1.3</th>
<th>-1.1</th>
<th>-0.9</th>
<th>-0.7</th>
<th>-0.5</th>
<th>-0.3</th>
<th>-0.1</th>
<th>0</th>
<th>0.1</th>
</tr>
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<tbody>
<tr>
<td>$\tau$</td>
<td>0.368</td>
<td>0.340</td>
<td>0.313</td>
<td>0.287</td>
<td>0.262</td>
<td>0.239</td>
<td>0.217</td>
<td>0.196</td>
<td>0.186</td>
<td>0.176</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>0.052</td>
<td>0.054</td>
<td>0.056</td>
<td>0.058</td>
<td>0.058</td>
<td>0.058</td>
<td>0.058</td>
<td>0.056</td>
<td>0.055</td>
<td>0.054</td>
</tr>
<tr>
<td>$U$</td>
<td>-4.68</td>
<td>-4.59</td>
<td>-4.55</td>
<td>-4.57</td>
<td>-4.64</td>
<td>-4.78</td>
<td>-4.98</td>
<td>-5.25</td>
<td>-5.41</td>
<td>-5.60</td>
</tr>
</tbody>
</table>

Table 1: Effect of Intervention

None of these three equilibria achieve the first-best outcome. The first-best is obtained by choosing the tax rates and growth rates simultaneously to maximize aggregate welfare. The outcome is a tax rate $\tau = 0.313$, a growth rate $\gamma = 0.0361$, and a utility level of $U = -4.428$. Table 1 reveals clearly why none of the other equilibria achieve this outcome. At a value of $\theta = -1.1$ the first-best tax rate is achieved but the households choose excessively high growth rates. Because of the choice process of the households the first-best cannot be attained.
5 Conclusions

We have analyzed an economy in which public sector expenditure is productive and there are spillovers of the benefits of public infrastructure between countries. This mechanism make it possible for an increase in taxation in one country to raise the growth rate in all countries. This effect will not be apparent in cross-country comparisons taken at one point, which may help explain why empirical data shows only a weak relationship between taxation and economic growth.

The policy implications of our analysis are that although public expenditure can assist growth there is no guarantee that the optimal level of growth will be achieved. The design of public expenditure has to take into account the infrastructural spillovers between countries. If the choices of individual countries are not coordinated then the outcome will be inefficient and growth will not be welfare-maximizing. A coordinating body, such as the European Commission, has a role to play in attaining a more efficient level of expenditure on public infrastructure. This role involves supporting the expenditure decisions of individual countries to raise the overall level of expenditure. However, even intervention will not achieve the first-best since households have an incentive to choose an excessively high rate of growth.

References


**A Appendix**

The necessary condition for the choice of the home household can be written

\[
\frac{\beta}{(1 - \beta)^2} \frac{1}{1 + \gamma} = \sum_{t=0}^{\infty} t \beta^t \rho (1 - \alpha) \frac{1}{A \left( \frac{G_0}{K_0} \right)} \left( 1 + \left( \frac{1 + \gamma}{1 + \gamma} \right)^t \frac{G_0}{G_0} \right) \left( \frac{1}{1 + \gamma} \right)^t \frac{K_0}{K_0} \frac{1}{1 + \gamma} + 1
\]

and that for the foreign household as

\[
\frac{\beta}{(1 - \beta)^2} \frac{1}{1 + \gamma} = \sum_{t=0}^{\infty} t \beta^t \rho (1 - \alpha) \frac{1}{A \left( \frac{G_0}{K_0} \right)} \left( 1 + \left( \frac{1 + \gamma}{1 + \gamma} \right)^t \frac{G_0}{G_0} \right) \left( \frac{1}{1 + \gamma} \right)^t \frac{K_0}{K_0} \frac{1}{1 + \gamma} + 1
\]

To calculate the derivatives of \( \gamma (\tau, \pi) \) and \( \pi (\tau, \gamma) \) we need to take the total differential of the system (38)-(39) with respect to \( \tau, \pi, \gamma \) and \( \pi \), assuming that \( G_0, G_0, K_0, \) and \( K_0 \) are taken as given. We then solve the resulting pair of equations for the partial derivatives. When the countries are assumed to be symmetric we can then evaluate the solution under the restriction that \( K_0 = K_0 \) and \( \gamma = \pi \). This is the process used to generate the expressions used in the simulations.