Strategic Ambiguity and Decision-making: An Experimental Study*

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Abstract

We conducted a set of experiments to compare the effect of ambiguity in single person decisions and games. Our results suggest that ambiguity has a bigger impact in games than in ball and urn problems. We find that ambiguity has the opposite effect in games of strategic substitutes and complements. This confirms a theoretical prediction made by Eichberger and Kelsey (2002). In addition, we note that subjects' ambiguity-attitudes appear to be context dependent: ambiguity-loving in single person decisions and ambiguity-averse in games. This is consistent with the findings of Kelsey and le Roux (2015).
Keywords: Ambiguity; Choquet expected utility; strategic complements; strategic substitutes; Ellsberg urn.

JEL Classification: **C**72, **C**91, **D**03, D81

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1 Introduction

We report an experimental study on the effects of ambiguity in single person decisions and games. Risks are said to be ambiguous if the probabilities of possible outcomes are unknown and it is difficult or impossible to assign subjective probabilities to them. There exists a substantial body of experimental literature which shows that ambiguity affects single person decisions. Most economic decisions, however, are not made by individuals, but by groups of individuals involved in strategic interactions. There is a small experimental literature studying ambiguity in games.[1] However, most of these papers do not test specific theories of behaviour in ambiguous games. Since many economic problems can be represented as games we believe this research will be useful for understanding how ambiguity affects the behaviour of economic systems.

Our paper is an experimental test of Eichberger and Kelsey (2002), which predicts that ambiguity has opposite effects in games of strategic complements and substitutes. In the case of strategic substitutes, increasing the level of ambiguity would shift the equilibrium strategies in an ex-post Pareto improving direction, whereas for strategic complements, an increase in ambiguity would have the opposite effect.[2] Thus it was hypothesised that ambiguity had an adverse effect in games with strategic complements, but was helpful in attaining an ex-post Pareto efficient outcome in games with strategic substitutes.

In addition we alternated the games with Ellsberg Urn type decision problems. This was done in order to test whether there was any difference in ambiguity-attitude between the types of decision. Moreover, it allowed us to elicit an independent measure of subjects' ambiguity-attitudes. Finally it acted to erase subjects' short term memory, so that they made decisions for each game round independently and without recall of their action in the previous game round.

[1] See for instance Calford (2016), Eichberger, Kelsey, and Schipper (2008), Ivanov (2011), Di Mauro and Castro (2011), Kelsey and le Roux (2015) or Kelsey and le Roux (2016).

[2] We refer to an ex-post Pareto improvement since this efficiency measure does not take into account any ex-ante losses in utility due to ambiguity-aversion.

Organisation of the Paper In Section 2, we describe the theory being tested. Section 3 and 4 describe the experimental model and design employed, Section 5 consists of data analysis and results, Section 6 reviews related literature and Section 7 provides a summary of results together with future avenues of research.

2 Preferences and Equilibrium under Ambiguity

In this section we shall explain how we model ambiguity in games. Our notation is as follows. A 2-player game $\Gamma = \langle \{1,2\}; X_1, X_2, u_1, u_2 \rangle$ consists of players, $i = 1, 2$, finite pure strategy sets X_i and payoff functions $u_i(x_i, x_{-i})$ for each player. The notation, x_{-i}, denotes the strategy chosen by i's opponent and the set of all strategies for i's opponent is X_{-i}. The space of all strategy profiles is denoted by X.

We shall model ambiguity by neo-additive preferences which have been axiomatised by Chateauneuf, Eichberger, and Grant (2007). These preferences may be represented by the function:

$$V_i(x_i) = \delta_i \alpha_i \min_{x_{-i} \in x_{-i}} u_i(x_i, x_{-i}) + \delta_i (1 - \alpha_i) \max_{x_{-i} \in X_{-i}} u_i(x_i, x_{-i}) + (1 - \delta_i) \mathbf{E}_{\pi_i} u_i(x_i, x_{-i}),$$

where $\mathbf{E}_{\pi_i}$ denotes conventional expectation with respect to the probability distribution π_i. This expression is a weighted average of the highest payoff, the lowest payoff and an average payoff. The response to ambiguity is partly optimistic represented by the weight given to the best outcome and partly pessimistic. These preferences are a special case of Choquet expected utility (CEU), Schmeidler (1989). Thus they may also be represented in the form

$$V_i(x_i) = \int u_i(x_i, x_{-i}) \, d\nu_i(x_{-i}),$$

where ν_i is a neo-additive capacity on X_{-i} and the integral is a Choquet integral, Choquet (1953-4).[3] We define the support of a neo-additive capacity to be the support of the additive

[3] A *neo-additive*-capacity ν_i on X_{-i} is defined by $\nu_i(X_{-i} | \alpha_i, \delta_i, \pi_i) = 1$, $\nu_i(\varnothing | \alpha_i, \delta_i, \pi_i) = 0$ and $\nu_i(A | \alpha_i, \delta_i, \pi_i) = (1 - \alpha_i)\delta_i + (1 - \delta_i)\pi_i(A)$ for $\emptyset \subsetneqq A \subsetneqq X_{-i}$, where $0 \leqslant \delta_i < 1$, π_i is an additive prob-

probability on which it is based, i. e. $\operatorname{supp} \nu_i = \operatorname{supp} \pi_i$.[4]

A player has a possibly ambiguous belief about what his/her opponent will do. In games, π_i is determined endogenously as the prediction of the players from the knowledge of the game structure and the preferences of others. In contrast, we treat the degrees of optimism, α_i and ambiguity, δ_i, as exogenous. Define the best-response correspondence of player i given that his/her beliefs are represented by a neo-additive capacity ν_i by $R_i(\nu_i) = \operatorname{argmax}_{x_i \in X_i} V_i(x_i)$. We can now define equilibrium under ambiguity.

Definition 2.1 (Equilibrium under Ambiguity) *A pair of neo-additive capacities (ν_1^*, ν_2^*) is an Equilibrium Under Ambiguity (EUA) if for $i = 1, 2$, $\operatorname{supp}(\nu_i^*) \subseteq R_{-i}(\nu_{-i}^*)$.*

An EUA will exist for any given ambiguity-attitude of the players, (see Eichberger and Kelsey (2014) for a proof). In equilibrium each player uses a strategy which is a best response given his/her beliefs. A player perceives ambiguity about the strategy of his/her opponent. This is represented by an ambiguous belief, which takes the form of a capacity over the opponent's strategy space. The support of a player's beliefs is itself an ambiguous event. This reflects some uncertainty about whether or not his/her opponents play best responses. Players respond to this ambiguity partly in an optimistic way by over-weighting the best outcome and partly in a pessimistic way by over-weighting the worst outcome.

Consistency between beliefs and actions is achieved by requiring that all strategies in the support of a player's beliefs be a best response for his/her opponent. This is a weaker notion of consistency than that used in Nash equilibrium (NE). However Greiner (2016) has experimental evidence which shows that behaviour does not satisfy stronger notions of consistency even in relatively simple games. Moreover the violations of consistency he finds are compatible with our theory.

A common interpretation of NE is that each player chooses a strategy which maximises his/her utility given the strategy of the other players. However it is also possible to view NE as an equilibrium in beliefs. From this viewpoint each player has a subjective belief about

ability distribution on X_{-i}.

[4]For a justification of this definition and its relation to other support notions see Eichberger and Kelsey (2014).

the actions of his/her opponents and chooses a best response to this belief. Definition 2.1 extends the interpretation of NE as an equilibrium in beliefs, by allowing these beliefs to be non-additive. We interpret the deviation from additivity as representing ambiguity about the opponent's strategy choice.

3 Experimental Model

In this section, we introduce the games used in our experiments, followed by the Ellsberg-style decision problems being studied by us. Henceforth we will use male pronouns he, his etc. to denote the Row Player, while female pronouns she, hers etc. will be used to denote the Column Player.[5]

3.1 The Games

A game has strategic complements if, when a given player increases his/her strategy, this gives other players an incentive to increase their strategies as well. Equivalently reaction functions are increasing. The notion of strategic complements is only generally defined for two player games. Strategic substitutes games have the opposite situation, where when one player increases his/her strategy, the opponent has an incentive to play a lower strategy. These terms were coined by Bulow, Geanakoplos, and Klemperer (1985). A game has positive (resp. negative externalities) if when a given player increases his/her strategies the pay-off of the remaining players increase (resp. decrease). The concepts of strategic complements and positive externalities both rely on there being a suitable ordering on the strategy spaces, e.g. if the strategies are described by real numbers as in a Cournot game.

The games used in our experimental sessions can be seen in Figure 1. Games ($SC1$) and ($SC2$) (as labelled in Figure 1) are games with strategic complements and positive externalities; while Games ($SS1$) and ($SS2$), were games with strategic substitutes. Game SS1 also has negative externalities.

[5]This convention is for the sake of convenience only and does not bear any relation to the actual gender of the subjects in our experiments.

Figure 1: The Games

(SC1)	Left	Middle	Right
Top	90, 85	90, 95	90, 0
Centre	0, 90	110, 95	110, 0
Bottom	5, 105	105, 95	105, 110

(SC2)	Left	Middle	Right
Top	115, 95	115, 100	115, 0
Centre	0, 95	135, 100	135, 0
Bottom	5, 105	105, 100	105, 135

(SS1)	Left	Middle	Right
Top	0, 0	70, 0	170, 150
Centre	100, 70	165, 165	165, 150
Bottom	150, 170	150, 165	150, 150

(SS2)	Left	Middle	Right
Top	95, 5	5, 0	5, 90
Centre	0, 5	100, 100	100, 90
Bottom	90, 95	90, 100	90, 90

The following result is the main theoretical prediction, which our experiments are designed to test. A formal proof can be found in Eichberger and Kelsey (2014).

Proposition 3.1 *If both players are ambiguity averse, i.e.* $\alpha = 1$, *and have neo-additive preferences then:*[6]

1. *In the case of games with strategic complements and positive (resp. negative) externalities, the equilibrium strategy under ambiguity of an agent with neo-additive beliefs, is decreasing (resp. increasing) in ambiguity.*

2. *In the case of games with strategic substitutes and negative (resp. positive) externalities, the equilibrium strategy under ambiguity of an agent with neo-additive beliefs, is decreasing (resp. increasing) in ambiguity.*

Games $(SC1)$ and $(SC2)$ are games with strategic complements and positive externalities. This can be verified if we fix the order $T < C < B$ and $L < M < R$. Both games have one pure Nash equilibrium: (C, M). The equilibrium under ambiguity for these games, when $\alpha = 1$, is (T, M).

Game $(SS1)$ is a strategic substitutes game with negative externalities and multiple Nash equilibria, if we fix $T > C > B$ and $L > M > R$. The game has three pure Nash equilibria: (T, R), (C, M) and (B, L), none of which are focal. The equilibrium under ambiguity for this

[6] It might be worth noting that Eichberger and Kelsey (2014) present a stronger result for more general CEU preferences.

game, when $\alpha = 1$, is (B, R). Game $(SS2)$ is a strategic substitutes game if we fix $T > C > B$ and $L > M > R$. The game has a unique Nash equilibrium: (C, M). The equilibrium under ambiguity for this game, when $\alpha = 1$, is (B, R). For both strategic substitutes games the Nash equilibrium Pareto dominates the equilibrium with ambiguity.

3.2 Ellsberg Urn Experiments

The game rounds were alternated with single person decision problems similar to the Ellsberg Urn experiment. Subjects were presented with an urn containing 90 balls, of which 30 were labelled X, and the remainder were an unknown proportion of Y or Z balls. The decisions put to the subjects took the following form:

"An urn contains 90 balls, of which 30 are labelled X. The remainder are either Y or Z.

Which of the following options do you prefer?

a) Payoff of λ if an X ball is drawn.

b) Payoff of 100 if a Y ball is drawn.

c) Payoff of 100 if a Z ball is drawn."

Payoff "λ" attached to the option X was changed from round to round, with $\lambda = 95$, 90, 80, 100, 105 (in that order), to measure the ambiguity threshold of subjects. When $\lambda = 100$, the choice equivalent to the 3-ball version of the Ellsberg Paradox. In the case $\lambda = 105$, we are testing whether some subjects would choose to bet on an ambiguous ball even though it has a lower expected return (according to a uniform prior). This is a feature not present in many previous experimental tests of the Ellsberg Paradox.

In our Ellsberg urn experiments, we use balls labelled X, Y and Z, rather than following the traditional practice of using Red, Blue and Yellow coloured balls.[7] This is because in a previous set of experiments, Kelsey and le Roux (2015), we used coloured balls and found that subjects often chose Blue (the ambiguous option), simply because they had a fondness for the colour blue. Similarly, we found a large number of Chinese subjects chose Red, because it was considered "auspicious" in Chinese culture. In this study we use balls labelled X, Y and Z, in

[7] The traditional Ellsberg urn contains Red, Blue and Yellow coloured balls. The number of Red balls in the urn is known, while the remaining Blue and Yellow coloured balls are ambiguous in number.

order to avoid biases of this sort.

4 Experimental Design

The games described above were used in paper-based experiments, conducted at St. Stephen's College in New Delhi, India, and at the Finance and Economics Experimental Laboratory in Exeter (FEELE), UK. All the subjects recruited at St. Stephen's College were Indian nationals, who (by assumption) had an Indian sociocultural upbringing. While sending out the invitations to recruit subjects at the University of Exeter, we took particular care to weed out any foreign students who were Indian.

The experiments were conducted with three different treatments.

- In Treatment I, subjects were matched against other local subjects (this treatment analyses data from Delhi vs. Delhi and Exeter vs. Exeter sessions).[8]
- Treatment II consisted of matching Exeter subjects against an Indian opponent. Subjects were told that the same experiments had been run in India and that they would be matched up against an Indian subject whose responses had already been collected.[9]
- In Treatment III, subjects were matched against both internationally as well as locally recruited subjects, for the purpose of payment.[10]

Treatments I and III consisted of 60 subjects each and Treatment II had 61 subjects. In total there were 181 subjects who took part in the experiment, 81 of whom were males and the remaining 100 were females. Each session lasted a maximum of 45 minutes including payment.

[8] A probit regression showed that the dummy variable for location ($Delhi/Exeter$) does not have a significant impact on choosing the ambiguity safe option. Thus for the purpose of analysing subject behaviour in Treatment I, we have combined the data from sessions where Delhi subjects played against other Delhi subjects (local vs. local) with data from the Exeter vs. Exeter session, without the loss of efficiency.

[9] Subjects knew that their choice would not affect the actual payoff of foreign players, while this was the case when they played the games against local opponents. As a result, when comparing behaviour in the games to test the role of ambiguity, we note that there may be a social preferences confound: subjects might have behaved differently when playing against foreign opponents simply because their choices did not affect the payoff of somebody else.

[10] We were initially testing whether foreign opponents would create more uncertainty. We had expected that difference in backgrounds would create ambiguity on the part of Exeter subjects.

Subjects first read through a short, comprehensive set of instructions at their own pace, following this the instructions were also read out to all the participants in general.[11] The subjects were asked to fill out practice questions to check that they understood the games correctly. Once the practice questions had been answered and discussed, the actual set of experimental questions were handed out to the subjects. Subjects were randomly assigned the role of either a Row Player or a Column Player at the beginning of the experiment, for the purpose of matching in the games, and remained in the same role for the rest of the experiment.

Each subject had to select one option per round: Top/Centre/Bottom if they were a Row Player or Left/Middle/Right if they were a Column Player, and in case of the Ellsberg urn rounds X, Y or Z. In the Ellsberg urn rounds, the pay-offs attached to drawing a Y or Z ball were held constant at 100, while those attached to drawing an X ball varied as 95, 90, $80, 100, 105$.

Once subjects had made all decisions, a throw of dice determined *one* game round and *one* Ellsberg urn round for which subjects would be paid. Subjects in India were paid a show-up fee of $Rs.200$ (£2.50), together with their earnings from the chosen round, where $100ECU = Rs.200$. Exeter subjects were paid a show-up fee of £3, together with their earnings from the chosen round, where $100ECU = £2$.[12] In order to prevent individuals from self-insuring against payoff risks across rounds, we picked one round at random for payment, see Charness and Genicot (2009).[13] Players' decisions were matched according to a predetermined matching, and pay-offs were announced.

Instead of using a real urn we used a computer to simulate the drawing of a ball from the urn.[14] The computer used a normal distribution to randomly assign the number of Y and Z balls in the urn so that they summed to 60, while keeping the number of X balls fixed at 30

[11] The experimental protocols can be found at the following link: http://saraleroux.weebly.com/experimental-protocols.html

[12] The experiments were conducted between November 2010 - February 2011. The exchange rate during the period was 1 GBP = 80 INR. Our aim was that the average earnings from our experiment which lasted a maximum of 30 minutes, should be able to afford subjects (university students) the chance purchase a meal and a non-alcoholic drink. The purchasing power parity that we were aiming for was a burger meal.

[13] If all rounds count equally towards the final payoff, subjects are likely to try and accumulate a high payoff in the first few rounds and then care less about how they decide in the following rounds. In contrast, if subjects know that they will be paid for a random round, they treat each decision with care.

[14] The computer simulated urn can be found at the following link: http://saraleroux.weebly.com/experimental-protocols.html

and the total number of balls in the urn at 90.[15] The computer then simulated an independent ball draw for each subject. If the label of the ball drawn by the computer matched that chosen by the subject, it entitled him to the payoff specified in the round chosen for payment.

5 Data Analysis and Results

5.1 Row Player Behaviour

See Figure 2, for a summary of row player behaviour. In Treatment I, we find that a large majority of our subjects, 63% in $SC1$ and 73% in $SC2$, chose the certain option T in our experiments. In comparison, only 13% ($SC1$) and 17% ($SC2$) of subjects chose C, the choice under Nash. *Binomial Test A* (See Table 1) finds that subjects chose the certain option significantly more often than either of the other options. Similarly, in $SS1$ and $SS2$ we find that 50% and 67% of subjects chose the certain option B. It is interesting to note that when multiple Nash equilibria are present (in $SS1$), 40% of the subjects select the Nash $(C, M)-$ which gives an equal payoff to both players. This might indicate that *fairness constraints* affect these subjects more than ambiguity. As such, *Binomial Test B* finds that subjects choose the certain option B, significantly more often in $SS2$, but fails to reject this hypothesis for $SS1$ (See Table 1, Row 5).

In Treatment II, we find that 83% and 87% of subjects chose the certain strategy T in $SC1$ and $SC2$, respectively, compared to 7% (2) and 13% of subjects who chose C (the choice under Nash equilibrium). As can be seen in Table 1 (Row 6), subjects chose the certain option significantly more often than the other two options available to them. In the strategic substitutes game $SS1$ and $SS2$, 50% and 67% of subjects chose B, the choice under EUA. Less than half the subjects (43%), opt for the Nash outcome which would result in equitable pay-offs for both players. *Binomial Test B* cannot be rejected for $SS1$, however, we do reject the null at a 5% level of significance for $SS2$.

[15] The number of Y balls in the urn were determined using the MSExcel command "=ROUNDDOWN(RAND()*61,0)", and the number of Z balls in the urn were simply = (60 minus the number of Y balls).

Figure 2: Row Player Behaviour

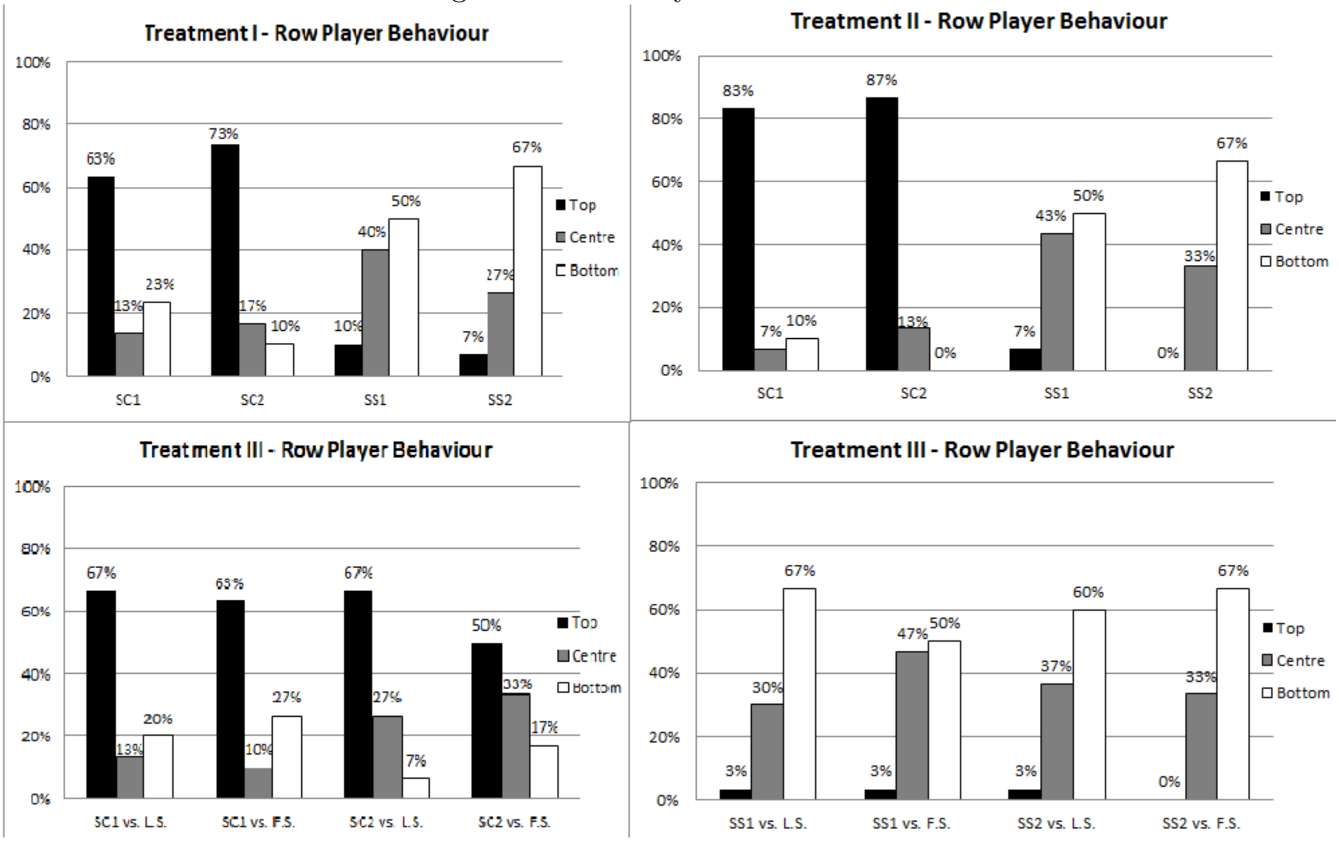


Table 1: Binomial Tests A and B - Results

Test:	*Z-score for Binomial Test A*		*Z-score for Binomial Test B*	
Null Hypothesis (H_0):	$prob(T) = prob(C+B) = 0.5$		$prob(B) = prob(T+C) = 0.5$	
Alt. Hypothesis (H_1):	$prob(T) > prob(C+B)$		$prob(B) > prob(T+C)$	
Game:	$SC1$	$SC2$	$SS1$	$SS2$
Treatment I	1.465059**	2.55604***	0	1.82574**
Treatment II	3.65148***	4.01663***	0	1.82574**
Treatment III vs. LS	1.82574**	1.82574**	1.82574**	1.09544
Treatment III vs. FS	1.46505*	0	0	1.82574**
*, **, *** indicate significance levels of 10%, 5% and 1% respectively.				

In Treatment III, Exeter subjects were asked to make decisions versus both the local (Exeter) as well as the foreign (Indian) opponent. They were allowed to choose different actions against the foreign opponent and the domestic one. See the bottom half of Figure 2, for a summary of Row Player behaviour. We find that *fewer* subjects chose the certain option T, against the foreign opponent than against the local opponent, in both $SC1$ and $SC2$. It is interesting to note that subjects chose the certain option significantly more often against the local opponent, but not against the foreign subject (See Table 1, Rows 7 and 8). We find similar behaviour in game $SS1$, where fewer subjects took the certain option versus the foreign subject than against the local subject. Play in $SS2$, was closer to our expectations, and subjects chose the certain option more against the foreign subject than the local one.

Table 2: Correlation in Row Player Behaviour between Games Rounds

Game/Action:	$SC1\_CO$	$SC2\_CO$	$SS1\_CO$	$SS2\_CO$
$SC1\_CO$	1.000			
$SC2\_CO$	0.724	1.000		
$SS1\_CO$	0.504	0.484	1.000	
$SS2\_CO$	0.637	0.619	0.559	1.000

We find that there is a significant amount of correlation between subjects' choice of the certain option (CO) between the four games (See Table 2). However, when we investigate whether there is any correlation between row players who consistently choose the certain option in the game rounds, with their choice in the Ellsberg Urn rounds, we find a weak negative correlation. The row players seemed to display a pessimistic attitude towards ambiguity in the game rounds, but displayed an optimistic attitude towards ambiguity in the Urn rounds.

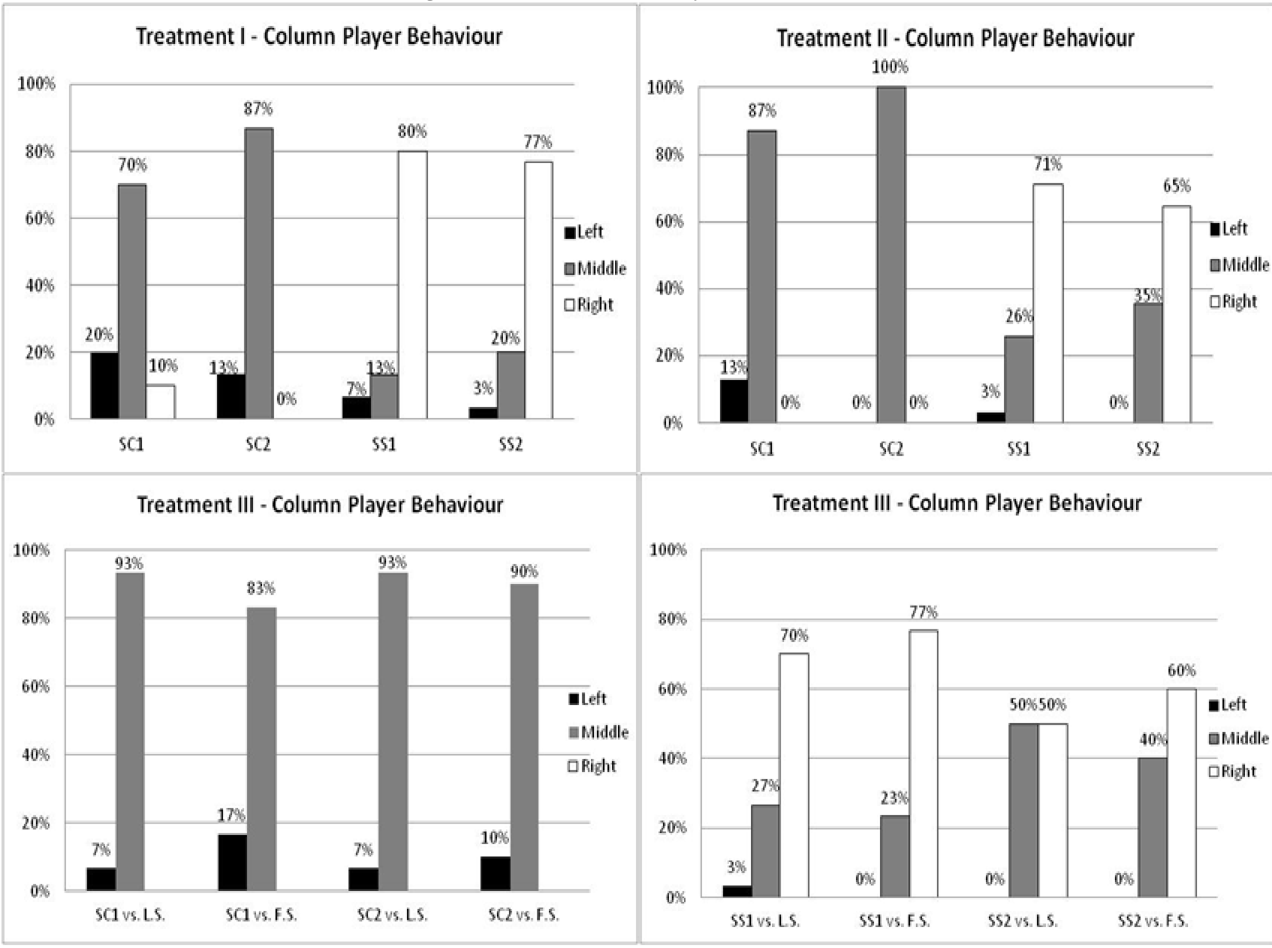
5.2 Column Player Behaviour

See Figure 3, for a summary of Column Player behaviour. In Treatment I, we find that 70% and 87% of subjects chose the Nash strategy M in $SC1$ and $SC2$, respectively.[16] *Binomial Test C* finds that subjects choose the certain option significantly more often than either of the other two choices (See Table 3, Row 5). In the strategic substitutes games $SS1$ and $SS2$, we find that 80% and 77% of subjects choose the certain option R, in comparison to 13% and 20% of subjects that chose M (the choice under Nash). *Binomial Test D* finds that subjects choose the certain option M, significantly more often than $L + R$ (Table 3, Row 5).

In Treatment II, 87% and 100% of subjects choose the certain option M, in $SC1$ and $SC2$ respectively. As might be expected, *Binomial Test C* can be rejected at a 1% level for both $SC1$ and $SC2$ (See Table 3, Row 6). In games $SS1$ and $SS2$, we find 71% and 65% of subjects chose the certain strategy R. *Binomial Test D* is rejected at 1% for $SS1$ and at 5% for $SS2$

[16] Note in the case of $SC1$ and $SC2$, the equilibrium action under ambiguity coincides with the Nash strategy, for the Column player.

Figure 3: Column Player Behaviour



(Table 3, Row 5). We had encouraged subjects to write a short account at the end of the experiment, about their reactions and what they were thinking about when they made their choices. A number of subjects mentioned that they preferred to stick with a safe (but definite) payoff rather than take a chance and lose out. Thus these subjects were willing to forego the possibility of a higher payoff, in order to avoid an option which they perceived as ambiguous.

Table 3: Binomial Tests C and D - Results

Test:	*Z-score for Binomial Test C*		*Z-score for Binomial Test D*	
Null Hypothesis H_0:	$prob(M) = prob(L+R) = 0.5$		$prob(R) = prob(L+M) = 0.5$	
Alt. Hypothesis H_1:	$prob(M) > prob(L+R)$		$prob(R) > prob(L+M)$	
Game:	$SC1$	$SC2$	$SS1$	$SS2$
Treatment I	2.19089**	4.01663***	3.28633***	2.92119***
Treatment II	4.13092***	5.56776***	2.33487***	1.61645**
Treatment III vs. LS	4.74693***	4.74693***	2.19089**	0
Treatment III vs. FS	3.65148***	4.38178***	2.92119***	1.09445
*, **, *** indicate significance levels of 10%, 5% and 1% respectively.				

In Treatment III, subjects were matched against both local as well as foreign opponents.

See Figure 3 (lower half of figure), for a summary of Column Player behaviour in Treatment III. *Binomial Test C* shows that the certain option was the significant choice of subjects, in both $SC1$ and $SC2$, and against both local and foreign opponents. In game $SS1$, we find that 70% and 77% of subjects chose the certain strategy R, against the local and foreign subject respectively. In $SS2$, half the subjects chose the certain strategy while the other half chose the Nash against the local opponent. When faced with the foreign opponent, 60% chose the certain option while 40% chose it under Nash. As before, we conduct *Binomial Test D* and reject the null at 5% against the local opponent and 1% against foreign opponents for $SS1$. We fail to reject the null for $SS2$, since the decisions were very close to the $50-50$ mark.

We ran probit regressions to ascertain what factors influenced subjects in their choice of the certain option. Dummy variables were defined to capture the characteristics of the data such as: $Quant = 1$, if the subject was doing a quantitative degree ($Quant = 0$, for degrees like English, History, Philosophy, Politics etc.); $Male = 1$, if gender is male (0, otherwise); and various dummies were constructed for the different game rounds (SC_1, SC_2, SS_1 and SS_2).

Table 4: Probit Regression Results Modelling the Choice of the Certain Option

$Certain\_Option$	Coefficient	Std. Err.	z	$P>\|z\|$	[95% Conf. Interval]	
$Male$	$-.4109032^{***}$	.1327117	-3.10	0.002	$-.6710134$	$-.1507931$
SC_1	$.6707215^{***}$	.171578	3.73	0.000	.3187552	1.022688
SC_2	1.101431^{***}	.2033642	5.42	0.000	.70228444	1.500017
SS_1	$.3362364^{***}$	.1707295	1.97	0.049	.0016127	.6708601
Constant	$.5022267^{***}$	.1336187	3.76	0.000	.240339	.7641145
*, **, *** indicate significance levels of 10%, 5% and 1% respectively.						

A probit regression of $Certain\_Option$ on $Male$ and the dummies for the game rounds, has a chi-squared ratio of 45.51 with a p-value of 0.0000, which shows that the model as a whole is statistically significant.[17] Regression results are seen in Table 4.[18] We note that subjects are

[17] A probit regression of $Certain\_Option$ (choice of the certain option) on the various dummies found that "$Quant$" was insignificant. It was thus dropped from the final regression. The dummy for SS_2 was dropped from the probit regression, in order to avoid the dummy variable trap.

[18] The coefficients from a probit regression do not have the same interpretation as coefficients from an Ordinary Least Squares regression. From the probit results, we can interpret that males are less likely to choose the certain option. If a subject is male, their z-score decreases by 0.41. Moreover, subjects are more likely to choose the certain option in $SC1$: the z-score increases by 0.67, in $SC2$: the z-score increases by 1.10, and for $SS1$: the z-score increases by 0.34, when compared to the base which is game $SS2$.

more likely to choose the certain option in games with strategic complements than those with strategic substitutes. Moreover, males are significantly less likely to take the certain option than females.

Table 5: Correlation in Column Player Behaviour between Games Rounds

Game/Action:	$SC1\_ASO$	$SC2\_ASO$	$SS1\_ASO$	$SS2\_ASO$
$SC1\_ASO$	1.000			
$SC2\_ASO$	0.801	1.000		
$SS1\_ASO$	0.622	0.725	1.000	
$SS2\_ASO$	0.537	0.619	0.729	1.000

Once again we found a strong correlation between subjects' choice of the certain option (CO) between the four games (See Table 5). However, the correlation choices in the game rounds and the Ellsberg Urn rounds, was weakly negative. Column player behaviour was consistent with row player behaviour, in that subjects displayed pessimism towards ambiguity in the game rounds, and optimism towards ambiguity in the Urn rounds.

5.3 Behaviour in Ellsberg Urn Rounds

The strategic complement and substitute games were alternated with Ellsberg Urn decisions, in order to elicit an ambiguity threshold of the subjects. Moreover, it enabled us to evaluate whether the ambiguity-attitude of subjects remained consistent between single person decisions, and situations where they were faced by ambiguity created by interacting with other players. The payoff on drawing X (the unambiguous event) was varied as $\lambda = 95,\ 90, 85, 100$ or 105 *ECU*, depending on the round being played.

As can be seen in Figure 5, for $\lambda = 100$ (the standard Ellsberg urn decision problem), 73% (133) of subjects chose X, while 27% (48) chose to bet on Y and Z.[19] This result is consistent with previous studies.

When there is a premium attached to X, i.e., when $\lambda = 105$, a majority of subjects (73%) opt for X. However, what is more interesting to note is that 27% of subjects opt for $Y + Z$. These subjects are willing to take a lower payoff, in order to choose Y or Z - the balls whose

[19] We consider the sum of the people who chose Y and Z, rather than the number of people who chose Y or Z balls individually, in order to negate any effect of people choosing Y just because it appeared before Z on the choice set.

Figure 4: Subject Behaviour in Ellsberg Urn Rounds

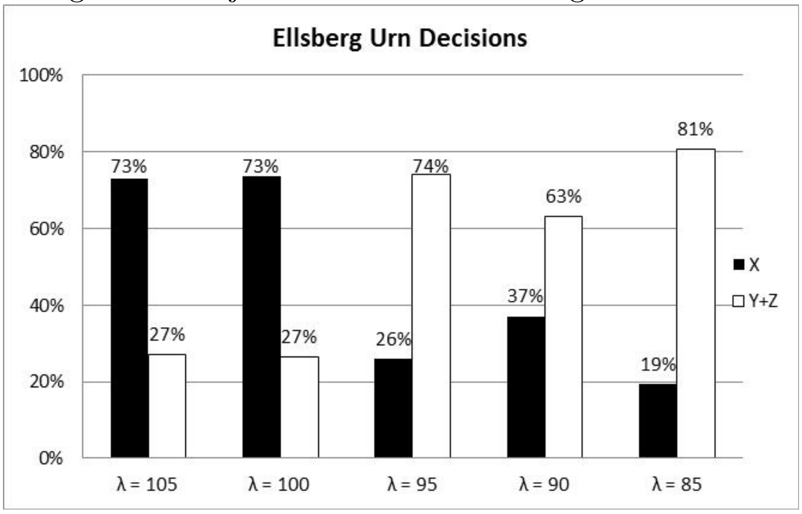


Figure 5:

proportion is unknown! We believe this captures ambiguity-seeking behaviour on the part of the subjects.

Even a small penalty on X from $\lambda = 100$ to $\lambda = 95$, leads to a big rise in the number of subjects choosing $Y + Z$. When $\lambda = 95$, 74% (134) of subjects choose $Y + Z$. This goes up substantially to 81% (146) of subjects choosing $Y + Z$, when $\lambda = 85$. Most subjects are not ambiguity-averse enough to bear a small penalty, in order to continue choosing X (the unambiguous event). It is significant that 19% (35) of subjects chose X, even when $X = 85$, thus displaying strong ambiguity-averse behaviour.

We ran probit regressions to ascertain what factors influenced subjects in their choice of X (the unambiguous ball). Dummy variables were defined to capture the characteristics of the data such as: $Quant = 1$, if the subject was doing a quantitative degree ($Quant = 0$, otherwise.); $Male = 1$, if gender is male (0, otherwise); λ_105, λ_100, λ_95, λ_90, $\lambda\_85 = 1$, depending on the value "λ" took in that particular round.

A probit regression of X on the various λ-value dummies λ_105, λ_95, λ_90, λ_85, has a chi-squared ratio of 201.29 with a p-value of 0.0000, which shows that our model as a whole is statistically significant.[20] Regression results are seen in Table 6.[21] There was no significant

[20] The dummy for λ_100 was dropped from the probit regression, in order to avoid the dummy variable trap. Dummies for $Quant$ and $Male$ were found to be insignificant, and were thus dropped from the final regression.

[21] From the probit results, we can interpret that when $\lambda = 105$: the z-score decreases by 0.03, for $\lambda = 95$: the z-score decreases by 1.27, for $\lambda = 90$: the z-score decreases by 0.96, for $\lambda = 85$: the z-score decreases by 1.49,

difference in the choice of X when $\lambda = 105$ and 100. However, for $\lambda = 95$, 90 and 85, subjects chose X significantly less often than when $\lambda = 100$.

Table 6: Probit Regression Results Modelling the Choice of X

X	Coefficient	Std. Err.	z	$P > \|z\|$	[95% Conf. Interval]	
λ_105	$-.033377$	.1410849	-0.24	0.813	$-.3098983$	.2431443
λ_95	-1.271783***	.1419004	-8.96	0.000	-1.549903	$-.9936638$
λ_90	$-.9588301$***	.1380596	-6.95	0.000	-1.229422	$-.6882383$
λ_85	-1.49296***	.146563	-10.19	0.000	-1.780218	-1.205701
Constant	.6274158***	.1001379	6.27	0.000	.431149	.8236825
*, **, *** indicate significance levels of 10%, 5% and 1% respectively.						

At the individual level, of the 133 subjects that chose X when $\lambda = 100$, 68 switched to $Y + Z$ at $\lambda = 95$, 7 switched to $Y + Z$ at $\lambda = 90$, 5 switched to $Y + Z$ at $\lambda = 85$, while 21 subjects chose X for all values of λ. Looking more closely at the choices of the subjects who always chose X, we find that 9 of them always chose the certain options in the game rounds.[22] Thus, a very small subset of our subject pool (5%), showed strong ambiguity-averse behaviour.

Looking at the 49 subjects who chose $Y + Z$ when $\lambda = 105$, we find that 11 of them never chose X in the Urn rounds. However, 12 of these 49 subjects always chose the certain options in the game rounds – these subjects seem to have a context-dependent ambiguity-attitude: ambiguity-loving in single person decisions and ambiguity-averse in the game environment.

Table 7: Binomial Tests E and F - Results

Test:	*Z-score for Binomial Test E*	*Z-score for Binomial Test F*
Null Hypothesis H_0:	$prob(X) = prob(Y + Z) = 0.5$	$prob(X) = prob(Y + Z) = 0.5$
Alt. Hypothesis H_1:	$prob(X) > prob(Y + Z)$	$prob(Y + Z) > prob(X)$
$\lambda = 105$	6.169***	
$\lambda = 100$	6.318***	
$\lambda = 95$		6.467***
$\lambda = 90$		3.493***
$\lambda = 85$		8.251***
*, **, *** indicate significance levels of 10%, 5% and 1% respectively.		

We conducted *Binomial Tests E* and *F* as described in Table 7. We find that subjects choose the unambiguous ball X significantly more often for $\lambda = 105$ and 100, but prefer the ambiguous balls $Y + Z$ for lower values of λ. On the whole, subjects seem to prefer "betting" on

when compared to the base which is $\lambda = 100$.

[22] Of these, 7 were Column Players and the remaining 2 were Row Players.

Y and Z. Responses gathered from the subjects showed that subjects viewed the urn rounds as "gambles". The justification given for this was that the computer could have picked any of the three options - thus Y or Z balls could have been more in number than X balls, that were capped at 30 balls. The subjects thus displayed an optimistic attitude towards ambiguity. Moreover, some subjects treated these rounds as based on luck rather than reasoning.[23]

5.4 Overall Results

We find that behaviour broadly supports our hypotheses. On average 69% of row players chose the certain option predicted by the Equilibrium under Ambiguity theory in games $SC1$ and $SC2$. In games $SS1$ and $SS2$, 54% and 65% of row players chose the certain option. Similarly, a large majority of column players chose the certain option - 83% in $SC1$, 93% in $SC2$, 74% in $SS1$ and 63% in $SS2$.

In the Ellsberg Urn rounds we find that for $\lambda = 105$ and $\lambda = 100$ subjects prefer to opt for X rather than Y or Z. A small drop in λ ($\lambda = 95$, 90, 85), leads to subjects choosing Y or Z significantly more often than X. We notice that the subjects are unwilling to bear even a small penalty in order to stick with the unambiguous choice.

A very small subset of our subject pool (5%), showed strong ambiguity-averse behaviour always choosing the certain option in the game rounds and the unambiguous ball in the Urn rounds. Among our subjects 27% displayed a mildly ambiguity-seeking behaviour in opting for the ambiguous ball, when there was a premium attached to the unambiguous choice.

An interesting observation from the data is that even though subjects display ambiguity-aversion when faced by other opponents, data from the Urn rounds leads us to conclude that they are often ambiguity-seeking when faced by nature/single-person decisions. This is consistent with an earlier study Kelsey and le Roux (2015), where subjects showed differences in ambiguity-attitudes based on the scenario they were facing.

[23] One subject in particular noted that– "The urn question is pure luck, because majority of the unmarked balls are either Y or Z, and choosing either is a gamble."

6 Related Literature

6.1 Ambiguity in games

Our study builds upon the theoretical paper by Eichberger and Kelsey (2002). They find that in a game with positive (resp. negative) externalities, ambiguity prompts a player to put an increased (resp. decreased) weight on the lowest of his opponent's actions. The marginal benefit that the player gets from his own action gets decreased (resp. increased) in the case of a game with strategic complements (resp. substitutes). In the presence of positive externalities, players often have the incentive to use a strategy below the Pareto optimal level, and so, the resultant Nash equilibrium is inefficient. In the case of strategic substitutes, increasing the level of ambiguity would cause a shift in equilibrium strategies towards an ex-post Pareto efficient outcome, whereas for strategic complements, an increase in ambiguity would cause a shift in equilibrium, away from the ex-post Pareto efficient outcome. Hence it was argued that ambiguity had an adverse effect in case of games with strategic complements, but was helpful in the case of games with strategic substitutes. Ambiguity thus causes a decrease in equilibrium actions in a game of strategic complements and positive externalities or the opposite case, i.e., strategic substitutes and negative externalities.

Eichberger, Kelsey, and Schipper (2008) tested whether ambiguity had the opposite effect in games of strategic complements and substitutes. Subjects were found to react to variations in the level of ambiguity, which was tested by altering the cardinal payoff in the game while keeping the ordinal payoff structure unchanged. It can thus be seen that subjects react not only to ambiguity on the part of the opponent, but also to subtle changes in the payoff structures of the experiment.

While our study concentrated on investigating individual behaviour in the presence of ambiguity, Keller, Sarin, and Sounderpandian (2007) investigate whether individuals deciding together as pairs display ambiguity averse behaviour. It was found that the pairs displayed risk averse as well as ambiguity averse preferences. Moreover, it was observed that the willingness-to-pay among pairs of individuals deciding together, was lower than the average of their individual willingness-to-pay for gambles. The authors concluded that ambiguity-averse behaviour is prevalent in group settings.

In our experiments, we did not allow subjects to interact with each other. We believed that this would reduce the level of ambiguity they would perceive, when asked to make decisions against each other. In contrast, Keck, Diecidue, and Budescu (2012), conduct an experiment in which subjects made decisions individually, as a group, and individually after interacting and exchanging information with others. They found that individuals are more likely to make ambiguity-neutral decisions after interacting with other subjects. Moreover, they find that ambiguity-seeking and ambiguity-averse preferences among individuals are eliminated by communication and interaction between individuals; and as such, groups are more likely to make ambiguity-neutral decisions.

6.2 Ambiguity in Single Person Decisions

Our Ellsberg urn experiments investigated whether there was any correlation between ambiguity attitude in games and single person decisions. Moreover, we wanted to evaluate whether there was any threshold at which individuals switched from being ambiguity averse to being ambiguity neutral (or seeking). For an extensive survey of the literature on Ellsberg experiments, see Trautmann and van de Kuilen (2016).

Eliaz and Ortoleva (2011) study a three-colour Ellsberg urn with increased ambiguity, in that the amount of money that subjects can earn also depends on the number of balls of the chosen colour in the ambiguous urn. The subjects thus face ambiguity on two accounts: the unknown proportion of balls in the urn as well as the size of the prize money. In their experiment, both winning and the amount that the subject could possibly win were both perfectly correlated - either positively or negatively, depending on which of the two treatments was run by them. In the experiment, most subjects preferred betting in the positively correlated treatment rather than the negative one. Moreover, subjects also showed a preference for a gamble when there was positively correlated ambiguity, as opposed to a gamble without any ambiguity. This behaviour of the subjects, is compatible with our findings that subjects preferred betting on Y/Z where there was ambiguity, rather than on X, the known choice.

Binmore, Stewart, and Voorhoeve (2012), test whether subjects are indeed ambiguity averse. They report that behaviour in their experiments is inconsistent with the Hurwicz criterion. Instead, they find that the principle of insufficient reason has greater predictive power with respect to their data, than ambiguity aversion. This may also explain why behaviour in games appears different to that in Ellsberg urn type experiments. It is harder to apply the principle of insufficient reason to games. Our results are consistent with these findings, since we find that subjects are not willing to pay even a moderate penalty to avoid ambiguity in the Ellsberg urn rounds where the payoff attached to X were $95/90/85ECU$. This might be because in the absence of information, subjects use the principle of insufficient reason and attach a $50:50$ probability to the remaining 60 Y and Z balls left in the urn. The principle of insufficient reason would imply that the probability distribution attached to the X, Y and Z balls in the urn is $(\frac{1}{3}, \frac{1}{3}, \frac{1}{3})$. It would thus be more rational to choose Y or Z and get a payoff of $100ECU$, than to choose X and suffer a penalty, i.e., get pay-offs $95/90/85ECU$.

7 Conclusions

Behaviour in our experiments was found to be consistent with our hypotheses. Nash equilibrium was a poor predictor of subject behaviour and the deviations from Nash behaviour were in the direction expected. In the games we find that subjects do indeed choose the equilibrium action under ambiguity more often than either of the other actions. As predicted, ambiguity had the opposite effect in games of strategic complements and substitutes.

One possible reason for the choice of the certain option in the game rounds is that it guarantees a certain income to the subject, irrespective of the opponent's choice. It could be conjectured that these subjects may be using a risk dominance criterion (in line with Harsanyi and Selten (1988)), to select a strategy that is inconsistent with Nash equilibria. Future research on this area should be more careful to control for subjects' risk attitude. However, given the relatively small stakes Expected Utility Theory would predict that subjects were approximately risk neutral. A similar argument does not apply to ambiguity. Choquet Expected Utility (CEU) or Maxmin Expected Utility (MEU) preferences have a kink. Consequently ambiguity-aversion may be seen even when the stakes are fairly small. Thus, it is not unreasonable to believe that most of the motivation for choosing the certain action is ambiguity-aversion.

In the Ellsberg Urn rounds we find that for $\lambda = 105$ and $\lambda = 100$ subjects prefer to opt for X rather than Y or Z, but even the smallest reduction in λ leads to subjects choosing Y or Z (which is the ambiguous choice). When the payoff attached to X was 95, 90, or 85, $Y + Z$ was chosen significantly more often than X. We notice that the subjects are unwilling to bear even a small penalty in order to stick with X balls (the unambiguous choice). At the individual level, we found steep drops in the number of subjects choosing X, for every reduction in the value of λ. A very small subset of our subject pool (5%), showed strong ambiguity-averse behaviour always choosing the certain option in the game rounds and X in the Urn rounds, while 27% of our subjects showed mildly ambiguity-seeking behaviour by opting for $Y + Z$ when $\lambda = 105$.

Our subjects appear to perceive more ambiguity and exhibit more ambiguity-aversion in games. In addition, we note that subjects' ambiguity-attitudes appear to be context dependent: ambiguity-loving in single person decisions and ambiguity-averse in games. This is consistent with our earlier study, Kelsey and le Roux (2015), where we found that the ambiguity-attitude of subjects was dependent on the scenario they were facing. It is our belief that subjects find it more ambiguous to make decisions against other people than against the random move of nature, over which everyone is equally powerless. This might even explain why people are more concerned with scenarios involving political turmoil or war - situations dependent on other people, but appear to discount the seriousness of possible natural disasters or climate change related catastrophes - which are beyond anyone's control.

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