

Contests with Ambiguity

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 - Tullock argues that the entire value of the prize will be expended during the contest.
 - In practice it seems that rent dissipation is significantly less than 100%.

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- Contestant A 's utility function:

$$u_A(x_A, x_B) = \frac{x_A}{x_A + x_B} V - x_A.$$

Nash Equilibrium

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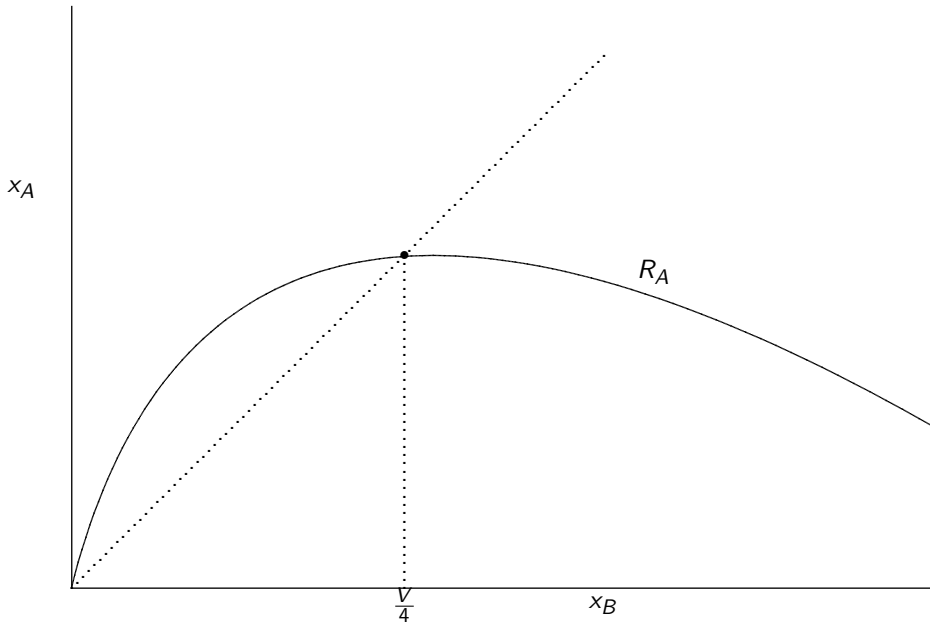
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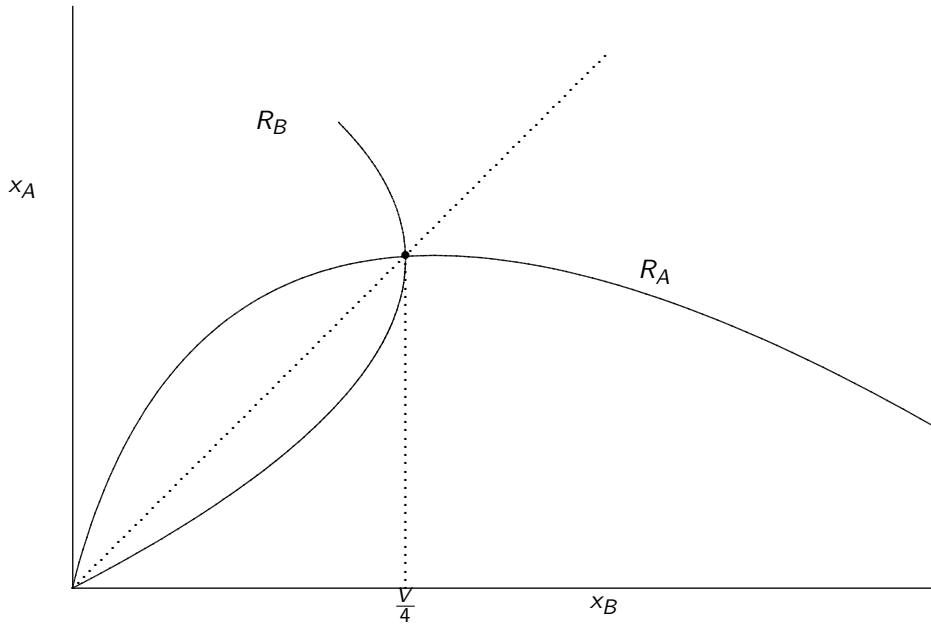
One can derive A 's best response function which is:

$$x_A = \sqrt{Vx_B} - x_B.$$

The best response function has the following properties:

- it is inverse U-shaped, (single peaked);
- the peak occurs where it crosses the 45° line;
- it is above (resp. below) the 45° line before (resp. after) the peak;
- There is a unique Nash equilibrium where $x_A^* = x_B^* = \frac{V}{4}$;
- half of the rent is dissipated in Nash equilibrium.





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- **Ambiguity is represented by assigning a set of probabilities to an event, e.g. the probability of winning the war is between 0.5 and 0.7.**

The Neo-additive Model of Ambiguity

We use the neo-additive model of ambiguity, axiomatised by Chateauneuf, Eichberger, and Grant (2007). They represent preferences by:

$$\alpha \delta M(a) + \delta (1 - \alpha) m(a) + (1 - \delta) \mathbf{E}_{\pi} u(a), \quad (1)$$

- $M(a)$ denotes the maximum utility of act a ,
- $m(a)$ denotes the minimum utility of act a ,
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- Only 2 additional parameters needed compared to expected utility.

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- Since ambiguity increases both optimism and pessimism it reduces the marginal benefit of effort.
- **The other major influence on behaviour is the intensity of competition. If one's opponents are providing similar effort levels competition is intense, which increases the incentive to provide effort.**

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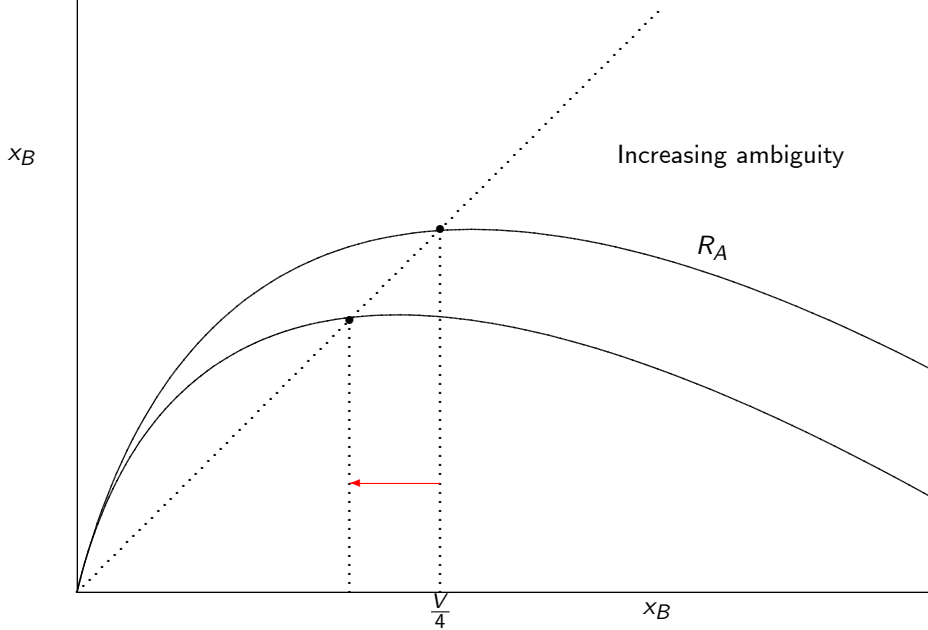
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Proposition

Assume $\delta_A = \delta_B = \hat{\delta} > 0$ and $\alpha_A = \alpha_B = \hat{\alpha}$. Then:

- 1 a symmetric equilibrium exists and is unique;
- 2 there is less effort than in Nash equilibrium;
- 3 the equilibrium effort level, \hat{x} , is a strictly decreasing function of $\hat{\delta}$.



Ambiguity-Attitude

The effect of a change in ambiguity-attitude is summarised by the following result.

Proposition

Consider the symmetric case where $\delta_A = \delta_B = \hat{\delta}$ and $\alpha_A = \alpha_B = \hat{\alpha}$ then an increase in ambiguity-aversion $\hat{\alpha}$ will reduce equilibrium effort provided $\lambda\kappa V^2 - \hat{\alpha}^2 > 0$.

Remark

Suppose that $\kappa \geq \frac{1}{16\lambda}$, then $\lambda\kappa V^2 - \hat{\alpha}^2 \geq \frac{V^2}{16} - \hat{\alpha}^2 > 0$ since $x_A < \frac{V}{4}$ by Proposition 4.1. Henceforth we shall assume $X_A = X_B = [\kappa V, \lambda V]$, where $\lambda > \frac{1}{4}, \frac{1}{4} > \kappa \geq \frac{1}{16\lambda}$.

Thus an increase in optimism (ambiguity-loving) usually leads lead to higher effort.

An decrease in α shifts decision weight from the worst outcome to the best outcome.

The inequality $\kappa \geq \frac{1}{16\lambda}$ implies that the best case is not too good.

Asymmetric Perceptions of Ambiguity

- Continue to assume the prize has the same value for both players.
- However we allow for asymmetric perceptions of ambiguity $\delta_A \neq \delta_B$ and different ambiguity-attitudes $\alpha_A \neq \alpha_B$.
- With ambiguity both players provide less than the Nash equilibrium level of effort.
- This is a possible explanation of why rent dissipation is not complete.

Proposition

Assume that both players perceive ambiguity, $1 \geq \delta_A > 0, 1 \geq \delta_B > 0$. Then in equilibrium both will make less than the Nash equilibrium level of contributions.

This result is not true if the value of winning is different for the two players, $V_A \neq V_B$.

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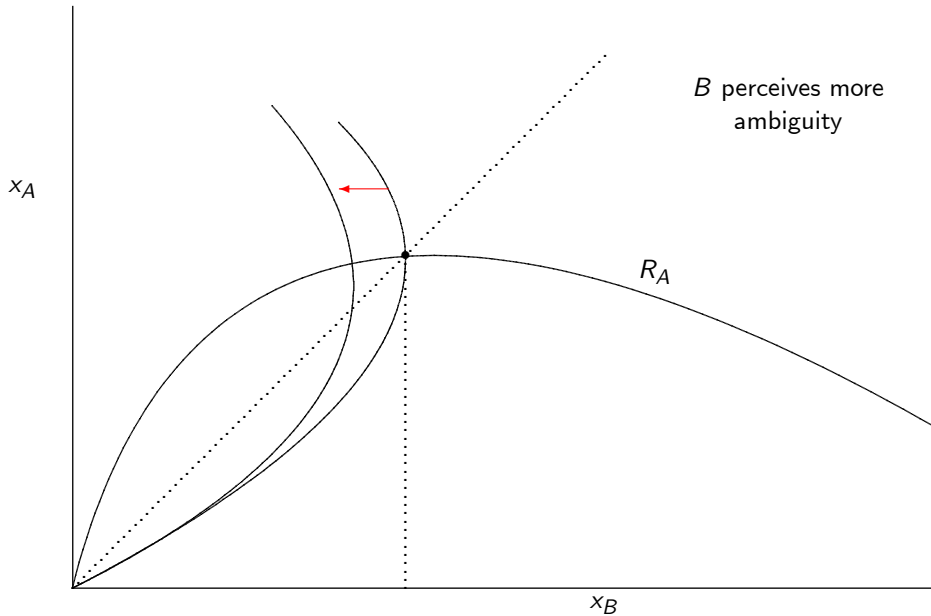
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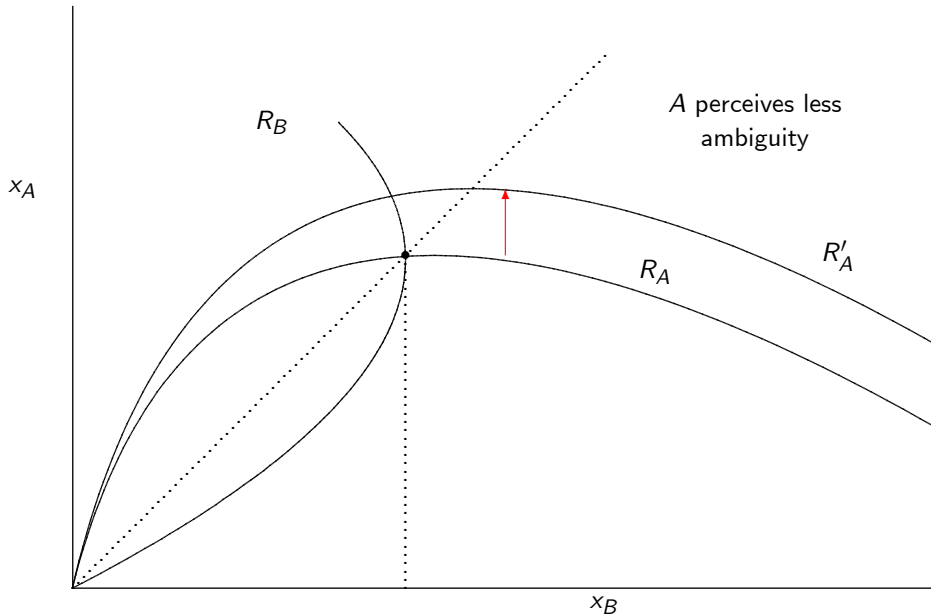
Comparative Statics of Ambiguity II

- Starting at a symmetric equilibrium assume that Player A perceives less ambiguity.
- This causes A 's equilibrium effort to rise.
- Player B 's effort will **fall** since the competition from A has become less intense. The competition is now biased in A 's favour, which reduces B 's marginal benefit of effort.

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- Recall Fudenberg and Tirole have decomposed the consequences of appointing an agent into a strategic effect and a direct effect.
- By the envelope theorem the direct effect is negligible for small changes.
- A more ambiguity-averse agent will provide less effort than you would. This has the strategic advantage of inducing your rival to expend less effort, which has a positive effect on your pay-off.

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- Directions for future research.
 - Other behavioural biases, e.g. overconfidence, loss aversion.
 - Can the results be generalised to a larger class of games, e.g. all games of aggregate externalities where marginal benefit is single peaked?

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