Price Normalisations in General Equilibrium Models of Imperfect Competition.

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Abstract: Price normalisations are an essential component of an economic model. The permissible class of normalisations for competitive models is well known but this is not the case for imperfect competition and a variety of approaches may be found in the literature. A formal definition of a permissible normalisation rule is given and the class of rules that satisfy this definition for imperfectly competitive models is derived. This class consists of all positive functions of the prices of goods traded on competitive markets. None of the functions in the class will guarantee the compactness of the space of possible prices, a result with implications for existence proofs.

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I. Introduction

A method of price normalisation is an essential component of any economic model. Without a normalisation rule, nominal indeterminancy can obscure analysis and prevent the application of standard mathematical theorems.\(^1\) In addition, applied economic theory, such as the analysis of optimal taxation, requires a price normalisation to provide a basis from which to measure government intervention.\(^2\) For competitive general equilibrium models, the permissible price normalisations are well known\(^3\): the price vector may be transformed in any way provided the ratios of prices remain unaffected. However the same claim cannot be made for models with imperfect competition and a variety of statements can be found in the literature. Our purpose in this paper is to investigate the nature of price normalisations in imperfectly competitive general equilibrium models and to derive the general class of permissible normalisation rules,\(^4\) as may be expected this class is more restricted than for the competitive model.

Since the paper of Gabszewicz and Vial (1972) it has been recognised that normalisation rules that would not affect a competitive equilibrium will have real effects upon equilibrium with imperfect competition as the following example demonstrates: consider an economy with one single-product monopolist producing at zero cost and whose profit maximising output is finite. Now imagine a model of this economy which, for analytical simplicity, uses a normalisation rule that selects the monopolist's good as the numeraire.\(^5\) The monopolist's profit maximising output in the model becomes unbounded and so the normalisation rule has altered the real equilibrium. Since this normalisation rule is known not to affect competitive

\(^1\) The requirement that the space of possible prices be compact in Gale's (1955) theorem is an immediate example.
\(^2\) See the careful discussions of this point in Diamond and Mirrlees (1971) and, especially, Munk (1978).
\(^3\) A thorough statement being given in Debreu (1959).
\(^4\) A formal definition of "permissible" is provided in section III.
\(^5\) If he produces good i, then all prices are multiplied by \((1/p_i)\).
equilibria, this example demonstrates that the class of normalisation rules that do not affect equilibrium is smaller for imperfect competition than for the competitive model. The actual rules employed by Gabszewicz and Vial are essentially arbitrary and were no doubt used for computational convenience. The same general comment applies to Dierker and Grodal (1986). In contrast, both Negishi (1961) and Cornwall (1977) restrict prices to the unit simplex in order to employ the resulting compactness properties of this set. Cornwall notes that this choice will affect the equilibrium except in the special case of the Negishi model with linear subjective demand. Roberts (1980) employs "leisure" as the numeraire and notes that the equilibrium is generally not numeraire free. It is not clear whether the market for the numeraire is competitive or not. Neither Guesnerie and Laffont (1978) nor Roberts and Sonnenschein (1977) use or discuss a price normalisation. Finally, each model in the survey paper of Hart (1985) employs a different method of price normalisation.

As the variety of price normalisations noted above illustrates, there is no common approach but there is general agreement that some price normalisations will affect the real equilibrium. The analysis of this paper aims to clarify these issues. Section II analyses a simple general equilibrium model and considers its homogeneity properties in detail. It is shown that the equilibrium is invariant to a normalisation rule that is defined as a function of competitive goods prices. Our concept of a permissible normalisation rule is defined in section III and the class of normalisation rules for which imperfectly competitive equilibria are invariant is derived. This class is a direct extension of that considered in section II. Section IV explores some implications of these results and conclusions are given in section V.

II. A Single Monopolist

In this section we study a simple example of a general equilibrium model with imperfect competition and use this to motivate and illustrate the more formal
reasoning of section III. The economy we consider has two consumption goods: one produced by a monopolist, the other by a perfectly competitive sector that exhibits constant returns to scale. Both goods are produced using labour as the only input. This model is used to establish the homogeneity properties of the equilibrium and to discuss the role of price normalisations.

The detailed structure of the model is as follows: there are three goods X, Y, and L which are monopoly output, competitive output and labour respectively. There are three types of agent: a monopolist, a perfectly competitive sector and a set of consumers. The consumers’ behaviour is characterised by an aggregate labour supply function \( L^s(q, p, w, \pi) \) and two demand functions \( X(q, p, w, \pi) \), \( Y(q, p, w, \pi) \), where \( \pi \) is profit income and \( q, p, w \) are the prices of the monopolist's product, the competitive good and labour. The technology in the competitive sector is constant returns to scale, using \( \sigma \) units of labour to produce one unit of output, and the monopolist's technology is described by the cost function \( C[X(q, p, w, \pi), w] \).

The monopolist chooses a price \( q \) to solve the optimisation problem

\[
\max_{\{q\}} \pi = qX(q, p, w, \pi) - C[X(q, p, w, \pi), w]
\]  (1)

The presence of \( \pi \) as an argument in the profit function reflects the income effects that occur in a general equilibrium model. Assuming the existence of a solution to the monopolist's optimisation,\(^6\) the existence of a general equilibrium is a trivial matter and can be verified by an accounting exercise. The monopolist is always producing on his demand curve so that his market is in equilibrium. The monopolist's budget constraint is satisfied by construction and the budget constraint of the competitive sector will also be satisfied. Furthermore, there will be a zero profit equilibrium in the competitive sector with \( p = \sigma w \). Equilibrium in the labour market then follows from the consumers' budget constraint by Walras' law, proving the existence of a general equilibrium.

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\(^6\) Sufficient conditions for the existence of an equilibrium are derived in Cripps and Myles (1988).
It is the homogeneity properties of economic models that permit the normalisation of prices. With perfect competition, demands and supplies are homogenous of degree zero with respect to prices and the real equilibrium is thus independent of the level of prices. To proceed with the analysis of the imperfectly competitive model, we now analyse the model's homogeneity properties and show that equilibrium output levels are determined by homogenous of degree zero functions of the wage rate. From this, we deduce the form of normalisations rules that do not affect the real equilibrium. The underlying reasoning is that the monopolist's price and profit level are both homogenous of degree one functions of the wage rate and the price of the competitively produced good, which because of constant returns is a constant multiple of the wage rate: in effect all prices and profits are determined by the level of the wage rate. Hence changes in the level of the wage rate will not affect relative prices and the equilibrium will be unaffected by a normalisation rule that consists of selecting a value for the wage rate.

From the maximisation of profits, the monopolist will condition his optimal price upon the price of the competitively produced good and upon the wage rate. We will write this choice as

\[ q = \Phi(p, w) . \]  

(2)

Similarly, maximised profits are written

\[ \pi = \Omega(p, w) . \]  

(3)

The competitive sector produces with constant returns to scale so that its labour demand is a constant fraction of output:

\[ L^d_y = \tau Y(q, p, w, \pi), \quad \tau = \frac{1}{\sigma} . \]  

(4)

The demand for labour from the monopolist is given, by Shephard's lemma, as the first derivative of the cost function
Labour market equilibrium requires

\[ L^d_X = \frac{\partial C[X(q, p, \pi, w), w]}{\partial w}. \]

The following two lemmas state the homogeneity properties.

**Lemma 1.** Profit, \( \pi \), is homogeneous of degree 1 in \( q, p \) and \( w \).

Proof: defining profits and multiplying \( q, p \) and \( w \) by \( \mu \) gives

\[ \pi(\mu) = \mu q X(\mu q, \mu p, \mu w, \pi(\mu)) - C[X(\mu q, \mu p, \mu w, \pi(\mu)), \mu w]. \]

As \( X(\mu q, \mu p, \mu w, \pi(\mu)) \) is homogeneous of degree 0 and the cost function is homogeneous of degree 1 in \( w \):

\[ \frac{\pi(\mu)}{\mu} = q X(q, p, w, \frac{\pi(\mu)}{\mu}) - C[X(q, p, w, \frac{\pi(\mu)}{\mu}), w]. \]

But, as \( \pi \) was the unique maximum level of profit,

\[ \frac{\pi(\mu)}{\mu} = \pi, \text{ or } \pi(\mu) = \mu \pi. \]

**Lemma 2.** The profit maximising price, \( q^* \), is homogeneous of degree 1 with respect to \( p \) and \( w \).

Proof: let \( q^* \) maximise profits for price \( p \), wage rate \( w \) and write the maximised value of profits as \( \pi^* \). Now assume for \( \mu p \) and \( \mu w \) that profits are maximised by \( q' \neq \mu q^* \).

As profits are homogeneous of degree 1 in \( q, p \) and \( w \),

\[ q' = \max_{\{q\}} \left\{ \frac{\pi(\mu)}{\mu} \right\}, \]

where

\[ \frac{\pi(\mu)}{\mu} = q X(q, p, w, \frac{\pi(\mu)}{\mu}) - C[X(q, p, w, \frac{\pi(\mu)}{\mu}), w]. \]
But \( q^* \) was the solution of this problem, so \( q' = \mu q^* \) and the profit maximising price is homogeneous of degree 1 in \( p \) and \( w \).

These lemmas can now be employed to demonstrate that the equilibrium of the model is independent of the level of the wage rate. In theorem 1 it should be recalled that labour market equilibrium is synonymous with general equilibrium.

**Theorem 1.** The equilibrium of the labour market equilibrium is independent of the level of the wage rate.

**Proof:** Labour market equilibrium requires

\[
\tau Y(q, p, w, \pi) + \frac{\partial C[X(q, p, w, \pi), w]}{\partial w} = L^*(q, p, w, \pi).
\]

This can be written in terms of the monopolist's behaviour as

\[
\tau Y(\Phi(p, w), p, w, \Omega(p, w)) + \frac{\partial C[X(\Phi(p, w), p, w, \Omega(p, w)), w]}{\partial w} = L'(\Phi(p, w), p, w, \Omega(p, w)).
\]

The assumption of zero profit in the competitive sector implies that \( p = \sigma w \). Using this and employing lemmas 1 and 2, the homogeneity properties allow the above to be expressed as

\[
\left( \frac{1}{\sigma} \right) Y(w\Phi(\sigma, 1), \sigma w, w\Omega(\sigma, 1)) + \frac{\partial C[X(w\Phi(\sigma, 1), \sigma w, w\Omega(\sigma, 1)), w]}{\partial w} = L'(w\Phi(\sigma, 1), \sigma w, w\Omega(\sigma, 1)).
\]

As \( Y(\cdot) \), \( X(\cdot) \) and \( L(\cdot) \) are derived from utility maximisation, they are homogenous of degree zero in their arguments so

\[
\left( \frac{1}{\sigma} \right) Y(\Phi(\sigma, 1), \sigma, 1, \Omega(\sigma, 1)) + \frac{\partial C[X(\Phi(\sigma, 1), \sigma, 1, \Omega(\sigma, 1)), w]}{\partial w} = L'(\Phi(\sigma, 1), \sigma, 1, \Omega(\sigma, 1)).
\]
Finally, as
\[ C[X(\Phi(\sigma, 1), \sigma, 1, \Omega(\sigma, 1)), w] \]
is homogenous of degree 1 in \( w \), its derivative is homogenous of degree zero. The equilibrium equation becomes:
\[
\left( \frac{1}{\sigma} \right) Y(\Phi(\sigma, 1), \sigma, 1, \Omega(\sigma, 1)) + \frac{\partial C[X(\Phi(\sigma, 1), \sigma, 1, \Omega(\sigma, 1)), 1]}{\partial w}
\]
\[ = L^*(\Phi(\sigma, 1), \sigma, 1, \Omega(\sigma, 1)). \]
This is independent of the wage rate, proving the theorem..

The conclusion to be drawn from theorem 1 is that starting from any equilibrium wage rate, we can normalise by scaling this wage rate up or down without affecting the real equilibrium. In particular, we can transform the wage rate \( w \) to a normalised value of \( w' \) using a mapping of the form \( w' = \beta(w).w,^7 \) or, more generally, of the form \( w' = \beta(p, w).w \) This normalisation then gives goods prices of \( p' = \beta(w).p \) and \( q' = \beta(w).q \) (or \( p' = \beta(p, w).p, q' = \beta(p, w).q \)). Here \( \beta(w) \) (or \( \beta(p, w) \)) represents our normalisation rule and the equilibrium of the economy is invariant with respect to the choice of \( \beta(w) \), provided it is positive. Notice that we do not include \( q \) as an argument of \( \beta(\cdot) \). As we show below, if this were done the rule would then affect the real equilibrium.\(^8\)

**III. The General Class of Price Normalisations.**

In this section we describe a general equilibrium model with quantity setting oligopolists and show how the normalisation procedure described above can be extended to this more general framework. The theorem given demonstrates that the

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^7 For instance, \( \beta(w) = 1/w \) will normalise the wage rate to unity.

^8 To pre-empt what follows, consider normalising prices with the rule \( \beta(q, p, w) \) and carry out the maximisation in (1). The result will obviously depend on the form of \( \beta(q, p, w) \).
set of equilibria of the economy is invariant with respect to a normalisation rule
dependent upon the prices of competitively produced goods and that this is the only
class of rule with this property.

We define a price normalisation with an adaptation of the approach of Dierker
and Grodal (1986). Prior to the application of a normalisation rule, the equilibrium
of a model representing an economy will define a vector of prices; we will term these
the "natural prices". However, the economic theorist typically cannot work with
natural prices and requires some structure, or restrictions, upon possible price vectors;
the restriction to the unit simplex being the typical example. This is the essence of a
normalisation rule: it is a scaling of the natural prices into an analytically tractable, or
desirable, form. A general description of a normalisation rule is that of a function
taking natural prices, which are members of some set P, to analytical prices, which
are in a set Q,

\[ a: P \rightarrow Q, \quad Q \subseteq \mathbb{R}_+^n \]

where \( n \) is the number of goods. To illustrate the possible forms of \( a \), take a vector \( p = (p_1, \ldots, p_n) \) from \( P \), then if

\[ a(p) = \left( \frac{1}{\sum_{i=1}^{n} p_i} \right) \cdot p, \]

\( Q \) is the unit simplex. Alternatively, if

\[ a(p) = \left( \frac{1}{\sum_{i=1}^{n} p_i^2} \right)^{1/2} \cdot p \]

then \( P \) is mapped into the positive quadrant of the unit sphere. Finally, the mapping

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9 In Dierker and Grodal the mapping is defined from relative prices (rates of exchange) to absolute
prices. In our view, the approach used in the text is more intuitive. The conclusions are not affected
by this since given a set of "natural prices", rates of exchange can always be defined.
\[ a(p) = \frac{1}{p_i} \cdot p. \]

selects good i as the numeraire. However, it is evident that the form of \( a(p) \) cannot be entirely arbitrary if it is to act as a normalisation rule. In order to proceed further it is necessary to move beyond an intuitive concept of a normalisation rule and to define formally our concept of a normalisation rule.

Definition 1: A normalisation rule is a function \( a(p) = (a_1(p),...,a_n(p)) \), \( a: P \rightarrow Q \), which satisfies

C1. \( p_i \cdot a_j(p) = p_j \cdot a_i(p) \), all i, j.

C2. i) \( x_h^* = \arg\max \{U^h(x_h') \text{ s.t. } x_h' \in B(p)\} = \arg\max \{U^h(x_h') \text{ s.t. } x_h' \in B(a(p))\} \)

ii) \( y_j^* = \arg\max \{\pi_j = p \cdot y_j' \} = \arg\max \{\pi_j = a(p) \cdot y_j' \} \)

C1. requires the normalisation rule to leave relative prices unchanged. C2. (i) requires the normalisation to leave the utility maximising choice of any consumer h unchanged, \( B(p) \) is the budget set at prices \( p \). (ii) restricts the normalisation to leave the profit maximising choice of any firm j unaffected. Evidently, if any firm, or consumer, has market power this must be incorporated in the maximisations involved in C2.

It is immediately apparent that C1. restricts the normalisation rule to the form \( a(p) = \beta(p) \cdot p \).\(^{10}\) Furthermore, under the competitive assumption that agents take prices as parametric, C1. necessarily implies C2. Hence for competitive models C2. does not need explicit recognition. This is not the case for imperfect competition and the class of normalisation rules that satisfy C1. and C2. is smaller for imperfect competition than for the competitive model. It appears that many of the difficulties in the literature are simply due to the denial of C2. or, equivalently, to the use without further consideration of normalisation rules that are applicable only to the competitive

\(^{10}\) Compare this with the examples.
model. Our task is now to delineate the class of rules that are permissible for imperfectly competitive models.

We begin by outlining the model, which is a slight generalisation of the Cournot-Walras model of Gabszewicz and Vial (1972).\textsuperscript{11} In this model prices are determined by a competitive economy. The endowments of the consumers engaging in trade are the sum of their initial endowment prior to economic activity and their transactions with imperfectly competitive firms. Each firm is assumed to maximise profits. The imperfectly competitive firms produce prior to the transactions in the competitive economy taking place, but in the knowledge of how their actions will affect equilibrium prices, and then assign consumers with predetermined shares of their production plans and profits. Competitive firms trade only in the competitive economy.

There are $n$ goods indexed $k = 1,2,\ldots,n$, the first $n_s$ are produced by competitive industries and goods $(n_{s+1},\ldots,n)$ are produced by oligopolistic industries. The oligopolistic firms are indexed $i = 1,2,\ldots,I$, have production sets $G_i$ which are convex subsets of $R^n$ and choose production plans $y_i \in G_i$. We assume the production sets satisfy irreversibility and free-disposal and, in addition, intersect the positive orthant only for the subspace determined by goods $n_{s+1}$ to $n$. The competitive firms are indexed $c = 1,2,\ldots,C$, and determine a production plan $z_c$ from production set $G_c$ in $R^n$. These firms are all assumed to produce with constant returns to scale and only have net outputs of goods $1,\ldots,n_s$.

The consumers have preferences representable by strictly quasi-concave continuous utility functions $U^h(x_h), h = 1,2,\ldots,H$, defined on the consumption set $X_h$, initial endowments $e_h$ and shares $\theta_{hi}$ in the profit $\pi_i$ of the $i$th firm.

For the competitive economy an equilibrium is defined by:

\textsuperscript{11} This model is employed for convenience. The results are not dependent upon this choice.
Definition 2: A Competitive Equilibrium relative to \((y, \pi)\) consists of a triple \((p, x, z)\) such that

(I) \(x_h \text{ max. } U^h(x_h') \text{ for all } x_h' \in X_h \text{ and } p.x_h' \leq p.e_h + \sum_{i=1}^{l} \theta_{hi} \pi_i, \ h = 1,...,H\)

(II) \(z_c \text{ max. } p.z_c \text{ s.t. } z_c \in G_c, \ c = 1,...,C\)

(III) \(\sum_{h=1}^{H} (x_h - e_h) = \sum_{c=1}^{C} z_c + \sum_{i=1}^{l} y_i\).

Introducing the imperfectly competitive sector, the Cournot-Walras equilibrium is defined as follows:

Definition 3: A Cournot-Walras Equilibrium consists of \((y^*, \pi^*, p^*, x^*, z^*)\) such that

1) \(p^*, x^*, z^*\) is a competitive equilibrium relative to \((y^*, \pi^*)\)

2) \(p^* . y_{i^*} = \pi_{i^*}\)

3) \(p^* . y_{i^*} \geq p^* . y_i^\prime, \text{ for all } y_i^\prime \in G_i \text{ and } (p', x', z') \text{ is a competitive equilibrium relative to } (y', \pi'), y' = (y_1^*,...,y_{i-1}^*,y_i^\prime,y_{i+1}^*,...,y_{I}^*)\), \(\pi_i' = p^* . y_i^\prime\).

This definition requires that the oligopolists choose their outputs, \(y_i^*\), to maximise profits given the outputs of other firms and subject to the constraint that the alternatives are on the objective demand function.\(^{13}\)

In view of theorem 1, we shall consider a normalisation rule defined on the prices of competitive goods. To simplify notation, let \(p_C = (p_1,...,p_{nC})\) and \(p^I = (p_{nC+1},...,p_n)\). It then follows:

Theorem 2. The set of Cournot-Walras equilibria is invariant with respect to the normalisation rule \(a: P \rightarrow Q\) if, and only if, \(a\) is of the form

\(^{12}\) Notation: \(x = (x_1,...,x_h,...,x_H), y = (y_1,...,y_i,...,y_I), z = (z_1,...,z_c,...,z_C), \theta = (\theta_{11},...,\theta_{1I},...,\theta_{HI})\) and \(\pi = (\pi_1,...,\pi_i,...,\pi_I)\).

\(^{13}\) See Hart (1985) for a discussion of objective demand in oligopoly models.
\[ q = \beta(p^C).p, \quad \beta(p^C) > 0. \]

Proof: to prove the "if" part, let \((y^+, \pi^+, \beta(p^{C^+}).p^+, x^+, z^+\) be a normalised Cournot-Walras equilibrium. This implies

1) \(\beta(p^{C^+}).p^+, x^+, z^+\) is a competitive equilibrium relative to \((y^+, \pi^+)\)

2) \(\beta(p^{C^+}).p^+.y_i^+ = \pi_i^+\)

3) \(\beta(p^{C^+}).p^+.y_i^+ \geq \beta(p^{C^*}).p^'.y_i'\), for all \(y_i' \in G_i\) and \((\beta(p^{C^*}).p^', x', z')\) is a competitive equilibrium relative to \((y', \pi')\), \(y' = (y_1^+,...,y_{i-1}^+,y_i',y_{i+1}^+,...,y_J^+)\), \(\pi_i^+ = \beta(p^{C^*}).p^'.y^+\).

Using standard results, with this normalisation rule the set of competitive equilibria determined by \(y^+, \pi^+\) is identical to that determined by \(y^*, \pi^*\) if \(y^+ = y^*\) and \(\pi^+ = \beta(p^{C^+}).\pi^*\).

To prove the first equality, we have to show that \(\{y_i^+ : \beta(p^{C^+}).p^+.y_i^+ \geq \beta(p^{C^*}).p^'.y_i'\} = \{y_i^* : p^*.y_i^* \geq p^'.y_i'\}\). However, as \(p^C\) represents the vector of competitive prices, these are independent of \(y\). Hence \(p^{C^+} = p^{C^*}\) and \(\beta(p^{C^+}) = \beta(p^{C^*}) = \beta(p^C)\). Eliminating the constant \(\beta(p^C)\) leaves two identical inequalities which must have the same solution as the production possibility sets are unaffected by the normalisation. For the second equality, 

\[ \pi_i^+ = \beta(p^{C^+}).p^+.y_i^+ = \beta(p^{C^*}).p^*.y_i^* = \beta(p^{C^+}).\pi_i^*. \]

The "only if" follows from observing that to satisfy C2.,

\[ \text{argmax } y_i.p(y_i, \pi_i) = \text{argmax } y_i.\beta(p(y_i, \pi_i)).p(y_i, \pi_i) \]

Any maximiser must satisfy the first-order condition

\[ \frac{\partial y_i.p(y_i, \pi_i)}{\partial y_i} = \frac{\partial y_i.\beta(p(y_i, \pi_i)).p(y_i, \pi_i)}{\partial y_i} = 0 \]

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14 The arguments of \(p(\cdot)\) are restricted to \(y_i\) and \(\pi_i\) to simplify notation.

15 This assumes differentiability.
Carrying out the differentiations

\[
y_i \frac{\partial p(y_i, \pi_i)}{\partial y_i} + p(y_i, \pi_i) = y_i p(y_i, \pi_i) \frac{\partial \beta(p(y_i, \pi_i))}{\partial p(y_i, \pi_i)} \frac{\partial p(y_i, \pi_i)}{\partial y_i} \\
+ \beta(p(y_i, \pi_i)) \left( y_i \frac{\partial p(y_i, \pi_i)}{\partial y_i} + p(y_i, \pi_i) \right) = 0
\]

These can only have identical solutions if

\[
\frac{\partial \beta(p(y_i, \pi_i))}{\partial p(y_i, \pi_i)}
\]

is identically zero for all goods for which firm \(i\) has market power. As this must hold for all firms \(i = 1, \ldots, I\), \(\beta(p(y_i, \pi_i))\) must be independent of \(p^i\).

**IV. Implications**

The results presented above carry several implications for future research in imperfect competition. On the positive side, we have derived the set of normalisation rules whose use will not affect the real equilibrium of imperfectly competitive economies. However, this is offset by the fact that of the rules, none has the property of always mapping into a compact set; the compactness of the price space typically being an essential part of an equilibrium existence proof. Our results show that with imperfect competition this compactness cannot be generated by the application of a normalisation rule in contrast, for example, to the use of the simplex mapping in the competitive model. Instead, any proof using our class of normalisation rules will have to show the prices that may be chosen by imperfectly competitive firms belong to compact sets due to the structure of their decision problems and not because of the normalisation rule.

Two further points are worth noting. Firstly, when there are no competitive markets in the model the only permissible form of normalisation is to multiply natural
prices by a positive constant, effectively just a change in the unit of account. Unfortunately, such a transformation serves none of the purposes for which normalisation rules are usually employed. Secondly, money can be used as a numeraire provided it is supplied "competitively".

V. Conclusions

The paper has discussed the role of normalisation rules in economic models and has provided a formal statement of the conditions such a rule must satisfy. In particular, we have argued that a normalisation rule must not affect the real equilibrium of the economic model. The normalisation rules employed in the analysis of competitive economies naturally satisfy both conditions.

For imperfectly competitive models the permissible class of normalisation rules is smaller. We have shown that this class consists of positive functions of the prices of goods traded on competitive markets. As a consequence, the normalisation rule cannot be employed to restrict prices to a compact set with obvious implications for existence proofs in models with imperfect competition.

References


