Distortionary Taxation and the Optimal Level of Public Good Provision

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Abstract: As an indirect tax system used to finance public good provision becomes more distortionary, it may be expected that optimal public good supply will fall. This paper assesses whether this contention is correct. A measure of the degree of differentiation of the tax system is introduced and inefficient (or distortionary) tax systems are modelled as those having less differentiation than the Ramsey tax rates. It is shown that an increase in allowable differentiation may reduce the optimal public good supply. This contradicts the contention that more distortion implies less public good.

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1. Introduction
The question of the relationship between the extent of distortion caused by the tax system and the level of public good provision is a very old one. Although discussion of this issue can be traced back to at least Pigou (1947), there is as yet no definitive answer. The results that have been derived to date are limited to the contrast between lump-sum taxes and commodity taxes, and even this case has not been completely solved. Atkinson and Stern (1974) and Wilson (1990, 1991) have constructed examples for which public good supply is higher when financed by lump-sum taxation relative to when financed by commodity taxation, but no general analytical results have been established. No results at all are available on the contrast between commodity tax systems with different degrees of distortion.

Despite this paucity of theoretical results, applied economists continue to calculate values for the marginal cost of public funds (MCPF). The interest in doing this is no doubt based on the reasoning that as taxes become more distortionary the MCPF rises, and that a higher MCPF implies optimal public good supply should be lower. An illustration of this argument is given in figure 1. Let the marginal cost of public funds be raised from MCPF to MCPF' by an increase in distortion. Assuming that the marginal utility of the public good, MUG, is unaffected by taxation, optimal public good supply would fall from $G_0$ to $G_1$. Expressed in this way, the outcome appears to be clear-cut. However, the argument is not as straightforward as this since the MCPF is not related to the level of distortion quite this simply. Indeed, it is this that has been at the heart of the failure to prove analytical results on the contrast in provision between lump-sum and commodity tax finance. From this observation, it can be seen that the truth, or otherwise, of this simple argument remains to be established.

Although the theoretical concentration to date on the contrast between lump-sum and commodity taxation, what is more interesting on a practical basis is the consideration of alternative non-optimal commodity tax schemes. In particular, the fundamental question that needs to be answered is whether it necessarily follows that as commodity taxes become more distortionary the chosen level of public good provision must fall. Expressed alternatively, should reforms that improve the efficiency of the tax system be accompanied by an increase in public good supply? If the answer is yes, then the implied simple link between the MCPF and optimal public

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1 Wilson (1991) has shown that the converse may occur in many-consumer economies when equity criteria become relevant but it is the single-consumer economy that is studied below.

2 See, in particular, the discussion of figure 1 in Atkinson and Stern (1974).
good supply will provide a useful "rule of thumb" for policy. Whether or not this is true is the question that is addressed in the present paper.

To answer this question, it is necessary to construct a meaningful method of contrasting tax systems with different degrees of distortion. To understand why this is so, note that if tax system 'A' raises more revenue than tax system 'B' then it will obviously lead to a greater public good supply (assuming all revenue is spent on public goods). Since this is regardless of which system is more distortionary, phrased in this general way the question of a relationship between distortion and provision is an empty one. What it is clearly necessary to do is to construct a relevant basis of comparison which gives the problem some content.

The approach adopted in this paper is explained in section 2 but can be described briefly as follows. Assume that the optimal tax system is non-uniform. Uniform taxes will then be less efficient, or more distortionary, than the optimal taxes in the sense that they will imply a higher excess burden. Uniform taxes also have the property that there is zero differentiation between the tax rates. A measure of the extent of differentiation in the tax system is then introduced and the tax rates are considered as being chosen subject to there being an allowable degree of differentiation. As the allowable degree of differentiation increases, the chosen tax rates will move away from being uniform and eventually reach the optimal set at which the excess burden is minimised. After this point, the restriction on differentiation will not be effective. From this perspective, an increase in distortion can be interpreted as a reduction in the allowable, and binding, degree of differentiation. The relation of public good supply to tax distortion can then be answered by calculating how it changes as allowable differentiation increases.
Adopting this approach, it is shown that an increase in distortion can actually lead to an increase in optimal public good supply. An example with this property is constructed (and so is another with the opposite property) and an explanation for why this can occur is given for the general case. The implication of these results is that the presumption that increased distortion always leads to a lower optimal public good supply is false. In fact, such a clear negative relation between public good supply and distortion can only be demonstrated under very restrictive assumptions. They also make the value of measuring the MCPF questionable. Although there may be some independent reasons why knowing this value is of interest in itself, the lack of any general relation shows that it carries no direct implication for public good provision.

The remainder of the paper is as follows. Section 3 of the paper derives a general expression for the effect of a change in the degree of differentiation upon public good supply. Section 4 evaluates this expression for a numerical example and section 5 for an analytical example. A discussion of the general case is given in section 6. Conclusions are offered in section 7.

2. Measurement of non-optimality

It is clear that the question of whether an increase in the distortion caused by taxes always leads to a lower level of public good provision is an empty one unless further structure is placed upon it. This is because if it is assumed that all tax revenue is spent on a public good, then any tax system that raises more revenue than the optimal taxes necessarily leads to greater public good supply, whether it is more distortionary or not. An interesting interpretation of the question has to involve the optimised level of the public good being related to the degree of distortion. Although this may seem straightforward, there is no natural or simple method of measuring the degree of distortion that can be varied in a parametric manner. The purpose of this section is to propose a re-interpretation of the question that leads to a tractable method of formulating the problem.

The first issue is to define clearly what is meant by the degree of distortion. Although strong preconceptions may exist as to what the phrase implies, no simple formal definition is currently available. Clearly, the degree of distortion cannot simply be related to the variation between taxes since optimal tax theory shows that efficiency generally requires taxes to be differentiated between goods. Instead, the degree of distortion must involve a measure of the deviation of the tax rates from those which

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3 It also shows that distortion should be measured in terms of quantities rather than in prices. A reflection of this is the “index of discouragement” (Mirrlees (1976)).
would be efficient under the Ramsey rule. Following this line of reasoning, assume that the optimal Ramsey tax system is known. Assume further that a measure exists of the extent of differentiation in the tax rates on different commodities. It then follows that any tax system that has a different level of differentiation than the Ramsey taxes (either more or less) must be more distortionary than the efficient system.

It is now possible to describe how the reasoning given above is operationalised. First, a measure of tax differentiation is introduced. Secondly, commodity taxes and public good supply are optimised subject to a constraint on the allowable degree of differentiation. If this degree of differentiation is less than that possessed by the Ramsey taxes, the constraint will be binding on the optimisation. Finally, assuming the constraint is binding, the effect of its relaxation upon public good supply is derived. Following the arguments above, this increase in allowable differentiation can be interpreted as the formalisation of a reduction in distortion.

To introduce the measure of differentiation that is employed, consider an economy that has two consumption goods which are subject to taxation and labour as the untaxed numeraire. As is well-known, the non-taxation of labour is a normalisation that does not impose any restrictions. Denoting the ad valorem commodity taxes by \( t_1 \) and \( t_2 \), the degree of differentiation of the tax system is measured by defining a function \( \rho(t_1, t_2) \) which has the following properties:

- **P1** (Non-negativity): \( \rho(t_1, t_2) \geq 0 \),
- **P2** (Zero at uniformity): \( \rho(t_1, t_2) = 0 \Longleftrightarrow t_1 = t_2 \),
- **P3** (Symmetry): \( \rho(t_1, t_2) = \rho(t_2, t_1) \),
- **P4** (Homogeneity): \( \rho(\lambda t_1, \lambda t_2) = \rho(t_1, t_2) \) for any \( \lambda > 0 \).

Properties 1 and 2 simply assert that the degree of differentiation must be non-negative and that the measure of differentiation is zero when taxes are uniform. Property 3 requires that the measure be symmetric in the tax rates and must evaluate equally deviations in either direction from uniformity. Property 4 makes the measure independent of the mean level of the tax rates. Given the assumption that the tax rates are ad valorem, these are all natural assumptions.

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4 Studying an economy with only two taxed goods is sufficient to establish the relationship between public good supply and distortions. Adding more goods would be possible but, as will be seen, it would not contribute to the results.
Although it is possible to conceive of further properties that the measure should satisfy, P1 - P4 are sufficient for the present purpose. It is obviously preferable to impose as few restrictions as possible. It should also be noted that these properties are very similar to those assumed for inequality measures. This is to be expected since measuring the dispersion of a set of incomes is a formally equivalent problem to measuring the differentiation of a set of tax rates.

Using the measure of differentiation, a given restriction upon the tax system can then be imposed by requiring that the set of potential tax rates must satisfy the weak inequality \( \rho(t_1, t_2) \leq r \). This will be binding upon the choice of taxes if the optimal unrestricted tax rates, \( t_1^*, t_2^* \), satisfy \( \rho(t_1^*, t_2^*) > r \). When \( r \) is set equal to zero, uniform taxes must be imposed, whereas \( r > 0 \) permits some degree of differentiation. This measure of differentiation is now applied to analyse the relationship between the level of public good supply and the allowable degree of differentiation.

3. Determination of provision

The level of provision of the public good when the tax system is restricted in the permissible degree of differentiation is determined by choosing the tax rates to maximise welfare whilst satisfying the differentiation constraint. Varying the allowable degree of differentiation then determines the relationship between this and public good supply. This section provides a general characterisation of this relationship, with a detailed analysis conducted in the following sections.

It is assumed that the economy has a single consumer, two privately provided consumption goods, a single public good and a labour service. All private markets are competitive and constant returns to scale prevail with labour being the only input into production. The wage rate is numeraire and labour is not taxed. As is well known, this involves no loss of generality. Under these assumptions, it is possible to choose units of the two available consumption goods so that their producer prices are both unity.\(^5\) With ad valorem tax rates \( t_1 \) and \( t_2 \) on goods 1 and 2 respectively, the relation between consumer prices, \( q_i \), and producer prices, \( p_i \), is given by

\[
q_i = \left[ 1 + t_i \right] p_i = 1 + t_i, i = 1, 2, \tag{1}
\]

\(^5\) The taxes are assumed to be ad valorem but given this normalisation, the same representation captures specific taxation.
since the normalisation gives \( p_i = 1, i = 1, 2 \). The preferences of the consumer can be represented by the indirect utility function

\[
U = v(t_1, t_2, G).
\] (2)

The demand function for good \( i \) is denoted by \( X^i = X^i(t_1, t_2, G) \), so that the budget constraint for the government is

\[
t_1 X^1(t_1, t_2, G) + t_2 X^2(t_1, t_2, G) = G.
\] (3)

Denoting partial derivatives by subscripts, it is assumed that \( 1 - \sum_{j=1}^{2} t_j X^j_G > 0 \) so the implicit function theorem can be applied to (3). This gives

\[
G = F(t_1, t_2),
\] (4)

where

\[
\frac{dG}{dt_i} = F_i = \frac{X^i + \sum_{j=1}^{2} t_j X^j_G}{1 - \sum_{j=1}^{2} t_j X^j_G},
\] (5)

The public good is said to be revenue enhancing if \( \sum_{j=1}^{2} t_j X^j_G > 0 \), revenue neutral if \( \sum_{j=1}^{2} t_j X^j_G = 0 \) and revenue reducing if \( \sum_{j=1}^{2} t_j X^j_G < 0 \).

In the absence of restrictions upon the tax rates, the optimal Ramsey taxes solve

\[
\max_{t_1, t_2, G} v(t_1, t_2, G) \text{ subject to } G = F(t_1, t_2).
\] (6)

The optimal tax rates arising from this maximisation are denoted by \( t_1^* \) and \( t_2^* \) and can be understood through the usual interpretation of the Ramsey rule with endogenous public good supply (see, for example, Atkinson and Stern (1974)). Similarly, the optimal tax rates when a limit is imposed upon the degree of differentiation solve the maximisation

\[
\max_{t_1, t_2, G} v(t_1, t_2, G) \text{ subject to } G = F(t_1, t_2) \text{ and } \rho(t_1, t_2) \leq r.
\] (7)
The essential point of the analysis is to derive a value of \( \frac{dG}{dr} \) from the solution of the maximisation in (7). If this were positive for all \( r \) for which the constraint were binding, this would indicate that public good supply was highest under the unrestricted tax system and decreased as restrictions were placed on the tax system. Although the problem can be approached directly from the specification above, it is now shown how an equivalent, but more tractable, representation can be derived.

The problem under consideration only has content if, at the solution of (6), \( t_1^* \neq t_2^* \). Assume that this is so, and choose the labelling of the goods so that \( t_2^* > t_1^* > 0 \). The structure of the optimisation in (7) is then illustrated in figure 2. Since properties \( P1 \) to \( P4 \) imply that \( \rho(t_1, t_2) \) is linearly homogenous (or conical), the set of tax rates satisfying \( \rho(t_1, t_2) \leq r \) is a convex cone with vertex at the origin. The effect of raising \( r \) is to pivot the boundary of the cone outwards. Under the assumption that \( t_2^* > t_1^* \), the locus of solutions as \( r \) varies is determined by the tangency between the upper boundary of the cone and the level sets of the indirect utility function (where (4) has been substituted in (2) for \( G \)). The question of whether the level of public good supply increases or decreases with \( r \) is then determined by the relative gradients of the solution locus and the level sets of \( G = F(t_1, t_2) \) as \( G \) varies.

![Figure 2: The structure of the solution](image)

From figure 2 it can be seen that, when the constraint is binding, the solution will occur on the upper boundary of the cone. It is therefore possible to replace the differentiation constraint \( \rho(t_1, t_2) \leq r \) by the equivalent condition

\[
t_2 = \lambda t_1 ,
\]

(8)
where \( \lambda \geq 1 \) is the gradient of the boundary of the cone. There is a one-to-one relationship between values of \( r \) and values of \( \lambda \) with \( r = 0 \Leftrightarrow \lambda = 1 \) so that an increase in \( r \) can be represented by an increase in \( \lambda \). This observation can be used to reformulate the maximisation in (6) as the simpler problem

\[
\max_{t} V(t; G; \lambda) \text{ subject to } f(t; \lambda) = G,
\]

where \( V(t; G; \lambda) = V(t, \lambda t, G) \) and \( f(t; \lambda) = F(t, \lambda t) \), for a given value of \( \lambda \). With this formulation, the interest is upon the effect of the parameter \( \lambda \) upon optimal provision.

An increase in \( \lambda \) represents a relaxation of the constraint upon the degree of differentiation so that it is equivalent to an increase in \( r \). Using (9), the necessary condition for the choice of \( t \) can be written

\[
V_t + V_G f_t = 0.
\]

The comparative statics of the optimum can be derived from (10) as

\[
\frac{dt}{d\lambda} = -\frac{V_{t\lambda} + V_{G\lambda} f_t + V_{GG} f_t f_{\lambda} + V_G f_{t\lambda}}{V_t + 2V_G f_t + V_{GG} f_t f_t},
\]

with \( V_t + 2V_G f_t + V_{GG} f_t f_t + V_G f_{tt} < 0 \) as the second-order condition for the maximisation. The effect upon the level of public good supply is then

\[
\frac{dG}{d\lambda} = f_{\lambda} + f_t \frac{dt}{d\lambda} = \frac{V_t f_{\lambda} + V_G f_t f_{\lambda} + V_{G\lambda} f_t f_t - V_{t\lambda} f_t - V_{G\lambda} f_t f_t - V_G f_{t\lambda}}{V_t + 2V_G f_t + V_{GG} f_t f_t + V_G f_{tt}}.
\]

The question originally posed of how the permissibile degree of differentiation, or the enforced distortion of taxes from their optimal levels, can now be answered by evaluating (12).

### 4. A numerical example

As already noted, an initial view of the problem would suggest that since moving closer to the optimal tax rates lowers the MCPF, this would raise the optimal level of public good provision. This section provides the first investigation of the issue by evaluating (12) for a numerical example which supports the basic presumption. But, as the following section will show, this does not provide a general finding.

Assume that the utility function is given by
\[ U = X^{1\alpha} + X^{2\beta} + \gamma G - \ell, \tag{13} \]

where \( \gamma > 1 \). With this specification, the optimal tax rates when there is no restriction upon the degree of differentiation can be calculated from (6) to satisfy

\[ \frac{t_1^*}{1 + t_1^*} = \frac{[\gamma - 1][1 - \alpha]}{\gamma}, \tag{14} \]

and

\[ \frac{t_2^*}{1 + t_2^*} = \frac{[\gamma - 1][1 - \beta]}{\gamma}. \tag{15} \]

Since good 2 is taken as the more highly taxed good, it follows from (14) and (15) that \( \alpha > \beta \). Adopting parameter values of \( \alpha = .75, \beta = .25 \) and \( \gamma = 2 \) gives optimal tax rates of \( t_1^* = 1/7 \) and \( t_2^* = 3/5 \). These tax rates lead to a level of public good supply of \( G = .07699 \).

Turning now to the outcome when the degree of differentiation is restricted, the values of the optimal tax rates calculated above show that \( \lambda \) can vary in the range \([1, 21/5]\), where a value of 1 represents uniformity and \( 21/5 \) the ratio \( t_2^*/t_1^* \). For \( \lambda \) above \( 21/5 \) the differentiation constraint will not be binding. Carrying out the optimisation in (9) for various values of \( \lambda \) provides the results reported in table 1.

<table>
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<tr>
<th>( \lambda )</th>
<th>( \xi )</th>
<th>( \sigma )</th>
</tr>
</thead>
<tbody>
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</tr>
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<td>1.5</td>
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</tr>
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<tr>
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</tr>
<tr>
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</tr>
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</table>

**Table 1: Tax rates and public good supply**

The results in table 1 show that in this case public good supply increases as the permissible degree of differentiation increases. The tax rate on good 1 (\( t_1 \)) falls as \( \lambda \) increases because when uniform taxes must be employed it is initially raised above its optimal level. In contrast, the tax on good 2, \( \lambda t_2 \), rises. The tax rates and the level of public good supply converge smoothly to the optimal values for the unconstrained

\[ \text{If } \gamma \leq 1 \text{ no public good would be supplied because the disutility of the labour required to produce it would more than offset the benefit.} \]
optimisation. With uniform taxation, public good supply is only 70% of the optimal level.

To give further insight into this solution, it can be calculated that at $\lambda = 2$, $f_t = 0.177584$ and $f_{\lambda} = 0.01242$. The latter of these shows that the economy is operating on the side of the Laffer curve where tax revenue is increasing in the tax rate on good 2. Since $dt/d\lambda < 0$, it can be seen from (12) that the two effects are offsetting and it is the latter that is dominant.

5. Counterexample

The example of section 4 presented a case that supports the presumption that if the tax system is allowed a greater degree of differentiation, then the level of public good supply will increase. This section presents a counter example that demonstrates this need not always occur.

The direct utility function is now assumed to be quadratic with

$$U = \alpha_1 X^1 - \frac{\beta_1}{2} [X^1]^2 + \alpha_2 X^2 - \frac{\beta_2}{2} [X^2]^2 + \gamma G - L,$$

which implies that the indirect utility function is

$$V = \sum_{i=1}^{2} \left[ \alpha_i - 1 - t_i \left[ 1 - \gamma \right] \left[ \frac{\alpha_i - 1 - t_i}{\beta_i} \right] - \left[ \frac{\alpha_i - 1 - t_i}{2\beta_i} \right]^2 \right].$$

When there is no restriction on the degree of differentiation, it follows from (17) that the optimal tax rates are given by

$$t_i^* = \left[ \frac{\alpha_i - 1}{\beta_i} \right] \left[ \frac{1 - \gamma}{\beta_i} \right], \quad i = 1, 2.$$  

Using (18) it can be seen that the optimal degree of differentiation of the taxes is

$$\frac{t_2^*}{t_1^*} = \frac{\alpha_2 - 1}{\alpha_1 - 1}.$$  

The ratio in (19) determines the maximum value of $\lambda$ for which the differentiation constraint will be binding.
Now assume that $\alpha_2 > \alpha_1 > 1$ and that $\lambda$ is less than the maximum value determined by (19). Carrying out the optimisation of welfare subject to the differentiation constraint determines the optimal tax rate $t$ as

$$t = \gamma - 1 \left[ \frac{\beta_2 \left( \alpha_1 - 1 \right) + \lambda \beta_1 \left( \alpha_2 - 1 \right)}{\beta_2 + \lambda^2 \beta_1} \right] \left[ 2 \gamma - 1 \right].$$

(20)

This tax rate is positive since $\gamma > 1$. Substituting (20) into the definition of government revenue gives the results reported in table 2.

<table>
<thead>
<tr>
<th>$\lambda$</th>
<th>$\tau$</th>
<th>$\sigma$</th>
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<td>1.45</td>
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<td>1.5</td>
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</tr>
<tr>
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<td>33.99</td>
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</tr>
</tbody>
</table>

Parameter values: $\alpha_1 = 100, \alpha_2 = 200, \beta_1 = 5, \beta_2 = 1, \gamma = 1.5$

Table 2: Counter example

It can be seen from table 2 that optimal public good supply is falling for values of $\lambda$ above 1.6. This result contradicts the presumption that an increase in allowable differentiation will increase public good supply. Neither the structure of utility or the production technology used in this example is in any way special. This indicates that this outcome is not especially unusual.

6. General case

The results in the previous two sections have shown that the relaxation of the restriction upon differentiation can lead to a change in public good supply in either direction. This section discusses this finding in terms of both the general expression (12) and the illustration of the problem in figure 2. The main purpose is to understand the nature of this indeterminacy.

The analytical expression in (12) can be usefully decomposed in order to understand the separate effects of the constituent parts. Since it is the second-order condition for the optimisation, the denominator is necessarily negative. The outcome
is therefore opposite in sign to the numerator. To obtain further insights, the Laffer curve assumption, $f_\lambda > 0$, is adopted. In addition, the indirect utility function is assumed separable between prices and public good supply. Under these assumptions, the numerator of (12) reduces to

$$[V_{tt}f_\lambda - V_{t\lambda}f_t] + V_G\left[f_{tt}f_\lambda - f_{t\lambda}f_t\right].$$  \hspace{1cm} (21)

Now consider the first term of (21). From (10), $f_t > 0$. Furthermore, as the indirect utility function $v(t_1, t_2,G)$ is necessarily quasi-convex in $t_1$ and $t_2$ (and may be convex, see Varian (1984)), it can be expected that $V_{tt}f_\lambda - V_{t\lambda}f_t > 0$. This is termed the welfare effect and it provides a tendency for public good supply to fall as the taxes move toward their efficient values. The effect arises because the welfare loss is convex in tax rates. Calculating $V_{tt}f_\lambda - V_{t\lambda}f_t$, it will be positive when

$$tF_2[v_{11} + \lambda v_{12}] - v_2[F_1 + \lambda F_2] - tF_1[\lambda v_{22} + v_{12}] > 0.$$  \hspace{1cm} (22)

The second term in (21) is termed the revenue effect. The marginal utility of public good, $V_G$, is positive while the term $f_{tt}f_\lambda - f_{t\lambda}f_t$ derived from the expression for tax revenue, will most likely be negative. The latter will be true when

$$f_{tt} = F_{11} + 2\lambda F_{12} + \lambda^2 F_{22} < 0, \quad f_{t\lambda} = F_2 + tF_{21} + t\lambda F_{22} > 0.$$  \hspace{1cm} (23)

The revenue effect therefore tends to result in an increased level of provision.

Consequently, there are two offsetting effects upon the direction of public good supply. The welfare effect will tend to reduce it whilst the revenue effect will tend to increase it. As has already been shown, the final outcome can move in either direction. When the indirect utility function is not separable, the additional effects that come into play can operate on either side of the balance. The same is also true if the Laffer curve assumption is not satisfied. What should be especially noticed from this discussion is that even a series of strong assumptions concerning separability etc. are not sufficient to provide a clear prediction.

Additional insight into the general indeterminateness of the outcome can be obtained by displaying the problem as represented in figure 2. In figure 3a the locus of solutions as $\lambda$ varies is shown to have a greater slope than the locus of tax rates that maintain the same level of public good supply as the solution for $\lambda = \lambda^0$. In this case, public good supply will increase as $\lambda$ is increased from $\lambda^0$. However, as figure 3b shows, a slight rearrangement of the level sets of the indirect utility function can reverse this finding. In the second case the solution locus is less steep than the constant public good supply locus and provision falls. Naturally, the constant public
good supply locus is related to the structure of the level sets of indirect utility but the examples have already shown that both cases are possible.

![Figure 3a: Increase in supply](image)

![Figure 3b: Reduction in supply](image)

7. Conclusions
The paper has investigated whether a less distortionary tax system will always lead to a greater level of public good supply. To provide some content to the question, the method has been adopted of considering tax systems which were optimal given an allowable degree of differentiation. This was intended to provide a workable and meaningful basis of comparison. A measure of differentiation was introduced and it was shown how this could be simply incorporated into the optimisation of the tax system.

The two examples show that the initial presumption that public good supply will rise when the tax system can become closer to optimal is not always correct. Even for a quadratic utility function it is possible for public good supply to decrease as the taxes are allowed to become increasingly differentiated. Discussion of the general case isolated the welfare and revenue effects which tend to have opposing consequences for the level of public good provision.

The results of this paper point to the conclusion that there is no simple relationship between the level of public good supply and the degree of distortion of the tax system. Although the simple argument about the marginal cost of public funds seems fairly compelling, it is not borne out by this analysis. In particular it is apparent that there are no simple restrictions that can be placed upon the indirect utility function to yield clear predictions.
References


