Ambiguity and Accident Law$^1$

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Abstract

This paper analyzes liability rules, when agents, both the potential injurer and the potential victim may perceive ambiguity, i.e., they are not aware of the probability of a possible environmental accident. Both the injurer and the victim invest in care, which is commonly observable, and they derive utility from an unobservable action, which may lead to the accident. Here we analyse the welfare implications of the tort rules. First, when agents only undertake investment in care and second, when agents choose care and the unobservable action. When agents only choose the level of care, under negligence ambiguity averse agents are more likely to choose the optimal amount of care. When agents choose both care and unobservable action, we propose a system of negligence plus punitive damages which gives optimal level of both care and unobserved action by both injurers and victims.
1 Introduction

Environmental disasters can cause severe private damage and harm. A number of such disasters are accidents and can be attributed to error or lack of care by agents involved as in the Bhopal Union Carbide gas leak, the Exxon Valdez oil spill or the British Petroleum (BP) Deepwater Horizon oil spill. Such events are regulated and governed broadly by law of tort.\textsuperscript{1} Tort and liability rules help society by giving potential environmental injurers and victims an incentive to endogenize the social cost of their actions (Calabresi, 1970, Posner, 2007, Shavell, 1980 and Calcott and Hutton, 2006). A significant part of the literature on tort has focused on the efficiency implications of using different tort rules, like negligence and strict liability. In the standard framework, the agents are assumed to know the relevant probabilities.

However this assumption can be criticised. In potential tort situations, we may have injurers and victims, who before choosing their actions, may not be able to correctly attribute the probability of an accident or estimate the damage caused as a result of the accident. Agents may not have sufficient information or time to assign precise probabilities to accidents. For example, an oil company, such as BP, may not be able to form correct beliefs about the probability of an accident given there was little previous experience of drilling at these depths. Since accidents are rare events companies may not have enough observations to base subjective probabilities on relative frequencies. Complexity can also make it hard to assign probabilities in certain cases, for example, nuclear technology is very complex. Moreover the chance of an accident often depends on the behaviour of other people.

\textsuperscript{1}Tort law decides the liability in case of private harm due to act of negligence or lack of duty of care by the injurer. Environmental accidents can cause both public and private harm. For the purpose of our analysis here we are going to ignore the difference between private and public harm. Victims are assumed to suffer damage and we are going to model them as a single representative agent.
something which is intrinsically hard to predict.

Risks with unknown probabilities are called ambiguous risks. The potential injurer, if he does not know the probability of the accident given his actions, will likely choose an action which is different from an injurer who knows the probability. Similarly ambiguity may change the behaviour of the victim. Thus ambiguity in analysing tort rules is relevant since if we accept that accidents are likely to be perceived as ambiguous events it is desirable that the tort rules are robust to ambiguity.

In the present paper we consider the case where the probability of an accident is unknown and study the implications of this assumption.

In the standard framework of tort, the agents involved are modelled as rational subjective expected utility (SEU from now on) decision makers (Savage, 1954). In this paper, we relax the assumption of SEU, more precisely we allow the agents to be affected by perceptions of ambiguity. We explain ambiguity below. We argue that when analysing tort law for environmental accidents, the notion of ambiguity allows us to make a more complete analysis of the tort rules. Agents involved are likely to have poor information about the probability of environmental events given the very low frequency of such accidents and major environmental accidents often have unique circumstances. Thus it is not implausible that they perceive the risks to be ambiguous.

We analyse welfare implications of the tort rules, negligence and strict liability, with ambiguity. We show that it may be possible to get closer to ex-post Pareto efficiency when there is ambiguity. We state the optimal tort rule with ambiguity. Our analysis here is motivated by examples similar to BP Deepwater Horizon oil spill, where the injurer due to the new and advanced technology being used, perceives ambiguity about the accident occurring. Moreover both the potential injurer (BP) and the victims also view the care levels taken by each other as ambiguous.

In most standard law and economics tort analysis, agents, potential injurers and victims, are
modelled as having subjective beliefs about the probability of the accident. However, this may not be fully realistic.\textsuperscript{2} Agents may fail to correctly consider the probability of the accident or in some cases may be unaware of the probability. This problem may be exacerbated for some cases of environmental accidents like the BP oil spill. This inability to correctly consider the probability of the accident may lead decision makers to over or under-weight the probabilities of the future events and thus behave optimistically or pessimistically. Such beliefs may cause agents to choose their actions differently from SEU agents and as a result cause a different outcome.

Experiments (Camerer and Weber, 1992) have shown that agents often do not follow SEU. Ellsberg (1962) showed that decision makers avoid risks with unknown probabilities and prefer risks with known probabilities. More generally he argued that decision makers behave differently when probabilities are unknown. For the rest of the paper, events with unknown probabilities are called ambiguous and decisions under unknown probabilities are called ambiguous decisions. In case of such unawareness of the probabilities, decision makers may under-weight and or over-weight the probability of the events. As a result if they over-weight the chances of good (bad) outcomes then they are likely to behave optimistically (pessimistically).

1.1 Modelling Ambiguity

We shall model ambiguity using neo-additive preferences, which are axiomatized in Chateauneuf, Eichberger, and Grant (2007). This is a special case of Choquet Expected Utility (CEU) developed by Schmeidler (1989). In this model, ambiguity has the effect of causing the best and worst outcomes

\textsuperscript{2}In law and economics of tort, Shuman (1993) argues that assumptions that agents are rational and are fully aware of the probability of the accident and the consequence of their actions may not be able to provide a complete analysis of the tort rules.
of any given action to be over-weighted, (compared to SEU). We believe this model is suitable for studying tort, since in an accident there are focal best and worst outcomes, i.e. no accident and being found liable for an accident respectively. More specifically we consider a situation where a potential injurer and victim undertake specific activities which may result in an accident and loss. Both may choose a care level and a unobservable action to reduce the chance of an accident. The injurer and the victim have ambiguous beliefs regarding the likelihood of an accident and damage resulting from their care level and unobservable action. Care levels can be observed by the third party but not the unobservable action.

The solution concept, we use, allows us to model both, ambiguity-averse and ambiguity-preferring agents. If an agent is pessimistic then he/she will over weight bad outcomes. This would also imply that an agent would expect the other agent to choose lower care levels (strategy) and if the game has increasing differences, then the agent will have a lower best response and thus choose a lower care level (Eichberger and Kelsey (2009)).\(^3\) If the agents have ambiguity preference then the equilibrium strategy will increase.

### 1.2 Tort Rules

Agents do not consider the social cost of their actions but only the private costs. The aim is to design tort rules to induce agents to take into account the externality due to their actions. They have to be provided incentives so that they take care and choose the optimal level of the activity. The design of the tort rules is based on the economic losses caused by the action of the injurer and

\(^3\)A function \( f (x, t) \) has increasing differences if the incremental gain in choosing a higher \( x \) is greater if \( t \) is higher, i.e., if \( \bar{x} > \bar{x} \), then \( f (\bar{x}, t) - f (\bar{x}, t) \) is increasing in \( t \). So \( \Pi_i(s_i, s_{-i}) \) will have increasing differences if the gain in profit from choosing a higher strategy \( s_i \) will be greater if the other player also chooses a higher strategy \( s_{-i} \). This is a form of strategic complementarity.
minimizing the social loss due to the action. In this paper we make the standard assumption that it is clear who is the injurer and who is the victim.

In this paper, potential injurers and victims choose an observable action and an unobservable action. The observable action may be the investment in care, for example, safety standards on the drilling rig and the unobservable could be depth of drilling. The tort regimes which have been analysed extensively over the last few decades have been strict liability and negligence and both strict liability and negligence with contributory negligence. Strict liability is when the injurer is liable for the damages from the accident independent of the investment in care by the victim or the injurer and negligence is when the injurer is liable for the damages when the injurer fails to take the assigned level of care. Shavell (1987) has shown that strict liability and negligence both result in optimal care by the potential injurer and the victim if activity levels or unobservable action in our case are not considered. But if unobservable actions of the agents are included in the analysis then both agents use higher than optimal levels of actions under negligence and the victim will undertake more than optimal unobservable action under strict liability.

We show that in case of strict liability, pessimistic injurers and victims will invest in more than optimal/stipulated care because pessimism causes them to overweight the bad outcome, which in this case means having an accident. If negligence is used then the pessimistic injurer invests the stipulated or optimal amount in care and the optimistic injurer may under invest. In case of strict liability, ambiguity aversion increases the injurer’s perceived liability, so he invests more in care. In case of negligence, since the injurer is held liable if he fails to provide the stipulated care, then the pessimistic injurer will invest the stipulated amount since this avoids the ambiguous risk of being

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4 Observability here means that the action is observable and verifiable by a third party like a court of law.

5 While overinvesting in care may not sound bad, it may cause the injurer not to invest in some projects which are socially desirable.
found liable for an accident. The optimistic injurer, under-weights the event where an accident occurs and ignores the cost of the damage. Hence he will find the expected cost of damages lower than the cost of investing the stipulated amount and thus may not invest the stipulated amount. Note that in case of negligence we observe non-convexity in the payoff of the players and because of this, for the pessimistic injurer we find that such an injurer will invest the stipulated amount.

We then proceed to discuss the optimal rule when agents choose both an observable action and an unobservable action at the same time. We show that negligence rule coupled with punitive damages can give optimal levels in both the observable and the unobservable action by both injurer and victim. Here the injurer is held liable for loss if he fails to take the stipulated observable care, and otherwise the victim bears the loss. The main difference from the standard negligence rule is that, if the loss is excessive, that is, higher than some threshold, then the injurer is held liable for the loss and also pays a punitive fine. The threshold is chosen so that if it is exceeded it is clear that both parties have chosen excessive unobservable actions. Note that if the loss is in excess of the threshold then the punitive damage dominates the negligence rule.

1.3 Organization of the paper

Next we briefly discuss some of the relevant literature, after which we discuss the model of ambiguity. In Section 3 we set up the model of accidents, first without unobserved actions and then with unobserved actions. Finally we discuss the optimal tort rule given ambiguity aversion and conclude.

All proofs are grouped in the appendix.
2 Literature

The literature on tort is extensive and old. With Coase (1960) providing the analysis how property rights can solve the problem of assigning correct incentives to internalize the social costs.\textsuperscript{6} Much of the analysis has been to what extent the injurer should be held liable; for example Lander and Posner (1987), and Shavell (1987) show that a negligent injurer should bear the full cost of damages caused. Apart from analysing the efficient tort rule, the literature has also focused on the issues of causation, and to what extent the actions by the injurer and the victim lead to the damages (Burrows (1999) and Ben-Shahar, (2000)). The analysis in this paper will focus on the efficiency of liability rules.

Over the last couple of decades, contributions to economic theory have modelled risk and ambiguity differently. Here we consider players who perceive ambiguity. Depending on their perception, they may over-weight the good outcomes, in case of optimistic beliefs, or over-weight the bad outcomes, in case of pessimism. In this paper, our analysis could be linked with that of Eichberger and Kelsey (2009) and Eichberger et al. (2009) which look at ambiguity in strategic settings including games where both parties choose quantity in Cournot oligopoly game and a public good contribution game.

With this paper we intend to add to the recent literature in behavioural law and economics (Jolls, Sunstein and Thaler (1998), Arlen (1998), Korobkin and Ullen, (2000) and Parisi and Smith (2005)) where decision making assumptions in mainstream economics are relaxed to include behavioural insights.\textsuperscript{7} For example, due to the endowment effect (Kahneman, Knetsch, and Thaler 1990),


\textsuperscript{7}Chakravarty and Kelsey (2008) provides an explanation using ambiguity why strict liability may not apply in
the Coase theorem may fail as the endowment effect may create biases in valuation by the agents. Thus behavioural assumptions made in the analysis may have important implication for legal policy recommendations. Various scholars such as Jolls, Sunstein and Thaler (1998) and Bar-Gill (2006) have pointed out that behavioural aspects such as loss aversion or optimism have a role in designing legal rules.

The paper closest to ours is Teitelbaum (2007), which analyses the tort rules, when the injurer has ambiguous beliefs. He specifically looks at the case when actions of the victim have no bearing on the outcome. The analysis is concerned with the effect of ambiguity on the decision making of the injurer and the implications for the efficient tort regime. Here in this paper we extend the analysis by modelling the interaction between the victim and the injurer as a game. So we also model the victims’s action, the amount of care he chooses and the amount of unobservable action, as they too influence the amount of damage. Thus we model the game between the parties using a theory of games with ambiguity developed by Eichberger, Kelsey and Schipper (2009). We derive the implication of this for tort rules.

3 Preferences

The Ellsberg paradox (Ellsberg (1961)) shows that there may be preferences which are not compatible with SEU or indeed any other plausible decision theory in which decision-makers assign conventional subjective probabilities to events. Decision makers can be averse to choices where the probabilities are not known. However Ellsberg argues that the main message of the paradox is not that individuals are uniformly ambiguity-averse but that ambiguity makes a difference.\footnote{This is based on an unpublished lecture which Ellsberg gave in Vienna, May 2010.}
Thus it is also possible that individuals may deviate from expected utility theory by behaving in an ambiguity-seeking way. Ambiguity-seeking is common in choices involving unlikely events and choices involving losses (Kilka and Weber (2001)).

Chateauneuf et al. (2007) axiomatize neo-additive preferences, which are able to exhibit both ambiguity-aversion and ambiguity-seeking. In their model, the set of all states of nature is denoted by $\Omega$, and the set of outcomes is denoted by $X$. An act is a function $a : \Omega \rightarrow X$, which assigns outcomes to states. Neo-additive preferences may be represented by the following function defined on the space of acts:

$$V(a) = \delta (1 - \alpha) M_i + \delta \alpha m_i + (1 - \delta) E_{\pi} u_i(a(s)),$$

where $E_{\pi} u_i$ denotes the expected utility of $u_i$ with respect to the (conventional) probability distribution $\pi$ on $\Omega$ and $M_i = \max_{s \in \Omega} u_i(a(s))$ and $m_i = \min_{s \in \Omega} u_i(a(s))$. Thus the decision-maker maximises a convex combination of the maximum pay-off, the minimum pay-off and the average pay-off. This is a special case of Choquet Expected Utility (CEU) preferences, which were axiomatized by Schmeidler (1989).

One may interpret $\pi$ as the decision-maker’s belief. However it is an ambiguous belief. The decision-maker does not give it full weight in his/her preferences, which are also influenced by ambiguity measured by $\delta$. He/she reacts to this ambiguity either in an ambiguity-averse way by over-weighting bad outcomes or in an ambiguity-seeking way by over-weighting good outcomes. Ambiguity-attitude is captured by the parameter $\alpha$, with higher values of $\alpha$ corresponding to more ambiguity-aversion. In case of $\delta > 0$ and $\alpha = 1$, we can interpret this as decision makers who are pessimistic. And if $\delta > 0$ and $\alpha = 0$ then the decision maker is optimistic and pays too much attention to the best outcome.
4 Model

There are three agents, a risk-neutral injurer, a risk-neutral victim and a court of law.\(^9\) The injurer undertakes an activity which may cause harm or damage to the victim. The injurer can take an action, care or precaution, \(x\). The cost of precaution is \(a(x)\), where \(a(\cdot)\) is increasing in \(x\) and is convex. The victim can also take an action, precaution \(y\), and incur a cost of \(b(y)\), and \(b(\cdot)\) is increasing in \(y\) and convex. The court of law implements the damage rule. The levels of precaution are observable by all parties including court of law. The injurer and the victim believe that accidents occur with probability \(\pi\), but the injurer and victim are unsure about their belief and do not trust their belief about the accident. The injurer and the victim, when they choose the level of precaution and their activity, view this probability as ambiguous. Let \(D(x,y)\) be the damage caused by the accident such that \(D\) decreases with \(x\) and \(y\). We assume that the marginal change in \(D\) due to \(x\) also decreases in \(x\) and the marginal change in \(D\) due to \(y\) also decreases in \(y\). Let the utility in case of no accident be \(U_o = 0\). So the social cost from the accident is

\[
L(x,y) = \pi D(x,y) + a(x) + b(y).
\]

Therefore the optimal precaution \(x^*\) will be given by the condition \(ML_x \leq 0\), where \(ML_x = \pi [D(x_1,y) - D(x_2,y)] + [a(x_1) - a(x_2)]\), is the marginal loss function for \(x_1 > x_2\). The optimal \(y^*\) is given by \(ML_y \leq 0\), where \(ML_y = \pi [D(x,y_1) - D(x,y_2)] + [b(y_1) - b(y_2)]\), is the marginal loss function for \(y_1 > y_2\).

The problem here is to design incentives so that the injurer chooses the optimal level of precaution. Let the expected utility of the SEU maximizing injurer be \(u_i(x,y)\) and that of the SEU

\(^9\)Here we have assumed risk neutrality of the agents. The implication is going to be that the wealth of the agents will not have any impact on our results.
maximizing victim be \( u_v(x, y) \). Note that here \( u_i(x, y) = -L_i(x, y) \) and \( u_v(x, y) = -L_v(x, y) \), where 
\( L_i \) and \( L_v \) are the expected loss functions. So the precaution taken by such an injurer will be \( x \),
which maximizes \( u_i(x, y) \) and the precaution taken by such a victim will be \( y \), which maximizes 
\( u_v(x, y) \).

### 4.1 Negligence and strict liability

In this very basic set up, let us analyse the effect of use of negligence and strict liability. Given 
that care levels are observable, under negligence, the injurer has to pay the damage caused by the 
accident if the care taken is less than the assigned or required care. Let the required care be \( x^* \), 
and assume that the court knows \( x^* \). So under negligence, the injurer and the victim have the 
following utility functions

\[
-L_i^n(x, y) = \begin{cases} -a(x) & \text{if } x \geq x^* \\ -\pi D(x, y) - a(x) & \text{if } x < x^* \\ \end{cases}
\]

\[
-L_v^n(x, y) = \begin{cases} -b(y) & \text{if } x < x^* \\ -\pi D(x, y) - b(y) & \text{if } x \geq x^* \\ \end{cases}
\]

Let \( x_n \) be the care chosen by the payoff maximizing injurer if the negligence rule is used and similarly 
\( y_n \) be the care chosen by the victim under negligence. So with negligence \( x_n \in \arg \max_i -L_i^n(x, y^*) \), 
\( x_n = x^* \) and \( y_n \in \arg \max_v -L_v^n(x^*, y) \), \( y_n = y^* \), so we get efficient investment in precaution by the 
injurer and the victim.

The second liability rule is strict liability, under which the injurer has to pay the damage no 
matter what level of ex ante precaution he takes. So the expected utility/loss under strict liability 
is \( -L_i^s(x, y) = -\pi D(x, y) - a(x) \) and \( -L_v^s(x, y) = -b(y) \), hence \( x_s \in \arg \max_i -L_i^s(x, y) \), \( x_s = x^* \)
and $y_s \in \arg \max -L_{x_s}^s(x, y)$, $y_s = y^*$ and the precaution taken by both the injurer and the victim under the rule of strict negligence will be efficient. For strict liability the injurer bears the full cost of the externality/accident he/she causes. The injurer invests in care until the marginal reduction in expected loss equals the marginal cost of care. In case of negligence, the injurer invests full care too. This is due to the fact that budget set of the injurer has a kink at the optimal care level and the injurer will find it beneficial to invest the optimal level rather than not invest the optimal level and be liable for the damage.

4.2 With ambiguity

Now let both the victim and the injurer perceive ambiguity, where both agents’ preferences are parameterized by ambiguity attitude $\alpha_j$ and degree of ambiguity $\delta_j$ (for $j = i, v$) such that if $\alpha_j = 1$ and $\delta_j > 0$ then the agents are pessimistic and if $\alpha_j = 0$ and $\delta_j > 0$ agents are optimistic. Here the victim and the injurer are ambiguity averse regarding the accident and each others’ actions. The loss from the worst realization for the injurer if he bears the cost of the accident is $a(x) + D(x, 0)$. Note here the minimum payoff (highest damage) for the injurer is $D(x, 0)$, that is, when the injurer bears liability for not investing $x^*$ and the victim invests zero in care. The utility/loss from the best realization is $a(x)$. For the victim the loss from the worst realization is $b(x) + D(x^*, y)$ which may arise in case of negligence rule. Here the minimum payoff for the victim is when he bears the cost of the damage since the injurer has invested in stipulated care $x^*$. The utility from the best realization is $b(y)$. The stipulated care for the agents, is going to be the optimal welfare maximizing amount $x^*$ and $y^*$. Under negligence, the injurer who perceives ambiguity will have the following

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10This is not true if $x_n > x^*$.  

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(Choquet) expected utility,

\[-L_{iA}^n(x, y) = -\delta_i \alpha_i D(x, 0) - (1 - \delta_i) \pi D(x, y) - a(x) \text{ if } x < x^*, \]

\[= -a(x) \text{ if } x \geq x^*. \]

For the victim under negligence we get

\[-L_{vA}^n(x, y) = -\delta_v \alpha_v D(x^*, y) - (1 - \delta_v) \pi D(x, y) - b(y) \text{ if } x \geq x^*, \]

\[= -b(y) \text{ if } x < x^*. \]

Note here that we assume that there is no ambiguity for the court regarding the measurement of $x^*$ and $y^*$. The marginal benefit, $MB_{iA}^n$, for the injurer for a change in $x$, for $x_1 > x_2$, is

\[\delta_i \alpha_i \left[ D(x_2, 0) - D(x_1, 0) \right] + (1 - \delta_i) \left( \pi \left[ D(x_2, y) - D(x_1, y) \right] \right) + [a(x_2) - a(x_1)] \text{ if } x_1 < x^*, \]

\[[a(x_2) - a(x_1)] \text{ if } x_2 \geq x^*. \]

So the injurer will increase $x$ as long as $MB_{iA}^n \geq 0$. Similarly, marginal benefit of the victim, $MB_{vA}^n$, for a change in $y$, $y_1 > y_2$, is

\[\delta_v \alpha_v \left[ D(x^*, y_2) - D(x^*, y_1) \right] + (1 - \delta_v) \left( \pi \left[ D(x^*, y_2) - D(x^*, y_1) \right] \right) + [b(y_2) - b(y_1)] \text{ if } x < x^*, \]

\[[b(y_2) - b(y_1)] \text{ if } x \geq x^*. \]

The victim will increase investment in care as long as $MB_{vA}^n \geq 0$. While under strict liability for the injurer the expected utility if he/she is ambiguity averse is

\[-L_{iA}^s(x, y) = -\delta_i \alpha_i D(x, 0) - (1 - \delta_i) \pi D(x, y) - a(x). \]

For the victim under strict liability we get

\[L_{vA}^s(x, y) = -b(y). \]
So the injurer will increase $x$ as long as

$$MB_{iA}^* = \delta_i \alpha_i [D(x_2, 0) - D(x_1, 0)] + (1 - \delta_i) (\pi [D(x_2, y) - D(x_1, y)]) + [a(x_2) - a(x_1)] \geq 0,$$

for $x_1 > x_2$ and the victim increases $y$ as long as $MB_{vA}^* \geq 0$, where $MB_{vA}^* = [b(y_2) - b(y_1)]$ for $y_1 > y_2$. So from above we get the propositions for strict liability and negligence.\footnote{Note, here that under strict liability, the game between the injurer and the victim is such that the payoffs of each agent is increasing in $(x, y)$. We use Theorem 3.1 from Eichberger and Kelsey (2009), which shows that with parameters $\delta$ and $\alpha$, and if the game is such that the payoff of player $i, \Pi_i(s_i, s_{-i})$, where $s_i \in S_i$ is the strategy of player $i, s_{-i} \in S_{-i}$ is the strategy of the other player $-i$, is increasing in $s_{-i}$, and has increasing differences in $(s_i, s_{-i})$ the equilibrium of the game will exist. For the game under negligence, if we consider mixed strategies then we can show existence of an equilibrium. Therefore the game under either damage rule will have an equilibrium. The definition of the equilibrium is provided in the Appendix.} First, we state the proposition about strict liability.

**Proposition 1** Under strict liability, pessimistic (respectively optimistic) injurers, are going to invest in more (respectively less) care than the socially optimal level, $x^*$, and the victims will invest in care level as long as $[b(y_2) - b(y_1)] \geq 0$ for any $y_1 > y_2$.

And for negligence we can state the following:

**Proposition 2** Under negligence, if both agents are pessimistic ($\alpha_j = 1$ and $\delta_j > 0$, for $j = i, v$) then the injurer will invest the stipulated amount in care and victim will invest in care as long as $[b(y_2) - b(y_1)] \geq 0$ for any $y_1 > y_2$. If both agents are optimistic ($\alpha_j = 0$ and $\delta_j > 0$, for $j = i, v$), then agents under-invest in care.

**Proof:** All proofs can be found in the Appendix.

If $\delta_i > 0$ and the injurer perceives ambiguity, then his/her marginal benefit of care is going to be higher than that of a SEU injurer. The marginal increase in benefit when care level goes up from $x_2$ to $x_1$ for the injurer who perceives ambiguity is $\delta_i \alpha_i [D(x_2, 0) - D(x_1, 0)] + (1 - \alpha_i [D(x_2, y) - D(x_1, y)]) + [a(x_2) - a(x_1)] \geq 0$. For the victim, the marginal benefit of care is $[b(y_2) - b(y_1)] \geq 0$ for any $y_1 > y_2$.
\[\delta_i (\pi [D(x_2, y) - D(x_1, y)])\] is greater than the marginal benefit, \((\pi [D(x_2, y) - D(x_1, y)])\), of the injurer who is a SEU maximizer. This is true as long as \(\alpha_i [D(x_2, 0) - D(x_1, 0)] \leq \pi [D(x_2, y) - D(x_1, y)]\).

So under strict liability the injurer will invest more in care. If the stipulated care is set at \(x^*\), then the SEU injurer will invest \(x^*\) in care but with ambiguity the injurer will invest \(x < x^*\). Since in strict liability the injurer bears the damage costs, the victim will invest \(y = 0\). For strict liability we can see from Figure 1, showing the best response functions of the ambiguity averse victim and the ambiguity averse injurer, the equilibrium is at point S. Under strict liability, since the injurer bears all the liability the victim’s best response is \(y = 0\) for all \(x\). The ambiguity averse injurer will over-invest at \(y = 0\), and as \(y\) increases will reduce the level of \(x\). Note here that as the injurer and the victim become more ambiguity averse, the equilibrium point \(S\) will shift to the right, since the best response of the victim will remain the same but the best response of the injurer will also shift further to the right.

For negligence, we present first the ambiguity-averse injurer’s best response function in Figure 2 and next the ambiguity-averse victim’s best response function Figure 3 and finally in Figure 4 we present the equilibrium which is at point N in the figure.

The best response function for the injurer suggests, he will invest the stipulated amount since this will ensure that it bears no liability. For high values of \(y\), since this will lower the probability of accident, the injurer will reduce \(x\). For the victim’s best response, first \(y\) remains the same as \(x\) increases, then at \(x^*\), \(y\) increases and then after a discontinuity resulting from the negligence rule \(y\) falls. Here as both, injurer and the victim become more ambiguity averse, the equilibrium \(N\) shifts up along the vertical line on \(x^*\). The increased ambiguity aversion will cause the best response function of the victim to shift up, while for the injurer only the section of the best response function sloping downwards will shift up.
Figure 1: Thick line $R_v(x)$ is the best response function of the victim and dashed line $R_i(y)$ is the best response function of the injurer. $S$ is the equilibrium.

Figure 2: The best response graph of the ambiguity-averse injurer
Figure 3: The best response graph of the ambiguity-averse victim

Figure 4: Equilibrium in case of ambiguity-averse injurer and victim. $R_i(y)$ is the best response for the injurer and $R_v(v)$ (dashed) is the best response for the victim. N is the equilibrium.
Figure 5: Injurer’s expected cost of care. Curve ABE: Expected cost under strict liability when injurer is SEU maximizer. Curve MON: Expected cost under strict liability when injurer is ambiguity averse. MOBCD: Expected cost under negligence when injurer is ambiguity averse.

In Figure 5, we can see how the pessimistic or the ambiguity averse injurer will choose the optimal care level under negligence. Such an injurer will have the expected cost MOBCD in Figure 5, and will therefore choose $x^\star$. While under strict liability such an ambiguity averse injurer will have expected cost MON and will choose $x' > x^\star$.

**Example 1** Let $a(x) = x^2$, $b(y) = y^2$, $D(x, y) = (1 - x - y)$, $\alpha_j = \alpha$, $\delta_j = \delta$ for $j = i, v$ and let $0 \leq x + y \leq 1$. The optimal investment level $x^\star = \frac{\pi}{2}$ and $y^\star = \frac{\pi}{2}$, where $\pi$ is the probability with which the injurer and the victim believe that accidents occur. If strict liability is used as the damage rule then $x^\star = \frac{\delta \alpha + \pi (1 - \delta)}{2}$ and $y$ such that $MB_{vA}^x \geq 0$ or $y = 0$. For $\alpha = 1$ and $\delta < 1, x^\star > x^\star$.

Note that this result is different from Teitelbaum (2007). Since the damages are influenced by the investment in care, the injurer due to ambiguity aversion does not correctly account for the expected loss due to his actions. So the care he takes, is higher ($x^\star > x^\star$, if $\alpha_i > \pi$) than the
optimal care. The victim expects that given strict liability he does not have to incur the damage of the accident, therefore minimizes just the cost of effort $b(y)$.

In case of negligence, the injurer bears the cost of damages if the care taken is lower than the stipulated care. If the care is $x^*$, the injurer will choose $x^*$ if the expected cost from choosing $x^*$ is less than the expected from choosing any other $x$. So if $\delta_i \alpha_i D(x, 0) + (1 - \delta_i) (\pi D(x, y)) + a(x) > a(x^*)$, the injurer will invest $x^*$. Now if we consider behavioural attitudes of the agents, if the injurer is pessimistic ($\alpha_i = 1, \delta_i > 0$), he/she can completely protect himself/herself from ambiguity by investing $x^*$, so such an injurer will invest $x^*$. Otherwise, if the injurer is optimistic, he will under weight the possibility of the damage, so he may choose $x < x^*$, if $\delta_i \alpha_i D(x, 0) + (1 - \delta_i) (\pi D(x, y)) + a(x) < a(x^*)$. If the optimism is not too great the injurer will still choose $x^*$ due to the kink in the pay-off function. Since the pessimistic injurer will invest $x^*$, the victim will increase $y$ as long as $MB_{VA}^n \geq 0$ or choose $y^*$ if $\delta_i \alpha_v D(x^*, y) + (1 - \delta_v) (\pi D(x, y)) + b(y) > b(y^*)$. In case of an optimistic injurer, the victim knows that the injurer will incur liability and therefore invest $y = 0$. Therefore, unless the victim is completely optimistic, he will invest $y < y^*$.

Example 2 Let the cost function of care be $a(x) = x^2$, $b(y) = y^2$, $\alpha_j = \alpha$ and $\delta_j = \delta$ (for $j = i, v$). Also let $D(x, y) = (1 - x - y)$. The optimal care is therefore, $x^* = \pi/2$ and $y^* = \pi/2$. Assume both the injurer and the victim perceive ambiguity. (i) Let $\alpha = 1$ and $\delta > 0$, then the cost function for the injurer is $\delta(1 - x) + (1 - \delta) (\pi (1 - x - y)) + x^2$ if he fails to take the stipulated care and the cost is $(x^*)^2$ if he takes stipulated care. If $\delta(1 - x) + (1 - \delta) (\pi (1 - x - y)) + x^2 > (x^*)^2$, the injurer will take stipulated care. (ii) Let $\alpha = 0$ and $\delta > 0$, for the injurer the loss function is $(1 - \delta) \pi (1 - x - y) + x^2$. So the care taken will be $x^*$ if $(1 - \delta) \pi (1 - x - y) + x^2 > x^*$. Note that while in case of strict liability $\alpha = 1$ and $\delta < 1$ results in excessive care by the injurer, here under
negligence if \( \alpha = 0 \) and \( \delta > 1 - \frac{x^{2} - x^{2}}{\pi(1-x-y)} \) then the injurer will under invest in care.

In case of negligence the injurer who is significantly optimistic may ignore the damage he would have to incur if he invests \( x < x^{*} \), as a result he ignores the damages while choosing care. In the case where injurer is pessimistic, he overweights the event where a loss occurs, and therefore will prefer to take stipulated care \( x^{*} \). We find that under negligence if the injurer is sufficiently optimistic then we move away from optimality towards under investment in care but if the injurer is slightly optimistic then, due to the kink in the injurer’s constraint set, the injurer still invests \( x^{*} \). Negligence, therefore is more robust to ambiguity preference than strict liability. So if the negligence rule is used, it is more likely to result in optimal care by the parties if we take into account possibility of ambiguity-seeking. Negligence is the most common tort rule used in common law in Britain and United States and the robustness to ambiguity may provide a possible reason.

4.3 With unobservable action

Next we introduce the case when the damage is not only influenced by the care taken by the parties but increases with another unobservable action of the injurer and the victim. This action is not observable by the court of law.\(^{12}\) In some cases we can think about this unobservable action as the activity level of the agents. For instance this can be the level of drilling in the BP oil spill, or the activity of the nuclear reactor at Windscale nuclear reactor in UK.\(^{13}\) Let us say the unobservable

\(^{12}\)Here we can also think about unobservability as something which the third party can find out but at a cost, and not as complete unobservability. Secondly, we undertake our analysis under this assumption because we believe that this gives us stronger results. Note if we relax this assumption of the action being observed then our results still hold.

\(^{13}\)The BP rig was one of the deepest oil wells in history [http://www.timesonline.co.uk/tol/news/world/us_and_americas/article7128842.ece]. Similarly in case of the Windscale accident in Cumbria, UK, the reactor was being used to build material for a H-bomb which it had not been constructed for [http://news.bbc.co.uk/1/hi/sci/tech/7030281.stm].
action of the injurer is $s$, and that of the victim is $t$. The maximum amount of the unobservable action for the injurer and the victim is $\bar{s}$ and $\bar{t}$ respectively. This gives the injurer utility $u(s)$ and victim utility $w(t)$ such that $u$ and $w$ are increasing in level of activity and concave. The damage increases with the activity such that the damage function is $\bar{D}(x,y,s,t)$, $\bar{D}$ is non decreasing in $s$ and $t$, and finally $\bar{D}(s,t)$ has increasing differences in $(s,t)$, $\bar{D}(x,s)$ has increasing differences in $(x,s)$ and $\bar{D}(y,t)$ has increasing differences in $(y,t)$. The last assumption means that lack of care is worse if the unobservable action is high. The cost for the injurer for undertaking care $x$ and the unobservable action $s$ is $sa(x)$ and likewise the cost of the victim for care $y$ and unobservable action $t$ is $tb(y)$.\(^{14}\) The social surplus is given by

$$S(x,y,s,t) = u(s) + w(t) - \pi \left[ \bar{D}(x,y,s,t) \right] - sa(x) - tb(y).$$

The first best care and the unobservable action level, $(x^*, y^*, s^*, t^*)$ is given by the following conditions, where $MS_x, MS_y, MS_s, and MS_t$ are the marginal changes in social welfare due to changes in $x, y, s$ and $t$ respectively.

$$MS_x \geq 0 : MS_x = s \left[ a(x_2) - a(x_1) \right] + \pi \left[ \bar{D}(x_2,y,s,t) - \bar{D}(x_1,y,s,t) \right],$$

for $x_1 > x_2$,

$$MS_y \geq 0 : MS_y = t \left[ b(y_2) - b(y_1) \right] + \pi \left[ \bar{D}(x,y_2,s,t) - \bar{D}(x,y_1,s,t) \right],$$

for $y_1 > y_2$,

$$MS_s \geq 0 : MS_s = [u(s_1) - u(s_2)] + \pi \left[ \bar{D}(x,y,s_2,t) - \bar{D}(x,y,s_1,t) \right] + [s_2 - s_1] a(x) = 0,$$

for $s_1 > s_2$,

$$MS_t \geq 0 : MS_t = [w(t_1) - w(t_2)] + \pi \left[ \bar{D}(x,y,s,t_1) - \bar{D}(x,y,s,t_2) \right] + [t_2 - t_1] b(y) = 0$$

for $t_1 > t_2$.

If the damage increases with the activity of the agents, and if the court applies strict liability, then with ambiguity the injurer and the victim, respectively, have the following expected utility

$$U_{x,A}^i(x,y,s,t) = u(s) + \alpha_i \delta_i \bar{D}(x,0,s,\bar{t}) + (1 - \delta_i) \pi \bar{D}(x,y,s,t) + sa(x),$$

\(^{14}\)Here we adopted the multiplicative cost function from Shavell (1987). This is done simply to stick to the Shavell (1987) structure as much as possible. This function can generalized without losing our results.
\[ U_{vA}^* (x, y, s, t) = w(t) + tb(y). \]

### 4.3.1 Strict Liability

First we discuss strict liability when damages increase with unobservable action and agents perceive the chances of accident to be ambiguous.

**Proposition 3**

(i) Under strict liability, when the damage increases with increase in unobservable action by the injurer and the victim, the ambiguity averse injurer will choose a lower unobservable action than optimal, but the victim will participate more in the unobservable action than the optimal level. (ii) Under strict liability the injurer invests more than stipulated care \( x^* \) and the victim invests \( y = 0 \), if they are sufficiently ambiguity averse.

The marginal benefit level of unobservable action of the injurer for \( s_1 > s_2 \), is given by the first order condition,

\[
MB^*_iA(s) = [u(s_1) - u(s_2)] + \alpha_i \delta_i \left[ \tilde{D}(x, 0, s_2, t) - \tilde{D}(x, 0, s_1, t) \right] + (1 - \delta_i) \pi \left[ \tilde{D}(x, y, s_2, t) - \tilde{D}(x, y, s_1, t) \right] + [s_2 - s_1] a(x).
\]

Note that under ambiguity the injurer faces an additional marginal cost compared to SEU if \( \alpha_i \delta_i \left[ \tilde{D}(x, 0, s_2, t) - \tilde{D}(x, 0, s_1, t) \right] - \delta_i \pi \left[ \tilde{D}(x, y, s_2, t) - \tilde{D}(x, y, s_1, t) \right] > 0 \). So under ambiguity the injurer decreases the level of unobservable action \( s \) in comparison to the SEU maximizing injurer if he/she is very pessimistic and the probability of accident is very small. The victim is going to undertake an unobservable action level \( t > t^* \), since he is not liable, he will ignore the externality caused by his unobservable action and does not consider the additional marginal cost his unobservable action causes, \( \pi \left[ \tilde{D}(x, y, s, t_2) - \tilde{D}(x, y, s, t_1) \right] \), for \( t_1 > t_2 \). The analysis of the choice of the care levels is quite similar to the case of strict liability without unobservable action.
levels. The injurer over-invests in care as \( x > x^* \) as long as \( \alpha_i \neq 0, \delta_i > 0 \), since the marginal benefit of an increase in \( x \) is given by

\[
MB^*_iA(x) = \alpha_i \delta_i \left[ \tilde{D}(x_2, 0, s, t) - \tilde{D}(x_1, 0, s, t) \right] + \\
(1 - \delta_i) \pi \left[ \tilde{D}(x_2, y, s, t) - \tilde{D}(x_1, y, s, t) \right] + s [a(x_2) - a(x_1)]
\]

for \( x_1 > x_2 \). Increasing differences implies \( MB^*_iA(x) \) is higher than the SEU marginal benefit. Thus the injurer’s best response curve is higher with ambiguity. Since the victim does not have to face the damage, he invests \( y = 0 \) in care.

Further, due to ambiguity-aversion, the injurer will use a lower unobservable action. While the victim will use a higher unobservable action since he does not pay for the externality he causes. The reduced unobservable action will compensate for the reduced care. The reason is that due to pessimism, for the injurer the marginal cost of the unobservable action will increase, since he will put extra weight on the damage.

### 4.3.2 Negligence

Now if the damage rule is negligence, then the agents have the following Choquet expected utility:

\[
U^*_{iA}(x, y) = \begin{cases} 
  u(s) - \alpha_i \delta_i \tilde{D}(x, 0, s, t) - (1 - \delta_i) \pi \tilde{D}(x, y, s, t) - sa(x) & \text{if } x < x^*, \\
  u(s) - sa(x) & \text{if } x \geq x^*, 
\end{cases}
\]

\[
U^*_{vA}(x, y) = \begin{cases} 
  w(t) - \alpha_v \delta_v \tilde{D}(x^*, y, s, t) - (1 - \delta_v) \pi \tilde{D}(x, y, s, t) - tb(y) & \text{if } x \geq x^*, \\
  w(t) - tb(y) & \text{if } x < x^*. 
\end{cases}
\]

Note that the negligence here behaves similar to the case of negligence in the previous section. We get the following proposition;
Proposition 4 There are two types of equilibria under negligence. (i) The injurer takes stipulated care $x \geq x^*$ and victim will invest $y$ such that the marginal benefit is greater than or equal to zero. The unobservable action of the injurer will be above the optimal level and the victim will choose a less than optimal unobservable action. (ii) The injurer takes less than stipulated care $x < x^*$ and the the victim will invest $y = 0$. The injurer’s unobservable action will be below the optimal level and the victim will choose a higher than optimal unobservable action.

The pessimistic injurer will choose $x = x^*$, since the cost of investing $x^*$ in care will be less than the expected cost of bearing the liability in case $x < x^*$. So the pessimistic injurer will choose $s > s^*$ since he will no longer be liable due to his choice of $x = x^*$. While the victim will choose $t < t^*$, as the pessimistic victim overweights the damage. In case the injurer is optimistic, then the injurer may ignore the possibility of the damage and hence invest $x < x^*$. The optimistic injurer who invests below $x^*$ will select $s < s^*$, since he will be facing liability. The victim will choose $t > t^*$ since the victim will realize that the injurer will choose $x < x^*$ and bear the damage. So the victim will ignore the marginal cost of damage caused due to $t$.

Here we see that the analysis regarding the investment in care is similar to the earlier case of negligence when the damage was not influenced by level of unobservable action. So what matters here is if the injurer is more pessimistic or more optimistic as earlier, where the pessimistic injurer will choose the stipulated care level. The victim in case of a pessimistic injurer will choose a higher level of care than when the injurer is more optimistic. In case the injurer is optimistic, he will ignore the damage in his decision making and therefore the level of care chosen is going to be $x < x^*$. This in turn will result in the victim ignoring the damage in his decision problem since, if $x < x^*$, after the accident the injurer will bear the cost of the accident. As a result this will not only reduce the level of care but also increase the level of unobservable action. So here we can see that if the injurer
is sufficiently optimistic negligence will do quite poorly in terms of social welfare if the stipulated care is the optimal level $x^*$. 

5 Optimal Tort

Shavell (1987) points out that it is not possible to achieve efficient levels of unobservable actions. The problem being that it is not possible to make both parties face the full marginal cost of their actions and balance the budget. So this would mean in our example tort rules, negligence and strict liability, would fail to provide incentives to BP and the victims of the accident to undertake the correct amount of care and level of unobservable action. In this section we show, if players are ambiguity averse ($\alpha_i = \alpha_o = 1$) then using the following liability rule, the optimal care and unobservable action by injurer and the victim can be implemented. We first discuss the case when only the unobservable action affects the damage level $\hat{D}(x, y, s, t)$, and the observed actions are held constant at $x$ and $y$. Note the level of damage is observed by all agents including the court.

Consider a damage rule which imposes a liability on the injurer if the damage is above a certain threshold $\hat{l}$, otherwise the victim is liable. The liability the injurer is held to, is the damage caused $\hat{D}$ and a fine $F$ which is paid by the injurer to the victim. This mechanism gives the optimal level of unobservable action, if the threshold $\hat{l}$ is appropriately chosen. The injurer faces an ambiguous liability if the damage is excessive, as a result he will limit the unobservable action to the optimal level. The ambiguity averse victim faced with the liability will choose his unobservable action in order to limit the size of the damage, and therefore internalize the damage in his unobservable action choice. Here the key thing is the ambiguous liability, which the victim and the injurer
consider while making their decision. The expected utility for the injurer is

\[ u(s) - \delta_i \left[ \tilde{D}(x, y, s, t) + F \right] - (1 - \delta_i) \pi \left[ \tilde{D}(x, y, s, t) + F \right] - sa(x) \text{ if } \tilde{D}(x, y, s, t) \geq \hat{l}, \]

\[ u(s) - sa(x) \text{ if } \tilde{D}(x, y, s, t) < \hat{l}. \]

For the victim, the expected utility, considering budget balance, is

\[ w(t) - \delta_v \tilde{D}(x, y, z, t) - (1 - \delta_v) \pi \left[ \tilde{D}(x, y, s, t) \right] - tb(y) \text{ if } \tilde{D}(x, y, s, t) < \hat{l}, \]

\[ w(t) + (1 - \delta_v)\pi F - tb(y) \text{ if } \tilde{D}(x, y, s, t) \geq \hat{l}. \]

Here the fine \( F \) is perceived as a bad outcome by the injurer and a good outcome by the victim. As a result it gets a positive weight when the injurer makes a decision but zero weight when the victim makes a decision. If the size of \( F \) is chosen correctly then it can induce both, injurer and victim, to choose the correct action. The injurer in order to avoid bearing the loss caused by the damage will choose the optimal level of action such that the size of the loss is not larger than the threshold loss \( \hat{l} \) and the fine is not triggered. Hence \( F \) would be paid only out of equilibrium and in the reduced form would not enter the agents’ utility. Since in equilibrium the victim bears the loss, the victim will choose the optimal level of unobservable action.

**Example 3** Let \( s \in \{\sigma_1, \sigma_2, \sigma_3\} \) and \( t \in \{\tau_1, \tau_2, \tau_3\} \) and let the optimal action \( s \) be \( \sigma_2 \) and optimal \( t \) be \( \tau_2 \). The damage from the unobservable action level is \( \tilde{D}(s, t) \), and \( \hat{l} \) is the threshold. Assume that without ambiguity aversion both the injurer and the victim choose \( \sigma_3 \) and \( \tau_3 \) respectively and assume \( w(\tau_1) = 0 \) and \( u(\sigma_1) = 0 \). Let \( F > 0 \). Choose \( \hat{l} \) such that \( \hat{l} = \tilde{D}(\sigma_3, \tau_3) \). The injurer will face liability if damage is greater than \( \hat{l} \). Since the following hold true

\[ u(\sigma_2) - \sigma_2a(x) \geq u(\sigma_3) - \delta_i \left[ \tilde{D}(\sigma_3, \tau_3) + F \right] - (1 - \delta_i)\pi \left[ \tilde{D}(\sigma_3, t) + F \right] - \sigma_3a(x) \]

\[ u(\sigma_2) - \sigma_2a(x) \geq -\sigma_1a(x) \]
where $t \in \{\tau_1, \tau_2, \tau_3\}$. So the injurer will choose $\sigma_2$. Given $\sigma_2$, the victim will choose $\tau_2$ since

$$w(\tau_2) - \delta_v \tilde{D}(\sigma_3, \tau_2) - (1 - \delta_v)\pi \left[ \tilde{D}(s, \tau_2) \right] - \tau_2 b(y)$$

$$\geq w(\tau_3) - \delta_v \tilde{D}(\sigma_3, \tau_3) - (1 - \delta_v)\pi \left[ \tilde{D}(s, \tau_3) \right] - \tau_3 b(y)$$

and

$$w(\tau_2) - \delta_v \tilde{D}(\sigma_3, \tau_2) - (1 - \delta_v)\pi \left[ \tilde{D}(s, \tau_2) \right] - \tau_2 b(y)$$

$$\geq \delta_v \tilde{D}(\sigma_3, \tau_1) - (1 - \delta_v)\pi \left[ \tilde{D}(s, \tau_1) \right] - \tau_2 b(y).$$

So that for the ambiguity averse victim, the choice will be $\tau_2$. The fine causes the injurer to choose the correct action, which in turn induces the victim to choose the correct action. Note that the threshold is chosen so that is clear that both, injurer and victim, have taken inefficient actions.

### 5.1 With variable care levels

Next we discuss the optimal tort rule if both the observed and the unobserved actions, are included in the analysis. So the liability rule has to be such that the injurer and the victim are induced to invest optimally in care as well as undertake the optimal level of unobservable action.

First, the victim will be liable for all his losses if the injurer takes the stipulated care or the size of the damage is below a threshold. Let us define the loss incurred due to the accident as $L \equiv \tilde{D}(x, y, s, t)$. So the victim with ambiguity averse preferences will have the following expected utility $w(t) - \delta_v \tilde{D}(x^*, y, s, t) - (1 - \delta_v)\pi \tilde{D}(x, y, s, t) - tb(y)$ only if $x \geq x^*$, otherwise he will have $w(t) - tb(y)$. The injurer bears the loss if he fails to take the stipulated care. So in case of $x < x^*$ and there is an accident the injurer faces a liability equivalent to the loss. In addition if the the loss is beyond a threshold $L \geq \hat{L}$, then the injurer faces a liability equivalent to the loss and a punitive fine $F > 0$ even if he has taken stipulated care. So the injurer knows the expected utility he will face if $x < x^*$, is $u(s) - \delta_i \tilde{D}(x, 0, s, \bar{t}) - (1 - \delta_i)\pi \left[ \tilde{D}(x, y, s, t) \right] - sa(x)$ and if the loss observed is $L \geq \hat{L}$ then
the expected utility he will face is $u(s) - \delta_i \left[ \tilde{D}(x, 0, s, t) + F \right] - (1 - \delta_i)\pi \left[ \tilde{D}(x, y, s, t) + F \right] - sa(x)$. The negligence rule that the injurer will face liability for the loss if $x < x^*$ will ensure that the injurer will invest $x^*$. This will be true since the injurer is pessimistic and the expected cost from investing $x^*$ will always be lower than the expected cost from not investing $x^*$ and bearing the liability.

Given that the injurer will invest in stipulated care, we now check if the injurer will undertake the optimal level of unobservable action. Since the injurer faces the punitive penalty and liability $\left( \tilde{D}(x, y, s, t) + F \right)$ if $L \geq \hat{L}$, this threat will result in a pessimistic injurer restricting himself from excessive unobservable action. The proposal works since it induces an ambiguity-averse injurer to take the correct action by making an ambiguous threat. The victim will bear all the losses unless there is excessive loss, $L \geq \hat{L}$, which only occurs out of equilibrium. So for $L < \hat{L}$, the marginal benefit from the care to the victim would be

$$\delta_v \left[ \tilde{D}(x^*, y_2, \bar{s}, t) - \tilde{D}(x^*, y_1, \bar{s}, t) \right] + (1 - \delta_v)\pi \left[ \tilde{D}(x, y_2, s, t) - \tilde{D}(x, y_1, s, t) \right]$$

$$+ t [b(y_2) - b(y_1)] \text{ for } y_1 > y_2$$

and the marginal benefit from increasing the unobservable action level will be

$$[w(t_1) - w(t_2)] + \delta_v \left[ \tilde{D}(x^*, y, \bar{s}, t_2) - \tilde{D}(x^*, y, \bar{s}, t_1) \right] + (1 - \delta_v)\pi \left[ \tilde{D}(x, y, s, t_2) - \tilde{D}(x, y, s, t_1) \right]$$

$$+ [t_2 - t_1] b(y) \text{ for } t_1 > t_2.$$  

For the victim with ambiguity averse preferences, he will invest optimally in care $y^*$ and undertake $t^*$. This will ensure that the victim invests optimally in care and unobservable action.

The analysis here can be related to example of an environmental accident like BP in Gulf of Mexico. An injurer like BP can be induced to take the optimal observable action or care in this
case by the rule of negligence and can be induced to take the optimal level of the unobservable action or activity by the threat of punitive damages. Perhaps the amount BP paid to the local businesses is in excess of the true loss incurred by the businesses.\textsuperscript{15} So the payment may have some punitive component it it.\textsuperscript{16}

**Proposition 5** The tort rule (a) victim is liable for the loss \( L \) below an appropriately set threshold \( \hat{L} \) and if the injurer invests in stipulated care, (b) the injurer is liable for the loss \( L \leq \hat{L} \) if he fails to invest in the stipulated care, (c) the injurer is liable for the loss \( L \) and punitive fine \( F \), if the loss is \( L > \hat{L} \), results in a sufficiently pessimistic injurer investing in the stipulated/optimal care levels, \( x^* \) and \( y^* \) respectively.

So we find that here while the negligence rule gives optimal care levels by the agents, a punitive fine borne by the injurer in case of excessive damage is results in optimal level of unobservable action from the injurer and the victim if they are sufficiently pessimistic\textsuperscript{17}. Since the unobservable action level is not observed by the court or the third party, a high enough fine levied on the pessimistic injurer will restrict the injurer’s unobservable action level while the negligence rule will induce the injurer to take stipulated or optimal care. So if we have agents who are ambiguity averse, we can get the optimal levels of care and unobservable action if in addition to negligence, punitive damages are used. We should note that ex-post optimality is achieved, and this is due to the threat of the punitive fine \( F \) which induces the injurer to choose the optimal unobservable

\textsuperscript{15}In the litigation after the Exxon Valdez oil spill the punitive damage imposed on Exxon was around five billion dollars which on appeal more recently the US Supreme Court has reduced it to half a billion dollars.(Exxon Shipping Co. v. Baker, 554 U.S. 471 (2008))

\textsuperscript{16}Note that this analysis can be extended to a multi-agent/victim similar to Kelsey and Spanjers (2004).

\textsuperscript{17}A related point is made in Kelsey and Spanjers (2004), who show that ambiguity may increase the set of outcomes which can be implemented in a context of team production. They show that with non-additive beliefs, it is possible to make both parties face the full marginal cost of their actions.
action level. Note this rule may not be ex-ante optimal, taking into account the any utility loss due to ambiguity aversion. Here the analysis assumes the agents are ambiguity averse, but this result can be extended to ambiguity-seeking if ambiguous rewards are used instead of ambiguous punishments.

Returning to our previous example of the BP oil spill. We can see how the liability rule may apply. If the damage is not substantial then the liability is borne by the victims as long as BP takes stipulated care, but if the damage is extremely large then BP would not only bear the liability but also a substantial fine. Further, even if the fine and the threshold is not set optimally the unobservable action and the care or observable action close to the optimal level can be achieved.

6 Conclusion

In an analysis of tort rules Shavell (1987) shows negligence and strict liability give efficient investment in care if the stipulated level of care is set at the correct level. At the same time, if the levels of unobservable action are included in the analysis, then with stipulated care equal to the optimal level, we cannot get optimal investment in care and optimal level of unobservable action by both the injurer and the victim. Here motivated by environmental accidents we analyse tort rules when the potential injurer and the victim perceives ambiguity and show that it is possible to design a liability scheme where both injurer and victim undertake optimal care and unobservable action.

In the first part of our analysis, without unobservable action, we find that with ambiguity, with strict liability, injurers invest more than optimal in care. With negligence, in case of pessimistic players, we see the optimal care by the injurer may be obtained. This is due to the fact that the pessimistic injurer will find it less costly to invest in the stipulated care than the expected cost when

\footnote{In a related model Kelsey and Spanjers (2004) show it can be possible to achieve ex ante optimality.}
he is liable. Stipulated care protects the injurer from ambiguity. But if the agents are optimistic then they may ignore the possibility of the damages and fail to take adequate care. So for social welfare, if the agents are pessimistic then negligence dominates strict liability.

From the analysis with unobservable actions, we find that if strict liability is used, ambiguity causes the injurer to restrict the amount of unobservable action. With ambiguity the injurer puts extra weight on the likelihood of the damage and this will increases its marginal cost. While the unobservable action may not be optimal, there may be an improvement over the case without ambiguity. For negligence, if the injurer is pessimistic then he will invest optimally in care but will increase his unobservable action. The victim will over invest in the care but will also reduce the unobservable action. Interestingly if the injurer is optimistic and does invest less than the optimal level in care, then the victim will not invest in any observable action and also increases the unobservable action. This suggests that with sufficiently optimistic injurers, negligence does not seem to work well, while with pessimistic injurers negligence seems to do better in terms of social welfare, subject to the earlier comments made on negligence and optimism.

Finally we present our result where we show that in case both injurers and victims perceive ambiguity then it is possible to get optimal care and unobservable action from both victim and injurer with a rule which couples negligence and punitive damages. In this section we assume the agents to be ambiguity averse. The injurer takes optimal care since he faces liability if he fails to take stipulated care and he restricts his unobservable action since if the damage is very large he faces a very large punitive damage. The victim will invest in care and restrict unobservable action since he is like to incur the damage from the accident if the injurer has undertaken stipulated care and the damage is not too large. In this liability scheme it is the threat of the large fine which induces optimal behaviour. The fine is not used in equilibrium. While the size of the optimal fine
may be difficult for the regulator or courts to compute, if the fine is significantly large then the use of the fine will be a Pareto improvement over not using it. The other qualification we would like to point out is that a large fine may lead to the ambiguity-averse injurer closing the business or not carrying out any activity.

Here we have attempted to raise the question how behavioural assumptions other than SEU may affect the analysis of optimal tort rule. Much of the law and economics literature assumes decision makers are SEU maximizers. However evidence such as Ellsberg (1961) suggests that decision makers may fail to act as SEU maximizers when probabilities are unknown. The law should deal with actual individual decisions and not idealised rational decisions. Hence we need to study how legal rules are affected by behavioural issues. Therefore to relax the assumption of SEU decision makers and replace it with decision makers who perceive ambiguity may be specifically relevant in case of tort. Since the decision maker, either the injurer or the victim, may not be aware of the probability distribution of accident and therefore this ambiguity will affect the optimal tort law. We find that the standard result that strict liability and negligence are efficient is no longer true but may be differently effective depending on ambiguity attitude of the agents. Instead we can show that a negligence rule coupled with punitive damages for excessive loss can give optimal care. Analysing legal rules by using a decision making model other than SEU will throw more light on the effectiveness of the legal rules.

References


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7 Appendix

Here is the definition of the equilibrium of the game when players can be ambiguity-seeking or ambiguity-averse. Consider a game, $\Gamma = \langle (i, v), (S_j), (u_j) \rangle$, between players $i$ and $v$, where each player $j$ has a finite strategy set $S_j$, and pay-off function $u_j(s_j, s_{-j})$, $s_{-j}$ is the notation used to denote the other player. The capacity on $S_j$ is denoted by $\nu_j$ and $R_j(\nu_j)$ denotes the best response of player $j$ given beliefs $\nu_j$ such that $R_j(\nu_j) = \arg \max_{s_j \in S_j} \int u_j(s_j, s_{-j}) \, d\nu_j(s_{-j})$. Finally, $\text{supp}\, \hat{\nu}_j = \text{supp}\, \hat{\pi}_j$, for justification and full characterization of the equilibrium see Eichberger and Kelsey (2009).

**Definition 1** For a game $\Gamma$ with externalities, an equilibrium is a pair of neo-additive capacities $\nu = \langle \nu_i, \nu_v \rangle$, where $\nu_j = \delta_j(1 - \alpha_j) + (1 - \delta_j)\pi$ for $j = i, v$ such that $\emptyset \neq \text{supp}\, \hat{\nu}_j \subseteq R_j(\nu_j)$. For each player $\hat{s}_j \in \text{supp}\, \hat{\nu}_j$, then $\hat{s} = \langle \hat{s}_i, \hat{s}_v \rangle$ is an equilibrium strategy profile.

**Proof of Proposition 1:** With ambiguity the expected losses for the injurer is $L^\nu_i(x, y) = (\delta_i\alpha_i + (1 - \delta_i)\pi)D(x, y) + a(x)$ he/she will increase $x$ as long as the marginal benefit from $x$ is $MB^\alpha_i \geq 0$. If $\delta_i\alpha_i + (1 - \delta_i)\pi > \pi$, the marginal cost to the CEU injurer is greater than $ML_x$, the marginal cost for the subjective expected utility maximizer, so $x > x^*$. Given the injurer bears the damage, the victim’s choice is given by $y$ as long as $MB^\alpha_v \geq 0$, or $y = 0$.

**Proof of Proposition 2:** Under negligence, the expected loss for the injurer and the victim is $L^\nu_i(x, y)$ and $L^\nu_v(x, y)$ respectively. The injurer will choose $x^*$ if

$$(\delta_i\alpha_i + (1 - \delta_i)\pi)D(x, y) + a(x) > a(x^*), x \neq x^*.$$ 

The victim’s investment in care is given by $MB^\alpha_v \geq 0$ and this implies $y = 0$. If the injurer is pessimistic, $\alpha = 1$ and $\delta \to 1$, then the loss function of the injurer is $D + a(x) > a(x^*)$. So the
completely pessimistic injurer will invest in stipulated \( x^* \) in care. In the case where the injurer is optimistic, \( \alpha = 0 \) and \( \delta \rightarrow 1 \), then \( a(x) < a(x^*) \) thus \( x < x^* \). If the victim knows that the injurer is optimistic then the care taken by the victim will be \( y = 0 \). And if the injurer is pessimistic then the victim will invest \( y \) such that \( MB_{vA}^* \geq 0 \) or \( y^* \) if \( (\alpha \epsilon_\epsilon + (1 - \delta) \pi) D(x, y) + b(y) > b(y^*) \).

**Proof of Proposition 3:**

The utilities, under strict liability, are,

\[
U_{iA}^s(s, t, x, y) = u(s) - \alpha_i \delta_i \tilde{D}(x, 0, s, t) - (1 - \delta_i) \pi \tilde{D}(x, y, s, t) - sa(x).
\]

\[
U_{vA}^s(s, t, x, y) = w(t) - tb(y).
\]

The injurer will choose \( s > 0 \) and will increase the unobservable action as long as \( MB_{iA}^s(s) \geq 0 \). But the level of unobservable action, \( s \), is lower than the optimal \( s^* \), since under ambiguity the injurer takes into account a higher marginal cost. The victim is going to choose \( t \) such that \( MB_{vA}^s(t) \geq 0 \) and this \( t > t^* \).

The injurer will increase the amount of care if \( MB_{iA}^s(x) \geq 0 \). If \( \delta_i > 0 \) and \( s > 0 \), then the marginal cost the injurer considers is higher than the actual marginal cost incurred. The victim invests \( y = 0 \), since the victim’s marginal benefit from increase in care is zero.

**Proof of Proposition 4:** The marginal benefit for change in \( x, y, s \) and \( t \), for \( x_1 > x_2, y_1 > y_2, t_1 > t_2 \) and \( s_1 > s_2 \) are given below. Note these are derived such that when choosing \( x \) and \( y \), unobservable actions \( s \) and \( t \) are taken as given, and when choosing \( s \) and \( t \), the levels of care \( x \) and \( y \), are taken as given.

\[ x : \alpha_i \delta_i \left[ \tilde{D}(x_2, 0, s, t) - \tilde{D}(x_1, 0, s, t) \right] + (1 - \delta_i) \pi \left[ \tilde{D}(x_2, y, s, t) - \tilde{D}(x_1, y, s, t) \right] + s \left[ a(x_2) - a(x_1) \right] \quad \text{if} \quad x_1 < x^*. \]
When agents choose unobservable action levels under negligence, the payoffs are $U_{Ax}^{n}(s, t, x, y)$ and $U_{vA}^{n}(s, t, x, y)$. Given a level of $x$ and $y$, the injurer chooses $s$ and the victim selects $t$. If $x \geq x^*$, then the level of unobservable action $s$ will be chosen such that the marginal benefit from the unobservable action $[u(s_1) - u(s_2)] + [s_2 - s_1]a(x) \geq 0$ and $s > s^*$. And in case $x < x^*$, then $s$ will be such that the marginal benefit, $[u(s_1) - u(s_2)] + \alpha_i \delta_i \left[ \tilde{D}(x, 0, s_2, \bar{t}) - \tilde{D}(x, 0, s_1, \bar{t}) \right] + (1 - \delta_i) \pi \left[ \tilde{D}(x, s_2, t) - \tilde{D}(x, s_1, t) \right] + [s_2 - s_1]a(x) \geq 0$. If $x \geq x^*$, the victim will choose $t < t^*$ as the pessimistic victim overweights the damage and therefore considers an increased marginal cost of $t$. In case $x < x^*$, then $t > t^*$. 

$$s \left[ a(x_2) - a(x_1) \right] \text{ if } x_2 \geq x^*,$$

$$y : \alpha_v \delta_v \left[ \tilde{D}(x^*, y_2, \bar{s}, t) - \tilde{D}(x^*, y_1, \bar{s}, t) \right] + (1 - \delta_v)\pi \left[ \tilde{D}(x, y_2, s, t) - \tilde{D}(x, y_1, s, t) \right] + t [b(y_2) - b(y_1)] \text{ if } x \geq x^*,$$

$$t [b(y_2) - b(y_1)] \text{ if } x < x^*,$$

$$s : [u(s_1) - u(s_2)] + \alpha_i \delta_i \left[ \tilde{D}(x, 0, s_2, \bar{t}) - \tilde{D}(x, 0, s_1, \bar{t}) \right] + (1 - \delta_i)\pi \left[ \tilde{D}(x, y, s_2, t) - \tilde{D}(x, y, s_1, t) \right] + [s_2 - s_1]a(x) \text{ if } x < x^*,$$

$$[u(s_1) - u(s_2)] + [s_2 - s_1]a(x) \text{ if } x \geq x^*,$$

$$t : [w(t_1) - w(t_2)] + \alpha_v \delta_v \left[ \tilde{D}(x^*, y, \bar{s}, t_2) - \tilde{D}(x^*, y, \bar{s}, t_1) \right] + (1 - \delta_i)\pi \left[ \tilde{D}(x, y, s, t_2) - \tilde{D}(x, y, s, t_1) \right] + [t_2 - t_1]b(y) \text{ if } x \geq x^*,$$

$$[w(t_1) - w(t_2)] + [t_2 - t_1]b(y) \text{ if } x < x^*.$$
The injurer will invest in care $x^*$ if $-s\alpha(x^*) < -\alpha_i\delta_i\tilde{D}(x, 0, s, \bar{t}) - (1 - \delta_i)\pi\tilde{D}(x, y, s, t) - sa(x)$. If $s > 0$ and the injurer is pessimistic (in the extreme $\alpha = 1, \delta > 0$) then the injurer will choose $x^*$. The victim therefore will choose $y$ such that $\alpha_i\delta_i\left[\tilde{D}(x, y_2, s, t) - \tilde{D}(x, y_1, s, t)\right] + (1 - \delta_i)\pi\left[\tilde{D}(x, y, s, t) - \tilde{D}(x_1, y, s, t)\right] + t[b(y_2) - b(y_1)] \geq 0$. In case the injurer is optimistic (in the extreme $\alpha > 0, \delta = 0$), he ignores the possibility of the damage, and the injurer chooses $x < x^*$ such that $\alpha_i\delta_i\left[\tilde{D}(x_2, 0, s, \bar{t}) - \tilde{D}(x_1, 0, s, \bar{t})\right] + (1 - \delta_i)\pi\left[\tilde{D}(x_2, y, s, t) - \tilde{D}(x_1, y, s, t)\right] + s[a(x_2) - a(x_1)] \geq 0$. The victim therefore knows that the injurer will be liable and selects $y = 0$ since the decision is given by the condition $t[b(y_2) - b(y_1)] \geq 0$.

**Proof of Proposition 5:**

The pay-off of the victim is

$$w(t) - \delta_i\tilde{D}(x^*, y, \bar{t}, s, t) - (1 - \delta_i)\pi\tilde{D}(x, y, s, t) - tb(y) \text{ if } x \geq x^* \text{ and } L \leq \hat{L},$$

$$w(t) - tb(y) \text{ if } x < x^*,$$

$$w(t) - tb(y) - F \text{ if } L \geq \hat{L}.$$

The pay-off of the injurer is

$$u(s) - sa(x) \text{ if } \left(x \geq x^* \text{ and } L \leq \hat{L}\right),$$

$$u(s) - \delta_i\left[\tilde{D}(x, 0, s, t)\right] - (1 - \delta_i)\pi\left[\tilde{D}(x, y, s, t)\right] - sa(x) \text{ if } x < x^* \text{ and } L < \hat{L},$$

$$u(s) - \delta_i\left[\tilde{D}(x, 0, s, t) + F\right] - (1 - \delta_i)\pi\left[\tilde{D}(x, y, s, t) + F\right] - sa(x) \text{ if } L \geq \hat{L}.$$

The injurer will invest in care $x^*$ if

$$u(s) - sa(x^*)\leq u(s) - \delta_i\left[\tilde{D}(x, 0, s, t)\right] - (1 - \delta_i)\pi\left[\tilde{D}(x, y, s, t)\right] - sa(x).$$
So a sufficiently pessimistic injurer will always find it beneficial to invest in care levels \( x^* \). If the damage level is too high, \( L \geq \hat{L} \), then the expected utility includes the fine \( F \). Hence

\[
u(s) - \delta_i \left[ \hat{D}(x, 0, s, \bar{t}) + F \right] - (1 - \delta_i) \pi \left[ \hat{D}(x, y, s, t) + F \right] - sa(x) .\]

The injurer is going to select \( s \) as long as \( MB_i \geq 0 \). If \( F \) can be set such that marginal benefit, \( MB_i \), is equal to \( MS_s \) at the efficient level of \( s^* \), then the level of \( s \) is going to be equal to the optimal level. Given that injurer is going to invest \( x^* \), the victim has an expected cost

\[
w(t) - \delta_v \hat{D}(x^*, y, \bar{s}, t) - (1 - \delta_v) \pi \hat{D}(x, y, s, t) - tb(y) .\]

Assume that the ambiguity perceived by the victim is not too high. So the victim’s marginal benefit from choosing \( t_1 > t_2 \) and \( y_1 > y_2 \) is

\[
\begin{align*}
t & : w(t_1) - w(t_2) + \delta_v \left[ \hat{D}(x^*, y, \bar{s}, t_2) - \hat{D}(x^*, y, \bar{s}, t_1) \right] + \\
& \quad (1 - \delta_v) \pi_v \left[ \hat{D}(x, y, s, t_2) - \hat{D}(x, y, s, t_1) \right] + (t_2 - t_1) b(y), \text{ if } L < \hat{L}, \\\n& \quad w(t_1) - w(t_2) + (t_2 - t_1) b(y) \text{ otherwise.}
\end{align*}
\]

\[
\begin{align*}
y & : \delta_v \left[ \hat{D}(x^*, y_2, \bar{s}, t) - \hat{D}(x^*, y_1, \bar{s}, t) \right] + \\
& \quad (1 - \delta_v) \pi \left[ \hat{D}(x, y_2, s, t) - \hat{D}(x, y_1, s, t) \right] + t \left[ b(y_2) - b(y_1) \right], \text{ if } L < \hat{L}, \\
& \quad t \left[ b(y_2) - b(y_1) \right] \text{ otherwise.}
\end{align*}
\]

The injurer will choose care and unobservable action levels such that the size of the damage, \( L < \hat{L} \).

The victim will increase \( t \) and \( y \) as long as the respective marginal benefit is greater than or equal to zero. The levels of marginal condition for the optimal choice, \( t^* \) and \( y^* \), is given by

\[
w(t^*) - w(t) + \pi \left[ \hat{D}(x, y, s, t) - \hat{D}(x, y, s, t^*) \right] + b(y) \geq 0 .\]
$$t [b(y) - b(y^*)] \geq -\pi \left[ \tilde{D}(x, y^*, s, t) - \tilde{D}(x, y, s, t) \right].$$

So if $L < \hat{L}$, and the victim will choose $y = y^*$ and $t = t^*$ as long as he/she is at least ambiguity neutral and not ambiguity loving.