TAX DESIGN IN THE PRESENCE OF IMPERFECT COMPETITION

An Example

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This paper studies the effects upon price, output and entry in an oligopolistic market induced by tax policy elsewhere in the economy. The results derived are applied to a model of copying to find conditions for which subsidisation of the manufacturers of originals is welfare improving. The critical factor is found to be the distribution of consumers' tastes.

1. Introduction

This paper analyses a model in which a tax placed on the output of a competitive industry, producing with constant returns to scale, has repercussions in a related imperfectly competitive market. The connection between the two industries is via consumers who view the outputs as perfect substitutes at a taste-dependent rate of transformation. Sufficient conditions are established for a tax increase to induce a higher price in the oligopolistic market; if the oligopoly permits free-entry it is shown that price increases and exit will not occur simultaneously. Furthermore, a range of parameter values exists for which entry accompanies price cuts. Applied to optimal tax design, these results indicate when it will be welfare improving to subsidise profit-making oligopolists, and monopolists, whilst taxing competitive firms.

Some work has already been completed on taxes in imperfectly competitive models. Seade (1985) considers the effect of taxes upon prices and profits whilst Stern (1987) analyses dual-pricing schemes for a variety of market structures. This paper has two elements that distinguish it from the above. Firstly, I study behaviour in one market induced by policy intervention in another and the implications of this for optimal taxes. Similar effects are also studied by Bulow et al. (1985) but their results are based upon the existence of multi-market firms. I assume each firm operates in a single market.

The second element is the demand structure imposed upon the model. This

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is derived by assuming that all consumers' indifference curves between the outputs of the two industries are linear but allowing the gradient to vary across the population. The aggregate demand function obtained is dependent upon prices and the distribution of tastes. The approach is similar to that employed in the theory of modal choice by Domencich and McFadden (1975).

The specific example which motivated the paper is the copying of records, using cassette tapes, about which there is in Britain a current policy debate. The debate is centred on whether a levy should be placed on blank cassette tapes and the proceeds used to subsidise record production. Two Government discussion documents [Department of Trade (1981, 1985)] have set out the background arguments in detail; I am concerned with the economic effects of such a levy and of assessing its validity on these grounds.

Production of records is characterised by a large initial recording cost and a small marginal cost. The industry is also clearly not perfectly competitive. For these reasons the paper concentrates upon oligopolistic equilibria with alternative entry assumptions.

Section 2 derives the equations describing the induced price and output changes and section 3 analyses these for a simple model of copying. In section 4 these results are combined with Seade's to study welfare-improving and optimal taxes. Section 5 presents some numerical simulations and section 6 contains conclusions.

2. Derivation of induced effects

This section establishes the results that will be employed in section 3. It is assumed throughout that the oligopolistic firms are identical, behave as Cournot competitors and produce with fixed cost $C$ and constant marginal cost $c$. For the present there are two goods in the model: $X$, the oligopolist’s output, and $Y$, produced at constant returns to scale. Their prices are $p_X$ and $p_Y$ respectively. The details of the derivations are omitted, since a detailed description of the techniques employed is found in Dixit (1986), but one point is worth noting: as the aggregate demand function derived in section 3 has no analytical inverse all results are given in terms of derivatives of direct demand, derivatives of inverse demand being eliminated by substitution.

Writing the industry demand function as

$$X = X(p_X, p_Y),$$

its inverse, assuming a symmetric equilibrium, is given by

$$p_X = p_X(X, p_Y) = p_X(nx, p_Y),$$

$$p_Y = p_Y(X, p_Y) = p_Y(ny, p_Y).$$
where \( x \) is the output of each of the \( n \) firms. Each firm chooses their output \( x \) to maximise

\[
\Pi_i = (p_x - c) \cdot x - C. \tag{3}
\]

The necessary conditions for a maximum of (3), with Cournot conjectures, are

\[
(p_x - c) + x \frac{\partial p_x}{\partial X} = 0. \tag{4}
\]

and

\[
2 \frac{\partial p_x}{\partial X} + x \frac{\partial^2 p_x}{\partial X^2} < 0. \tag{5}
\]

Employing (4) and the relationships between the partial derivatives of direct and inverse demand, (5) can also be written:

\[
(\frac{\partial p_x}{\partial X})^2 \cdot (2 \frac{\partial X}{\partial p_x} + (p_x - c) \cdot \frac{\partial^2 X}{\partial p_x^2}) < 0. \tag{6}
\]

Now considering differential changes in \( p_x \) and \( p_y \), whilst maintaining each firm at a profit maximum, the total derivative of (4) gives:

\[
\frac{dp_x}{dp_y} = -\frac{\frac{1}{n} \frac{\partial X}{\partial p_x} + (p_x - c) \cdot \frac{\partial^2 X}{\partial p_x \partial p_y}}{(n+1)n^{-1} \frac{\partial X}{\partial p_x} + (p_x - c) \cdot \frac{\partial^2 X}{\partial p_x^2}}. \tag{7}
\]

From (6), when \( n = 1 \) the denominator of (7) is negative. When \( n \neq 1 \) Seade (1980) has shown that negativity of the denominator is a sufficient condition for stability. Although it cannot be ensured that this condition will be satisfied, its negativity will be assumed below. The numerator of this expression will be studied in detail for a specific demand structure in section 3; in general it is evident that (7) may be positive or negative. Hence a tax increase on a competitive market may induce a price reduction in the oligopolistic market. The converse is also possible.

If the oligopoly admits free entry, equilibrium is described by the pair of equations

\[
p_x - c + x \cdot \frac{\partial p_x}{\partial X} = 0, \tag{8}
\]

\[
(p_x - c) \cdot x - C = 0. \tag{9}
\]

Eq. (8) describes profit maximisation of individual firms and (9) the zero profit entry constraint, where \( n \) is treated as a continuous variable. The solution to (8) and (9) is an \( x, n \) pair which then determine \( p_x \). By
differentiating (8) and (9), the effect of a differential increase in \( p_y \) upon the equilibrium can be expressed as:

\[
\frac{dp_x}{dp_y} = \frac{-(p_x-c) \cdot \partial^2 X/\partial p_y \partial p_x}{2 \partial X/\partial p_x + (p_x-c) \cdot \partial^2 X/\partial p_x^2},
\]

(10)

\[
\frac{dx}{dp_y} = \frac{-(p_x-c) \cdot \partial X/\partial p_x \cdot \partial^2 X/\partial p_y \partial p_x}{2 \partial X/\partial p_x + (p_x-c) \cdot \partial^2 X/\partial p_x^2},
\]

(11)

\[
\frac{dn}{dp_y} = \frac{1}{x} \cdot \frac{\partial X}{\partial p_y} \cdot \frac{(n-1) \cdot \partial^2 X/\partial p_y \partial p_x}{2 \partial X/\partial p_x + (p_x-c) \cdot \partial^2 X/\partial p_x^2}.
\]

(12)

Using (6) it can be seen that the denominator in (10) and (11) is the second-order condition multiplied by \((\partial X/\partial p_x)^2\) and hence is assumed negative. With \( \partial X/\partial p_x < 0 \) it is apparent that price and output will move in opposite directions, note that this is the individual firm's output, the direction of the price change being given by the sign of the second cross-derivative.

In eq. (12) the second term has the same sign as (10) but the first term may reinforce or diminish this effect depending upon the sign of the first cross-derivative.

These results are now analysed for a model of copying and then employed to analyse welfare-improving and optimal taxes.

3. An application to a model of copying

The model of copying is built upon the assumptions that consumers vary in their appreciation of the superior sound reproduction of records in comparison with cassette tapes and that each consumer's marginal rate of substitution between records and tapes is constant. It is also assumed that records are an homogenous product; to do otherwise would introduce the problem of locational choice but add little to the results. The constancy of the marginal rate of substitution implies linear indifference curves so that at given prices a consumer will never buy both records and tapes; a corner solution will always prevail. Describing the population by a distribution function of tastes allows discontinuous individual behaviour to become continuous as an aggregate.

Introducing a numeraire \( Z \) which remains untaxed throughout, treating \( X \) and \( Y \) as records and tapes respectively, the utility of a consumer with taste parameter \( \mu \) is

\[
U = u(\mu x + y) + z, \quad u' > 0, \quad u'' < 0,
\]

(13)
where lower-case letters represent individual choices. The population is described by a distribution function $F(\mu)$ with density $f(\mu)$. $f(\mu)$ has support on $(0, \infty)$.

If each consumer is endowed with income $M$ he seeks to maximise utility subject to

$$M = p_x x + p_y y + z.$$  

(14)

For consumers with $\mu > p_x/p_y$, only $x$ is consumed, individual demand is hence

$$x = \mu^{-1} g(p_x/\mu),$$  

(15)

where $g()$ is the inverse of marginal utility and $g'(u)^{-1} < 0$.

Aggregate demand for $x$ is derived by integrating from $\mu = p_x/p_y = \mu^*$ to $\mu = \infty$, denoting this by $X$

$$X = \int_{\mu^*}^{\infty} \mu^{-1} g(p_x/\mu) \cdot f(\mu) \, d\mu.$$  

(16)

It is now possible to evaluate the equations derived in section 2.

Using (16) the numerator of (7) is

$$- n^{-1} g(p_x) f(\mu^*) p_y^{-1} - (p_x - c) g(p_y) f'(\mu^*) p_y^{-2}. $$  

(17)

If the denominator of (7) is negative, which is always the case for monopoly and is assumed to be true for oligopoly, $dp_x/dp_y > 0$ when

$$f(\mu^*) / f'(\mu^*) > - n(p_x - c)/p_y.$$  

(18)

Two observations are immediate: if $\mu$ is distributed uniformly, or if the distribution is such that $f' > 0$ for all $\mu$, then $dp_x/dp_y$ is always positive. Uniformity of distributions is a common simplifying assumption; in this model it would have an important consequence.

For the model with entry the denominator of (10) is negative so that the sign of $dp_x/dp_y$ is the same as that of $\partial^2 X / \partial p_x \partial p_y$. From (15)

$$\partial^2 X / \partial p_x \partial p_y = g(p_y) f'(\mu^*) p_y^{-2}.$$  

(19)

As $g(p_y) > 0$, the gradient of the density function at $\mu^*$ determines the direction of the induced effect: if the gradient is positive, an increase in $p_y$ will result in $p_x$ increasing. In contrast to oligopoly without entry, the result is
also unambiguous when \( f' \) is negative. Recalling (10), the output of each firm will always move in the opposite direction to price.

Regarding entry, the first term in (12) is positive so that \( f' > 0 \) is sufficient to guarantee entry. Consequently if the price rises entry occurs. Furthermore there exists a range of values of \( f' \) for which the price will fall and entry will occur. A sufficiently negative \( f' \) will ensure exit. However it is not possible to have price increases and exit occurring simultaneously.

Hence for
\[
0 < f' > \frac{-p_x^2 \frac{\partial X}{\partial p_x} \cdot (2 \frac{\partial X}{\partial p} + (p_x - c) \cdot \frac{\partial^2 X}{\partial p_x^2})}{x(n-1)g(p_x)},
\]  

(20)
a tax on \( Y \) will induce entry, reduce the price of \( X \) and increase the output of each firm. These are the effects that the proponents of the blank cassette levy would like to see occur. That they occur only for a range of values of \( f' < 0 \) suggests that they should not be seen as the inevitable consequences of the levy.

4. Welfare improving and optimal taxes

In the presence of imperfect competition and non-constant returns to scale the calculation of optimal tax rates involves two difficulties that are not present in the competitive constant returns model. Firstly, after-tax prices are not linear functions of the tax rates, and secondly, prices are functionally related across industries. The nature of the functional relations has been the subject matter of section 2 above and Seade (1985) has provided results on the relation of pre- to post-tax prices. These two strands are now combined to analyse the direction of welfare improving and optimal taxes for the model of section 3.

The policy proposed in the Department of Trade Green Papers is a levy on cassette tapes, the proceeds being used to subsidise the production of records. This is modelled by considering only tax systems \( t_x, t_y \) that will result in a balanced budget. \( Z \) remains untaxed throughout. The aim is to investigate whether conditions exist for which the policy proposal can be justified. For the model without entry the policy is justified if the tax changes increase welfare; if entry is possible they must also increase the number of firms. Recognising this distinction the two are treated separately.

To derive the results the indirect utility function is required. For consumers with \( \mu < \mu^* \) this can be calculated as
\[
V(p_y, 1, M) = u(g(p_y)) + M - p_y g(p_y),
\]
and if $\mu > \mu^*$ indirect utility is

$$V(p_x, 1, M; \mu) = u(g(p_x/\mu)) + M - p_x \mu^{-1} g(p_x/\mu).$$

Note that this specification assumes that profits accrue to an actor outside the model.

As $Y$ is produced with constant returns to scale the post-tax price $q_y = p_y + t_y$. In line with the analysis above, the post-tax price of $X$ is written

$$q_x = h(q_y, t_x),$$

with partial derivatives $\partial q_x/\partial q_y = h_1$ and $\partial q_x/\partial t_x = h_2$.

For the model without entry, welfare-improving tax changes, from an initial state with $t_x = t_y = 0$, are the solution to:

**WII.** Find $dt_x, dt_y$ such that $dSW > 0$, and $dR = 0$, where

$$SW = \int_0^{\mu^*} V(q_y, 1, M) f(\mu) d\mu + \int_{\mu^*}^{\infty} V(q_x, 1, M; \mu) f(\mu) d\mu,$$

$$R = X \cdot t_x + Y \cdot t_y.$$

Using the definitions of indirect utility,

$$dSW = - Y \cdot dt_y - h_2 X \cdot dt_x - h_1 X \cdot dt_y,$$

and after substituting from the revenue constraint,

$$dt_x > 0, \quad dt_y < 0 \quad \text{if} \quad (1 - h_2)X + h_1 X^2 Y^{-1} > 0, \quad (22)$$

$$dt_x < 0, \quad dt_y > 0 \quad \text{if} \quad (1 - h_2)X + h_1 X^2 Y^{-1} < 0. \quad (23)$$

It is the second of these that represents the suggested policy. For oligopoly Seade (1985) has shown that

$$h_2 = \frac{n \partial X/\partial p_x}{(n + 1) \partial X/\partial p_x + (p_x - c)n \partial^2 X/\partial p_x^2},$$

which is the degree of shifting of the tax and may be greater than 1, whilst $h_1$ is given by (7). From (23), $dt_x < 0$ if

$$h_1 < (h_2 - 1) XY^{-1}. \quad (25)$$

Consequently, for the proposed policy to be welfare improving when record production is by a fixed number of oligopolists, $h_1$ must be below the upper
bound described by (25). This in turn implies an upper bound on $f'(\mu^*)$ and $f(\mu^*)$. Note also that the possibility of overshifting implies that the right hand side may be greater than zero.

Optimal taxes will be the solution to:

$$\max_{t_x, t_y} SW \quad \text{s.t.} \quad R = 0.$$  

The solution for $t_x$ is

$$t_x = \frac{X[(1 - h_2)Y + h_1X]}{h_2[\partial X/\partial q_x - (X/Y) \cdot \partial Y/\partial q_y] - Y(\partial X/\partial q_x - (X/Y) \cdot \partial X/\partial q_x)}.$$  \hspace{1cm} (26)

For the demand structure of this model the denominator of (26) is positive and the numerator is a simple translation of the term given in (22) and (23). Hence the sign of the optimal tax rates is always given by the direction of welfare-improving changes and the discussion following (22) and (23) is relevant here.

To derive welfare-improving taxes for the model with free-entry, account must be taken of the effect of the tax scheme upon the number of firms in the industry. Following the arguments in Department of Trade (1981, 1985), it is taken as given that policy should attempt to increase the number of firms. If entry is to be induced, and welfare increased, the tax changes must be the solution to:

$$W12. \text{ Find } dt_x, dt_y \text{ such that } dSW > 0, dn > 0, dR = 0.$$  

The possible solutions to W12 are characterised by the following theorem:

**Theorem 1.** Problem W12 has for solution either a pair $dt_x < 0, dt_y > 0$ or a pair $dt_x = dt_y = 0$. $dt_x > 0, dt_y < 0$ will never be a solution.

**Proof.** As above $dSW = - Y dt_y - h_2X dt_x - h_1X dt$ and, additionally, $dn = \partial n/\partial t_x \cdot dt_x + \partial n/\partial t_y \cdot dt_y$. Noting that

$$h_2 = \frac{2 \partial X/\partial q_x}{2 \partial X/\partial q_x + (q_x - t_x - c) \partial^2 X/\partial q^2_x} > 0$$

and

$$\partial n = \frac{1}{x} \cdot \frac{\partial X}{\partial q_x} \cdot \frac{(n - 1) \cdot \partial^2 X/\partial q^2_x}{2 \partial X/\partial q_x + (q_x - t_x - c) \partial^2 X/\partial q^2_x},$$

substituting from the budget constraint and evaluating at $t_x = 0, dt_x > 0, dt_y < 0$ if
where \( a = 2 \frac{\partial X}{\partial p_x} + (p_x - c) \frac{\partial^2 X}{\partial p_x^2} < 0 \) and \( d_{tx} < 0, d_{ty} > 0 \) if both inequalities are reversed. It is evident that the two left-hand sides need not have the same sign; when the signs differ there will be no welfare-improving tax changes.

After simplifying (27) and (28), \( d_{tx} > 0, d_{ty} < 0 \) will be a solution when

\[
Y \cdot \frac{\partial^2 X}{\partial p_x^2} - X \cdot \frac{\partial^2 X}{\partial p_x \partial p_y} < 0
\]

and

\[
\left[\frac{a}{x(n-1)}\right] \left[ Y \cdot \frac{\partial X}{\partial p_x} - X \cdot \frac{\partial X}{\partial p_y} \right] \]

\[
- \left[ Y \cdot \frac{\partial^2 X}{\partial p_x^2} - X \cdot \frac{\partial^2 X}{\partial p_x \partial p_y} \right] < 0,
\]

which can be written together as

\[
\left[\frac{a}{x(n-1)}\right] \left[ Y \cdot \frac{\partial X}{\partial p_x} - X \cdot \frac{\partial X}{\partial p_y} \right] \]

\[
< \left[ Y \cdot \frac{\partial^2 X}{\partial p_x^2} - X \cdot \frac{\partial^2 X}{\partial p_x \partial p_y} \right] < 0.
\]

However, as \( a < 0, \frac{\partial X}{\partial p_x} < 0 \) and \( \frac{\partial X}{\partial p_y} > 0 \) this inequality cannot be satisfied. Hence \( d_{tx} > 0, d_{ty} < 0 \) will never be a solution.

Alternatively, \( d_{tx} < 0, d_{ty} > 0 \) is the solution when

\[
\left[\frac{a}{x(n-1)}\right] \left[ Y \cdot \frac{\partial X}{\partial p_x} - X \cdot \frac{\partial X}{\partial p_y} \right] \]

\[
> \left[ Y \cdot \frac{\partial^2 X}{\partial p_x^2} - X \cdot \frac{\partial^2 X}{\partial p_x \partial p_y} \right] > 0.
\]

Note from (19) that the second part of this inequality is always satisfied if \( \frac{\partial^2 X}{\partial p_x^2} > 0 \) and \( f'(\mu^*) < 0. \)

The implication of this theorem is that taxation of record production, with subsidisation of cassettes, will never raise both welfare and the number of active firms simultaneously and therefore cannot be a solution to W12. However the choice remains between subsidisation of records and the initial zero-tax state; in the final event this choice must be based on empirical analysis of (29).
5. Examples

To illustrate the reasoning above I will now state and discuss the results of a number of numerical simulations. Due to computational factors these are restricted to the case of monopoly production of $X$ with constant marginal cost.

The density functions are chosen to represent three of the basic possibilities. For the first the density function decreases with $\mu$, the second increases with $\mu$, while the third has a uniform distribution. As noted above, only for the first is it possible to have $h_2 < 0$.

For all three examples $p_y = 3$ and $c = 3$, also from (22) and (23) I will define $H \equiv (1 - h_2)Y + h_1X$. Hence from (23) a negative $H$ indicates that tax policy should move towards the subsidisation of the monopoly.

Remains, however, whether to subsidise records or to retain the initial position of zero taxation; it would appear that zero taxes may be the solution to the problem for a number of taste distributions. When attempting to raise welfare and induce entry the maintenance of a balanced budget implies that the two targets are being pursued with only one instrument. It is this that makes success unlikely.

References


Department of Trade, 1981, Reform of the law relating to copyright, design and performers’ protection, Cmnd. 8302.

Department of Trade, 1985, The recording and rental of audio and video copyright material, Cmnd. 9445.


given by (18) appears fairly weak, it may in practice prove to be restrictive. The results also indicate that overshifting is not an unlikely feature of imperfectly competitive industries.

6. Conclusions

I have discussed above the induced effects that occur in imperfectly competitive markets when the tax system is modified elsewhere in the economy, and evaluated these effects for a demand system derived from a model of copying. These results, combined with results on the direct price effects of taxes, have then been used to discover the circumstances for which the policy proposal to tax cassette tapes and subsidise record production is welfare-improving. The conditions prove to be restrictive: the gradient of the taste distribution function must be below some upper bound, a point that is illustrated by the simulations reported in section 5. From the analysis it must be concluded that the choice of policy for the fixed number model cannot be inferred from a theoretical analysis alone but must be based on close scrutiny of the relevant data.

In contrast, when the size of the industry is variable, the constraint that a welfare-improving policy must also increase the number of active firms permits stronger conclusions to be drawn. As shown in Theorem 1, taxation of record production can be ruled out as a candidate for policy. The decision

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Table 2
Density function $f(\mu) = 1000 + 100\mu'$, upper limit = 10.

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Table 3
Density function $f(\mu) = 100$. 


remains, however, whether to subsidise records or to retain the initial position of zero taxation; it would appear that zero taxes may be the solution to the problem for a number of taste distributions. When attempting to raise welfare and induce entry the maintenance of a balanced budget implies that the two targets are being pursued with only one instrument. It is this that makes success unlikely.

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