Ambiguity and the Centipede Game

Jürgen Eichberger, Simon Grant and David Kelsey
Heidelberg University,
Australian National University,
University of Exeter.

University of Exeter.

June 2016

The Centipede Game I

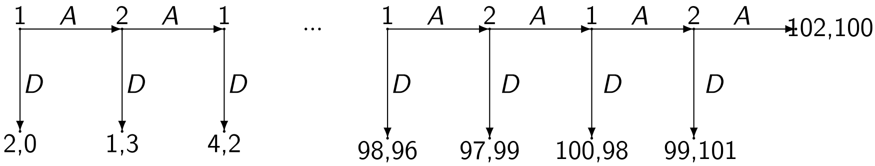


Figure: Centipede

The Centipede Game II

- There are two people A and B. Between them is a table which contains $2M$ one-pound coins and a single two-pound coin. They move alternately.

The Centipede Game II

- There are two people A and B. Between them is a table which contains $2M$ one-pound coins and a single two-pound coin. They move alternately.
- At each move there are two actions available.

The Centipede Game II

- There are two people A and B. Between them is a table which contains $2M$ one-pound coins and a single two-pound coin. They move alternately.
- At each move there are two actions available.
  - The player whose move it is may either pick up the two-pound coin in which case the game ends;

The Centipede Game II

- There are two people A and B. Between them is a table which contains $2M$ one-pound coins and a single two-pound coin. They move alternately.
- At each move there are two actions available.
  - The player whose move it is may either pick up the two-pound coin in which case the game ends;
  - or (s)he may pick up two one-pound coins keep one and give the other to his/her opponent; in which case the game continues.

The Centipede Game II

- There are two people A and B. Between them is a table which contains $2M$ one-pound coins and a single two-pound coin. They move alternately.
- At each move there are two actions available.
  - The player whose move it is may either pick up the two-pound coin in which case the game ends;
  - or (s)he may pick up two one-pound coins keep one and give the other to his/her opponent; in which case the game continues.
- In the final round there is one two-pound coin and two one-pound coins remaining. The player whose move it is may either:

The Centipede Game II

- There are two people A and B. Between them is a table which contains $2M$ one-pound coins and a single two-pound coin. They move alternately.
- At each move there are two actions available.
  - The player whose move it is may either pick up the two-pound coin in which case the game ends;
  - or (s)he may pick up two one-pound coins keep one and give the other to his/her opponent; in which case the game continues.
- In the final round there is one two-pound coin and two one-pound coins remaining. The player whose move it is may either:
  - pick up the two-pound coin, in which the game ends and nobody gets the one-pound coins;

The Centipede Game II

- There are two people A and B. Between them is a table which contains $2M$ one-pound coins and a single two-pound coin. They move alternately.
- At each move there are two actions available.
  - The player whose move it is may either pick up the two-pound coin in which case the game ends;
  - or (s)he may pick up two one-pound coins keep one and give the other to his/her opponent; in which case the game continues.
- In the final round there is one two-pound coin and two one-pound coins remaining. The player whose move it is may either:
  - pick up the two-pound coin, in which the game ends and nobody gets the one-pound coins;
  - (s)he may pick up the two one-pound coins keep one and give the other to his/her opponent, in which case the opponent also gets the two pound coin and the game ends.

The Centipede Game II

- There are two people A and B. Between them is a table which contains $2M$ one-pound coins and a single two-pound coin. They move alternately.
- At each move there are two actions available.
  - The player whose move it is may either pick up the two-pound coin in which case the game ends;
  - or (s)he may pick up two one-pound coins keep one and give the other to his/her opponent; in which case the game continues.
- In the final round there is one two-pound coin and two one-pound coins remaining. The player whose move it is may either:
  - pick up the two-pound coin, in which the game ends and nobody gets the one-pound coins;
  - (s)he may pick up the two one-pound coins keep one and give the other to his/her opponent, in which case the opponent also gets the two pound coin and the game ends.
- There is a unique iterated dominance equilibrium. At any node the player whose move it is picks up the 2-pound coin and ends the game.

The Centipede Game II

- There are two people A and B. Between them is a table which contains $2M$ one-pound coins and a single two-pound coin. They move alternately.
- At each move there are two actions available.
  - The player whose move it is may either pick up the two-pound coin in which case the game ends;
  - or (s)he may pick up two one-pound coins keep one and give the other to his/her opponent; in which case the game continues.
- In the final round there is one two-pound coin and two one-pound coins remaining. The player whose move it is may either:
  - pick up the two-pound coin, in which the game ends and nobody gets the one-pound coins;
  - (s)he may pick up the two one-pound coins keep one and give the other to his/her opponent, in which case the opponent also gets the two pound coin and the game ends.
- There is a unique iterated dominance equilibrium. At any node the player whose move it is picks up the 2-pound coin and ends the game.
- The centipede game could represent two countries disarming in stages. An illegal drug producer and a dealer building cooperation in stages.

Centipede III

- Intuition suggests that individuals will cooperate to achieve the high pay-offs towards the end of the centipede rather taking a small gain and ending the game near the beginning.

Centipede III

- Intuition suggests that individuals will cooperate to achieve the high pay-offs towards the end of the centipede rather taking a small gain and ending the game near the beginning.
- Experimental evidence also shows that subjects manage to cooperate, at least for a while.

Centipede III

- Intuition suggests that individuals will cooperate to achieve the high pay-offs towards the end of the centipede rather taking a small gain and ending the game near the beginning.
- Experimental evidence also shows that subjects manage to cooperate, at least for a while.
- Ambiguity (unknown probabilities) can cause individuals to overweight bad outcomes. This is know as ambiguity-aversion.

Centipede III

- Intuition suggests that individuals will cooperate to achieve the high pay-offs towards the end of the centipede rather taking a small gain and ending the game near the beginning.
- Experimental evidence also shows that subjects manage to cooperate, at least for a while.
- Ambiguity (unknown probabilities) can cause individuals to overweight bad outcomes. This is know as ambiguity-aversion.
- Less frequently (but still quite common) ambiguity can cause individuals to over-weight good outcomes. This is know as ambiguity-loving.

Centipede III

- Intuition suggests that individuals will cooperate to achieve the high pay-offs towards the end of the centipede rather taking a small gain and ending the game near the beginning.
- Experimental evidence also shows that subjects manage to cooperate, at least for a while.
- Ambiguity (unknown probabilities) can cause individuals to overweight bad outcomes. This is know as ambiguity-aversion.
- Less frequently (but still quite common) ambiguity can cause individuals to over-weight good outcomes. This is know as ambiguity-loving.
- Given the high pay-offs towards the end of the centipede it is plausible that ambiguity-loving behaviour may be the reason that people do not follow the Nash equilibrium.

Extensive Form Games with Ambiguity

- There are a number of papers on games with ambiguity. However nearly all of these focus on normal form games.

Extensive Form Games with Ambiguity

- There are a number of papers on games with ambiguity. However nearly all of these focus on normal form games.
- Extensive form games pose a number of new problems.

Extensive Form Games with Ambiguity

- There are a number of papers on games with ambiguity. However nearly all of these focus on normal form games.
- Extensive form games pose a number of new problems.
- How are beliefs updated as new information is received?

Extensive Form Games with Ambiguity

- There are a number of papers on games with ambiguity. However nearly all of these focus on normal form games.
- Extensive form games pose a number of new problems.
- How are beliefs updated as new information is received?
- Does the updating create dynamic consistency problems?

Extensive Form Games with Ambiguity

- There are a number of papers on games with ambiguity. However nearly all of these focus on normal form games.
- Extensive form games pose a number of new problems.
- How are beliefs updated as new information is received?
- Does the updating create dynamic consistency problems?
- Dynamic consistency is not restrictive if we do not require consequentialism, Machina JEL (1989), Hanany and Klibanoff, TE, 2007.

Extensive Form Games with Ambiguity

- There are a number of papers on games with ambiguity. However nearly all of these focus on normal form games.
- Extensive form games pose a number of new problems.
- How are beliefs updated as new information is received?
- Does the updating create dynamic consistency problems?
- Dynamic consistency is not restrictive if we do not require consequentialism, Machina JEL (1989), Hanany and Klibanoff, TE, 2007.
    - However non-consequentialist preferences will depend on past decisions and the choice set.

Extensive Form Games with Ambiguity

- There are a number of papers on games with ambiguity. However nearly all of these focus on normal form games.
- Extensive form games pose a number of new problems.
- How are beliefs updated as new information is received?
- Does the updating create dynamic consistency problems?
- Dynamic consistency is not restrictive if we do not require consequentialism, Machina JEL (1989), Hanany and Klibanoff, TE, 2007.
  - However non-consequentialist preferences will depend on past decisions and the choice set.
  - Moreover the updates will not typically satisfy the MEU axioms even if the priors satisfy them.

Extensive Form Games with Ambiguity

- There are a number of papers on games with ambiguity. However nearly all of these focus on normal form games.
- Extensive form games pose a number of new problems.
- How are beliefs updated as new information is received?
- Does the updating create dynamic consistency problems?
- Dynamic consistency is not restrictive if we do not require consequentialism, Machina JEL (1989), Hanany and Klibanoff, TE, 2007.
  - However non-consequentialist preferences will depend on past decisions and the choice set.
  - Moreover the updates will not typically satisfy the MEU axioms even if the priors satisfy them.
- Ambiguity is compatible with dynamic consistency if we restrict to a single decision tree and individuals have recursive multiple priors preferences, Wakker (1998), Epstein and Schneider JET (2003).

Dynamic Ellsberg Paradox

		30	60		
		R	B	Y	
Choice 1	a	100	0	0	✓
	b	0	100	0	
Choice 2	c	100	0	100	
	d	0	100	100	✓

- Suppose an individual is offered Choice 2.

Dynamic Ellsberg Paradox

		30	60		
		R	B	Y	
Choice 1	a	100	0	0	✓
	b	0	100	0	
Choice 2	c	100	0	100	
	d	0	100	100	✓

- Suppose an individual is offered Choice 2.
- He/she chooses d;

Dynamic Ellsberg Paradox

		30	60		
		R	B	Y	
Choice 1	a	100	0	0	✓
	b	0	100	0	
Choice 2	c	100	0	100	
	d	0	100	100	✓

- Suppose an individual is offered Choice 2.
- He/she chooses d;
- then (s)he is told the ball is not yellow and given the opportunity to reconsider;

Dynamic Ellsberg Paradox

		30	60		
		R	B	Y	
Choice 1	a	100	0	0	✓
	b	0	100	0	
Choice 2	c	100	0	100	
	d	0	100	100	✓

- Suppose an individual is offered Choice 2.
- He/she chooses d;
- then (s)he is told the ball is not yellow and given the opportunity to reconsider;
- ambiguity-aversion may cause her to choose c, which is now the option with known probability.

		30	60		
		R	B	Y	
Choice 1	a	100	0	0	✓
	b	0	100	0	
Choice 2	c	100	0	100	
	d	0	100	100	✓

- Suppose an individual is offered Choice 2.
- He/she chooses d;
- then (s)he is told the ball is not yellow and given the opportunity to reconsider;
- ambiguity-aversion may cause her to choose c, which is now the option with known probability.
- This is a violation of dynamic consistency. The ex-ante optimal choice is d, but it is not chosen ex-post.

Consistent Planning

		R	B	Y
Choice 3	c	100	0	100
	d	0	100	100
	e	x	x	x

- Suppose now the individual faces the following decision problem.

Consistent Planning

		R	B	Y
Choice 3	c	100	0	100
	d	0	100	100
	e	x	x	x

- Suppose now the individual faces the following decision problem.
- At time $t = 0$, (s)he faces the choice between c, d and e which yields a certain amount of money, x, such that $d \succ x \succ c$.

Consistent Planning

		R	B	Y
Choice 3	c	100	0	100
	d	0	100	100
	e	x	x	x

- Suppose now the individual faces the following decision problem.
- At time $t = 0$, (s)he faces the choice between c, d and e which yields a certain amount of money, x, such that $d \succ x \succ c$.
- At time $t = 1$,

Consistent Planning

		R	B	Y
Choice 3	c	100	0	100
	d	0	100	100
	e	x	x	x

- Suppose now the individual faces the following decision problem.
- At time $t = 0$, (s)he faces the choice between c, d and e which yields a certain amount of money, x, such that $d \succ x \succ c$.
- At time $t = 1$,
  - (s)he is told whether or not the ball is yellow.

Consistent Planning

		R	B	Y
Choice 3	c	100	0	100
	d	0	100	100
	e	x	x	x

- Suppose now the individual faces the following decision problem.
- At time $t = 0$, (s)he faces the choice between c, d and e which yields a certain amount of money, x, such that $d \succ x \succ c$.
- At time $t = 1$,
  - (s)he is told whether or not the ball is yellow.
  - if (s)he chose c or d she has the opportunity to reconsider;

Consistent Planning

		R	B	Y
Choice 3	c	100	0	100
	d	0	100	100
	e	x	x	x

- Suppose now the individual faces the following decision problem.
- At time $t = 0$, (s)he faces the choice between c, d and e which yields a certain amount of money, x, such that $d \succ x \succ c$.
- At time $t = 1$,
  - (s)he is told whether or not the ball is yellow.
  - if (s)he chose c or d she has the opportunity to reconsider;
  - if (s)he chose e, (s)he has to stick with her previous choice.

Consistent Planning

		R	B	Y
Choice 3	c	100	0	100
	d	0	100	100
	e	x	x	x

- Suppose now the individual faces the following decision problem.
- At time $t = 0$, (s)he faces the choice between c, d and e which yields a certain amount of money, x, such that $d \succ x \succ c$.
- At time $t = 1$,
  - (s)he is told whether or not the ball is yellow.
  - if (s)he chose c or d she has the opportunity to reconsider;
  - if (s)he chose e, (s)he has to stick with her previous choice.
- If the decision-maker anticipates that (s)he will prefer c when told a ball is not yellow she may decide to choose e at $t = 0$.

Consistent Planning

		R	B	Y
Choice 3	c	100	0	100
	d	0	100	100
	e	x	x	x

- Suppose now the individual faces the following decision problem.
- At time $t = 0$, (s)he faces the choice between c, d and e which yields a certain amount of money, x, such that $d \succ x \succ c$.
- At time $t = 1$,
  - (s)he is told whether or not the ball is yellow.
  - if (s)he chose c or d she has the opportunity to reconsider;
  - if (s)he chose e, (s)he has to stick with her previous choice.
- If the decision-maker anticipates that (s)he will prefer c when told a ball is not yellow she may decide to choose e at $t = 0$.
- We refer to this as consistent planning, Siniscalchi TE (2011).

Consistent Planning

		R	B	Y
Choice 3	c	100	0	100
	d	0	100	100
	e	x	x	x

- Suppose now the individual faces the following decision problem.
- At time $t = 0$, (s)he faces the choice between c, d and e which yields a certain amount of money, x, such that $d \succ x \succ c$.
- At time $t = 1$,
  - (s)he is told whether or not the ball is yellow.
  - if (s)he chose c or d she has the opportunity to reconsider;
  - if (s)he chose e, (s)he has to stick with her previous choice.
- If the decision-maker anticipates that (s)he will prefer c when told a ball is not yellow she may decide to choose e at $t = 0$.
- We refer to this as consistent planning, Siniscalchi TE (2011).
- The decision-maker anticipates how the receipt of new information will change her preferences and takes this into account when making decisions.

The Neo-additive Model of Ambiguity

We use the neo-additive model of ambiguity, which represents preferences by:

$$\int u(a)\,d\nu = \alpha\delta M(a) + \delta(1-\alpha)\,m(a) + (1-\delta)\mathbf{E}_{\pi}u(a), \qquad (1)$$

- ν is a neo-additive capacity,
- $M(a)$ denotes the maximum utility of act a,
- $m(a)$ denotes the minimum utility of act a,
- $\mathbf{E}_{\pi}u(a)$ denotes the expected utility of act a.
- This is a special case of the Choquet expected utility model, which represents beliefs as capacities.

The Neo-additive Model of Ambiguity

We use the neo-additive model of ambiguity, which represents preferences by:

$$\int u(a)\, d\nu = \alpha\delta M(a) + \delta(1-\alpha)\, m(a) + (1-\delta)\mathbf{E}_{\pi} u(a), \tag{1}$$

- ν is a neo-additive capacity,
- $M(a)$ denotes the maximum utility of act a,
- $m(a)$ denotes the minimum utility of act a,
- $\mathbf{E}_{\pi} u(a)$ denotes the expected utility of act a.
- This is a special case of the Choquet expected utility model, which represents beliefs as capacities.

- δ is a measure of perceived ambiguity;

The Neo-additive Model of Ambiguity

We use the neo-additive model of ambiguity, which represents preferences by:

$$\int u(a)\, d\nu = \alpha\delta M(a) + \delta(1-\alpha)\, m(a) + (1-\delta)\mathbf{E}_{\pi} u(a)\,, \tag{1}$$

- ν is a neo-additive capacity,
- $M(a)$ denotes the maximum utility of act a,
- $m(a)$ denotes the minimum utility of act a,
- $\mathbf{E}_{\pi} u(a)$ denotes the expected utility of act a.
- This is a special case of the Choquet expected utility model, which represents beliefs as capacities.

- δ is a measure of perceived ambiguity;
- α measures ambiguity-attitude, $\alpha = 1$ (resp. $\alpha = 0$) corresponding to pure optimism (resp. pessimism).

The Neo-additive Model of Ambiguity

We use the neo-additive model of ambiguity, which represents preferences by:

$$\int u(a)\, d\nu = \alpha\delta M(a) + \delta(1-\alpha)\, m(a) + (1-\delta)\mathbf{E}_{\pi} u(a), \qquad (1)$$

- ν is a neo-additive capacity,
- $M(a)$ denotes the maximum utility of act a,
- $m(a)$ denotes the minimum utility of act a,
- $\mathbf{E}_{\pi} u(a)$ denotes the expected utility of act a.
- This is a special case of the Choquet expected utility model, which represents beliefs as capacities.

- δ is a measure of perceived ambiguity;
- α measures ambiguity-attitude, $\alpha = 1$ (resp. $\alpha = 0$) corresponding to pure optimism (resp. pessimism).
- Only 2 additional parameters needed compared to expected utility.

Suppose that event $E \subseteq S$ is observed.

Let ν_E denote the updated capacity then:

$$a \succsim_E b \Longleftrightarrow \int u(a(s))d\nu_E(s) \succcurlyeq \int u(a(s))d\nu_E(s).$$

Generalized Bayesian Updating (GBU)

$$\nu_E(A) = \frac{\nu(A \cap E)}{\nu(A \cap E) + 1 - \nu(E^c \cup A)} = \frac{\nu(A \cap E)}{\nu(A \cap E) + \bar{\nu}(A^c \cap E)},$$

where $\bar{\nu}(E) = 1 - \nu(E^c)$.

Updating Ambiguous Beliefs I

Suppose that event $E \subseteq S$ is observed.

Let ν_E denote the updated capacity then:

$$a \succsim_E b \Longleftrightarrow \int u(a(s))d\nu_E(s) \succsim \int u(a(s))d\nu_E(s).$$

Generalized Bayesian Updating (GBU)

$$\nu_E(A) = \frac{\nu(A \cap E)}{\nu(A \cap E) + 1 - \nu(E^c \cup A)} = \frac{\nu(A \cap E)}{\nu(A \cap E) + \bar{\nu}(A^c \cap E)},$$

where $\bar{\nu}(E) = 1 - \nu(E^c)$.

Note that $\nu_E(A)$ is defined even if $\nu(E) = 0$. Thus it is possible to update beliefs off the equilibrium path.

Updating Ambiguous Beliefs II

If ν is neo-additive both the ex-ante and the ex-post preferences may be given a multiple priors representation:

$$\int f d\nu = \alpha \cdot \inf_{\pi \in B} \mathbf{E}_{\pi}(f) + (1-\alpha) \cdot \mathbf{E}_{\pi} \sup_{\pi \in B} (f),$$

$$\int f d\nu_E = \alpha \cdot \inf_{\pi \in B_E} \mathbf{E}_{\pi}(f) + (1-\alpha) \cdot \mathbf{E}_{\pi} \sup_{\pi \in B_E} (f),$$

where B_E is the set of Bayesian updates of B, i.e.

$$\left\{ p \in \Delta(E) : \exists q \in B, \forall A \subseteq S, p(A) = \frac{q(A \cap E)}{q(E)} \right\}.$$

Consistent Planning Equilibrium

In a consistent planning equilibrium

- Players have CEU/MEU preferences.

Definition

A *Consistent-Planning Equilibrium Under Ambiguity (CPEBUA)* for an extensive-form game with perfect information is a strategy-belief-tuple $((s_1, b_1), (s_2, b_2))$ such that for each player i and each history $h \in H_i$,

$$s_i^h \in \operatorname{supp} b_j(S_i^h | h) \subseteq \underset{\hat{s}_i^h \in S_i^h}{\arg\max} V_i(\hat{s}_i^h, b_i(S_j^h | h)).$$

Consistent Planning Equilibrium

In a consistent planning equilibrium

- Players have CEU/MEU preferences.
- They update beliefs using Generalized Bayesian Updating.

Definition

A *Consistent-Planning Equilibrium Under Ambiguity (CPEBUA)* for an extensive-form game with perfect information is a strategy-belief-tuple $((s_1, b_1), (s_2, b_2))$ such that for each player i and each history $h \in H_i$,

$$s_i^h \in \operatorname{supp} b_j(S_i^h|h) \subseteq \underset{\hat{s}_i^h \in S_i^h}{\arg\max} V_i(\hat{s}_i^h, b_i(S_j^h|h)).$$

Consistent Planning Equilibrium

In a consistent planning equilibrium

- Players have CEU/MEU preferences.
- They update beliefs using Generalized Bayesian Updating.
- In any sub-game they play a best response to their beliefs.

Definition

A *Consistent-Planning Equilibrium Under Ambiguity (CPEBUA)* for an extensive-form game with perfect information is a strategy-belief-tuple $((s_1, b_1), (s_2, b_2))$ such that for each player i and each history $h \in H_i$,

$$s_i^h \in \operatorname{supp} b_j(S_i^h|h) \subseteq \underset{\hat{s}_i^h \in S_i^h}{\arg\max} V_i(\hat{s}_i^h, b_i(S_j^h|h)).$$

Consistent Planning Equilibrium

In a consistent planning equilibrium

- Players have CEU/MEU preferences.
- They update beliefs using Generalized Bayesian Updating.
- In any sub-game they play a best response to their beliefs.
- They anticipate how information they may receive in the future will change their preferences. (consistent planning)

Definition

A *Consistent-Planning Equilibrium Under Ambiguity (CPEBUA)* for an extensive-form game with perfect information is a strategy-belief-tuple $((s_1, b_1), (s_2, b_2))$ such that for each player i and each history $h \in H_i$,

$$s_i^h \in \operatorname{supp} b_j(S_i^h|h) \subseteq \underset{\hat{s}_i^h \in S_i^h}{\arg\max} V_i(\hat{s}_i^h, b_i(S_j^h|h)).$$

The Centipede Game: Cooperation

If there is sufficient ambiguity and players are sufficiently ambiguity-loving, the equilibrium involves playing "continue" until the final node.

Proposition

There will exist a pure strategy equilibrium in which the action played at node ρ is

$$\begin{cases} c_\rho, 1 \leqslant \rho \leqslant 2M-1; \\ s_\rho, \rho = 2M-1; \end{cases}$$

provided: $\delta(1-\alpha) \geqslant \frac{1}{3}$.

- At the final node Player 2 chooses stop since it is a dominant strategy.

The Centipede Game: Cooperation

If there is sufficient ambiguity and players are sufficiently ambiguity-loving, the equilibrium involves playing "continue" until the final node.

Proposition

There will exist a pure strategy equilibrium in which the action played at node ρ is

$$\begin{cases} c_\rho, 1 \leqslant \rho \leqslant 2M-1; \\ s_\rho, \rho = 2M-1; \end{cases}$$

provided: $\delta(1-\alpha) \geqslant \frac{1}{3}$.

- At the final node Player 2 chooses stop since it is a dominant strategy.
- Recall δ measures ambiguity, α represents ambiguity-attitude.

The Centipede Game: Non-Cooperation

- With high levels of ambiguity-aversion the only equilibrium is playing stop at every node.

The Centipede Game: Non-Cooperation

- With high levels of ambiguity-aversion the only equilibrium is playing stop at every node.
- Similar to the Nash equilibrium.

The Centipede Game: Non-Cooperation

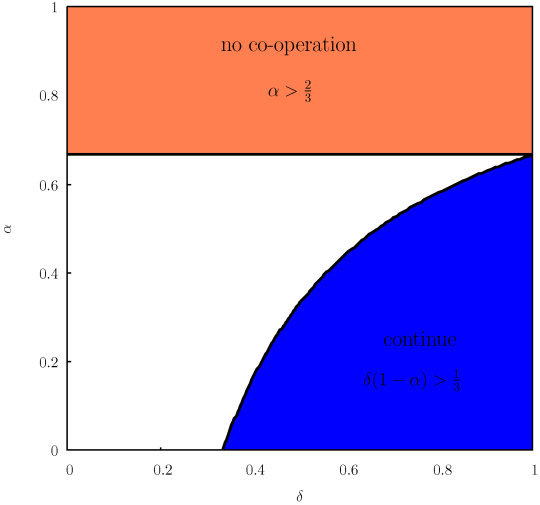
- With high levels of ambiguity-aversion the only equilibrium is playing stop at every node.
- Similar to the Nash equilibrium.

- With high levels of ambiguity-aversion the only equilibrium is playing stop at every node.
- Similar to the Nash equilibrium.

Proposition

Let Γ be a $2M$ stage centipede game, where $M \geqslant 2$, there will exist a consistent planning equilibrium in which s_ρ is played at every node ρ provided $\alpha \geqslant \frac{2}{3}$. This equilibrium will be unique provided the inequality is strict.

Cooperation versus Non-Cooperation



Mixed Equilibria I

Proposition

Let Γ be a $2M$ stage centipede game, where $M \geqslant 2$, then Γ does not have a pure strategy equilibrium when $\alpha < \frac{2}{3}$ and $\delta(1-\alpha) < \frac{1}{3}$.

- Under these assumptions there is a mixed strategy equilibrium. The two players randomly choose stop at one of the final two nodes at which they have to play.

Mixed Equilibria I

Proposition

Let Γ be a $2M$ stage centipede game, where $M \geqslant 2$, then Γ does not have a pure strategy equilibrium when $\alpha < \frac{2}{3}$ and $\delta(1-\alpha) < \frac{1}{3}$.

- Under these assumptions there is a mixed strategy equilibrium. The two players randomly choose stop at one of the final two nodes at which they have to play.
- Thus the game may end at $2M-2, 2M-1$ or $2M$.

Mixed Equilibria I

Proposition

Let Γ be a $2M$ stage centipede game, where $M \geqslant 2$, then Γ does not have a pure strategy equilibrium when $\alpha < \frac{2}{3}$ and $\delta(1-\alpha) < \frac{1}{3}$.

- Under these assumptions there is a mixed strategy equilibrium. The two players randomly choose stop at one of the final two nodes at which they have to play.
- Thus the game may end at $2M-2, 2M-1$ or $2M$.
- Kilka and Weber, (2001) experimentally estimate neo-additive preferences and find on average $\alpha = \delta = \frac{1}{2}$. This is compatible with the present case.

Mixed Equilibria II

Proposition

Given GB-updating, $\delta(p) := \frac{\delta}{\delta+(1-\delta)p}$*, there is a (CPBEUA)* $\langle (s_1^*, \nu_1(\mu_1^*, \alpha, \delta)), (s_2^*, \nu_2(\mu_2^*, \alpha, \delta)) \rangle$ *in which*

- *player 1 believes with degree of ambiguity* δ *that player 2 will choose his strategies with probability* μ_1*,*
  $\mu_1(s_2) = p$*, for* $s_2 = s_2^{2M}$; $1 - p$ *for* $s_2 = s_2^{2M-2}$; 0 *otherwise, where* $p = \frac{\delta(2-3\alpha)}{(1-\delta)}$.
- *player 2 believes that player 1 will choose her strategies with probability* μ_2*;*
  $\mu_2(s_1) = q$*, for* $s_1 = s_1^{2M+1}$; $\mu_2(s_1) = 1 - q$*, for* $s_1 = s_1^{2M-1}$; $\mu_2(s_1) = 0$ *otherwise, where* $q = \frac{1-3\delta(1-\alpha)}{3(1-\delta)}$.

The game will end either at $2M - 2$ *or* $2M$ *with player 2 exiting, or* $2M - 1$ *with player 1 exiting.*

Exponential Utility

- So far we have assumed utility is linear in money, (risk neutrality).

Exponential Utility

- So far we have assumed utility is linear in money, (risk neutrality).
- Similar results apply if utility is convex $u(x) = e^{\lambda x}$.

Exponential Utility

- So far we have assumed utility is linear in money, (risk neutrality).
- Similar results apply if utility is convex $u(x) = e^{\lambda x}$.
- or concave $u(x) = -e^{-\lambda x}$;

Exponential Utility

- So far we have assumed utility is linear in money, (risk neutrality).
- Similar results apply if utility is convex $u(x) = e^{\lambda x}$.
- or concave $u(x) = -e^{-\lambda x}$;
- Convex utility increases (resp. reduces) the parameter range in which cooperation is possible (resp. an equilibrium with zero cooperation exists).

Exponential Utility

- So far we have assumed utility is linear in money, (risk neutrality).
- Similar results apply if utility is convex $u(x) = e^{\lambda x}$.
- or concave $u(x) = -e^{-\lambda x}$;
- Convex utility increases (resp. reduces) the parameter range in which cooperation is possible (resp. an equilibrium with zero cooperation exists).
- Concave utility has the opposite effect.

Exponential Utility

- So far we have assumed utility is linear in money, (risk neutrality).
- Similar results apply if utility is convex $u(x) = e^{\lambda x}$.
- or concave $u(x) = -e^{-\lambda x}$;
- Convex utility increases (resp. reduces) the parameter range in which cooperation is possible (resp. an equilibrium with zero cooperation exists).
- Concave utility has the opposite effect.
- This extends our analysis to those experimental studies of the centipede game have increasing or decreasing differences between successive legs.

Extensions/Work in Progress

- Extend to a larger class of games e.g. multi-stage games.

Extensions/Work in Progress

- Extend to a larger class of games e.g. multi-stage games.
- Add a type space.

Extensions/Work in Progress

- Extend to a larger class of games e.g. multi-stage games.
- Add a type space.
- We need a general result about existence of equilibrium.

Extensions/Work in Progress

- Extend to a larger class of games e.g. multi-stage games.
- Add a type space.
- We need a general result about existence of equilibrium.
- Extend to related games.

Extensions/Work in Progress

- Extend to a larger class of games e.g. multi-stage games.
- Add a type space.
- We need a general result about existence of equilibrium.
- Extend to related games.
    - Alternating offers bargaining.

Extensions/Work in Progress

- Extend to a larger class of games e.g. multi-stage games.
- Add a type space.
- We need a general result about existence of equilibrium.
- Extend to related games.
  - Alternating offers bargaining.
  - Chain store paradox.

Extensions/Work in Progress

- Extend to a larger class of games e.g. multi-stage games.
- Add a type space.
- We need a general result about existence of equilibrium.
- Extend to related games.
    - Alternating offers bargaining.
    - Chain store paradox.
    - Repeated Games

Extensions/Work in Progress

- Extend to a larger class of games e.g. multi-stage games.
- Add a type space.
- We need a general result about existence of equilibrium.
- Extend to related games.
  - Alternating offers bargaining.
  - Chain store paradox.
  - Repeated Games
  - Herding in financial markets.

Extensions/Work in Progress

- Extend to a larger class of games e.g. multi-stage games.
- Add a type space.
- We need a general result about existence of equilibrium.
- Extend to related games.
    - Alternating offers bargaining.
    - Chain store paradox.
    - Repeated Games
    - Herding in financial markets.
    - Asset price bubbles.

Pure Equilibria

- Finite games of complete and perfect information, always have a Nash equilibrium in pure strategies.

Pure Equilibria

- Finite games of complete and perfect information, always have a Nash equilibrium in pure strategies.
- This is found by backward induction.

Pure Equilibria

- Finite games of complete and perfect information, always have a Nash equilibrium in pure strategies.
- This is found by backward induction.
- We show, by example, that a finite game of complete and perfect information may not have a pure strategy equilibrium if there is ambiguity.

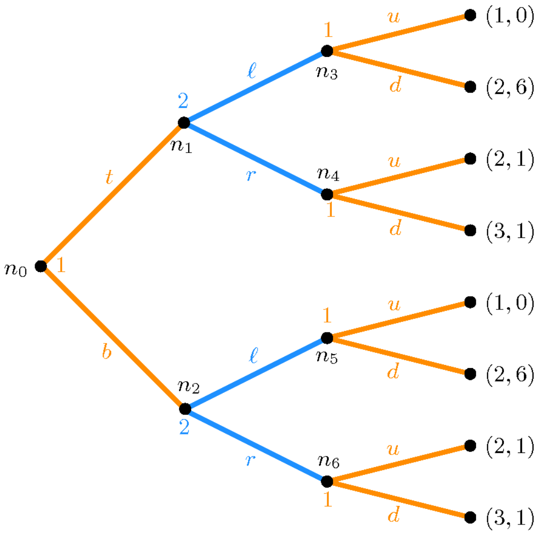
Pure Equilibria

- Finite games of complete and perfect information, always have a Nash equilibrium in pure strategies.
- This is found by backward induction.
- We show, by example, that a finite game of complete and perfect information may not have a pure strategy equilibrium if there is ambiguity.
- Thus a well-known property of Nash equilibria may no longer apply when there is ambiguity.

Pure Equilibria

- Finite games of complete and perfect information, always have a Nash equilibrium in pure strategies.
- This is found by backward induction.
- We show, by example, that a finite game of complete and perfect information may not have a pure strategy equilibrium if there is ambiguity.
- Thus a well-known property of Nash equilibria may no longer apply when there is ambiguity.
- Moreover the backward induction technique may fail when there is ambiguity.

Pure Equilibria



Equilibria

Proposition

Provided $\alpha > \frac{5}{6} \geqslant \alpha\delta$, Game A has no pure strategy equilibrium with ambiguity.

- The parameter restrictions, $\alpha > \frac{5}{6} \geqslant \alpha\delta$, imply that both players perceive positive degrees of ambiguity and have high levels of ambiguity-aversion.

Equilibria

Proposition

Provided $\alpha > \frac{5}{6} \geqslant \alpha\delta$, Game A has no pure strategy equilibrium with ambiguity.

- The parameter restrictions, $\alpha > \frac{5}{6} \geqslant \alpha\delta$, imply that both players perceive positive degrees of ambiguity and have high levels of ambiguity-aversion.
- Player 2 has different ordinal preferences at node n_2 depending whether it is on or off the equilibrium path.

Equilibria

Proposition

Provided $\alpha > \frac{5}{6} \geqslant \alpha\delta$, Game A has no pure strategy equilibrium with ambiguity.

- The parameter restrictions, $\alpha > \frac{5}{6} \geqslant \alpha\delta$, imply that both players perceive positive degrees of ambiguity and have high levels of ambiguity-aversion.
- Player 2 has different ordinal preferences at node n_2 depending whether it is on or off the equilibrium path.
- Ambiguity-aversion causes Player 2 to choose the relatively safe action r off the equilibrium path.

Equilibria

Proposition

Provided $\alpha > \frac{5}{6} \geqslant \alpha\delta$, Game A has no pure strategy equilibrium with ambiguity.

- The parameter restrictions, $\alpha > \frac{5}{6} \geqslant \alpha\delta$, imply that both players perceive positive degrees of ambiguity and have high levels of ambiguity-aversion.
- Player 2 has different ordinal preferences at node n_2 depending whether it is on or off the equilibrium path.
- Ambiguity-aversion causes Player 2 to choose the relatively safe action r off the equilibrium path.
- If a small perturbation were applied to all the pay-offs the same reasoning would still imply the non-existence of a pure equilibrium. Thus one cannot say the example is non-generic.

Equilibria

Proposition

Provided $\alpha > \frac{5}{6} \geqslant \alpha\delta$, Game A has no pure strategy equilibrium with ambiguity.

- The parameter restrictions, $\alpha > \frac{5}{6} \geqslant \alpha\delta$, imply that both players perceive positive degrees of ambiguity and have high levels of ambiguity-aversion.
- Player 2 has different ordinal preferences at node n_2 depending whether it is on or off the equilibrium path.
- Ambiguity-aversion causes Player 2 to choose the relatively safe action r off the equilibrium path.
- If a small perturbation were applied to all the pay-offs the same reasoning would still imply the non-existence of a pure equilibrium. Thus one cannot say the example is non-generic.
- There are two Nash equilibria in pure strategies, which are compatible with backward induction.

Equilibria

Proposition

Provided $\alpha > \frac{5}{6} \geqslant \alpha\delta$, Game A has no pure strategy equilibrium with ambiguity.

- The parameter restrictions, $\alpha > \frac{5}{6} \geqslant \alpha\delta$, imply that both players perceive positive degrees of ambiguity and have high levels of ambiguity-aversion.
- Player 2 has different ordinal preferences at node n_2 depending whether it is on or off the equilibrium path.
- Ambiguity-aversion causes Player 2 to choose the relatively safe action r off the equilibrium path.
- If a small perturbation were applied to all the pay-offs the same reasoning would still imply the non-existence of a pure equilibrium. Thus one cannot say the example is non-generic.
- There are two Nash equilibria in pure strategies, which are compatible with backward induction.
  - Player 1 may either play t or b at node n_o, and plays d at nodes n_3-n_6.

Equilibria

Proposition

Provided $\alpha > \frac{5}{6} \geqslant \alpha\delta$, Game A has no pure strategy equilibrium with ambiguity.

- The parameter restrictions, $\alpha > \frac{5}{6} \geqslant \alpha\delta$, imply that both players perceive positive degrees of ambiguity and have high levels of ambiguity-aversion.
- Player 2 has different ordinal preferences at node n_2 depending whether it is on or off the equilibrium path.
- Ambiguity-aversion causes Player 2 to choose the relatively safe action r off the equilibrium path.
- If a small perturbation were applied to all the pay-offs the same reasoning would still imply the non-existence of a pure equilibrium. Thus one cannot say the example is non-generic.
- There are two Nash equilibria in pure strategies, which are compatible with backward induction.
    - Player 1 may either play t or b at node n_o, and plays d at nodes n_3-n_6.
    - Player 2 chooses ℓ at node n_1 and at node n_2.

Equilibria with Ambiguity I

Since CEU preferences respect dominance Player 1 will choose action d at nodes n_3, n_4, n_5 and n_6.
Suppose that node n_1 is on the equilibrium path. Then player 2's (Choquet) expected utility from his actions at node n_1 are:

$$V^2(\ell|n_1) = 6\delta(1-\alpha) + \delta\alpha.0 + (1-\delta)6 = (1-\delta\alpha)6$$
$$V^2(r|n_1) = 1.$$

Thus 2 will choose ℓ at node n_1 provided $\frac{5}{6} \geqslant \alpha\delta$.
Now node n_2 must be off the equilibrium path. Applying the formula for GBU updating we find that 2's preferences at node n_2 are represented by

$$\alpha \min\left\{u^2(\ell,u), u^2(r,d)\right\} + (1-\alpha)\max\left\{u^2(\ell,u), u^2(r,d)\right\}.$$

Thus $V^2(\ell|n_2) = \alpha.0 + (1-\alpha)6 = 6 - 6\alpha$ and $V^2(r|n_2) = \alpha.1 + (1-\alpha)1 = 1$.
Hence 2 will choose action r provided $1 > 6 - 6\alpha \Leftrightarrow 6\alpha > 5 \Leftrightarrow \alpha > \frac{5}{6}$.

Equilibria with Ambiguity II

- Now consider 1's decision at node n_o.

Equilibria with Ambiguity II

- Now consider 1's decision at node n_o.
- His (Choquet) expected utilities are: $V^1(t|n_o) = 2$, $V^1(b|n_o) = 3$. Hence 1's best response is b.

Equilibria with Ambiguity II

- Now consider 1's decision at node n_o.
- His (Choquet) expected utilities are: $V^1(t|n_o) = 2$, $V^1(b|n_o) = 3$. Hence 1's best response is b.
- However this contradicts the original assumption that n_1 is on the equilibrium path.

Equilibria with Ambiguity II

- Now consider 1's decision at node n_o.
- His (Choquet) expected utilities are: $V^1(t|n_o) = 2$, $V^1(b|n_o) = 3$. Hence 1's best response is b.
- However this contradicts the original assumption that n_1 is on the equilibrium path.
- Thus there is no pure equilibrium in which n_1 is on the equilibrium path.

Equilibria with Ambiguity II

- Now consider 1's decision at node n_o.
- His (Choquet) expected utilities are: $V^1(t|n_o) = 2$, $V^1(b|n_o) = 3$. Hence 1's best response is b.
- However this contradicts the original assumption that n_1 is on the equilibrium path.
- Thus there is no pure equilibrium in which n_1 is on the equilibrium path.
- A similar argument establishes that we cannot have a pure strategy equilibrium in which node n_2 is on the equilibrium path.