Ambiguity and the Centipede Game

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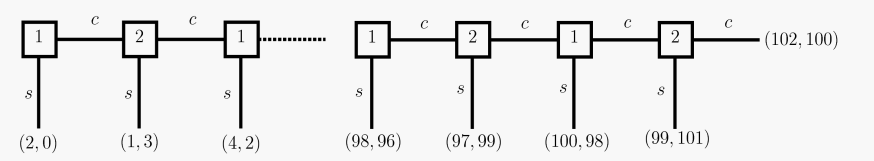
July 2017

The Centipede Game I



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- The centipede game could represent two countries disarming in stages. An illegal drug producer and a dealer building cooperation in stages.

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- Ambiguity (unknown probabilities) can cause individuals to overweight bad outcomes. This is know as ambiguity-aversion.
- Less frequently (but still quite common) ambiguity can cause individuals to over-weight good outcomes. This is know as ambiguity-loving.
- Given the high pay-offs towards the end of the centipede it is plausible that ambiguity-loving behaviour may be the reason that people do not follow the Nash equilibrium.

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- But non-consequentialist preferences will depend on past decisions and the choice set.
- We assume consistent planning. Agents take into account that future preferences may be different when making their initial decisions.

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		B	$G \cap L$	$G \cap W$
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- We refer to this as **consistent planning** (Siniscalchi [2011]).

The Neo-additive Model of Ambiguity

We use the neo-additive model of ambiguity, which represents preferences by:

$$\int u(a)\, d\nu = \alpha\delta m(a) + \delta(1-\alpha) M(a) + (1-\delta)\mathbf{E}_{\pi} u(a). \qquad (1)$$

- ν is a neo-additive capacity,
- $M(a)$ denotes the maximum utility of act a,
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- Only 2 additional parameters needed compared to expected utility.

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    - S_i^h, S_{-i}^h & S^h are the corresponding continuation sets.

Conditional neo-expected Pay-offs

If Player i's initial theory about how her opponent is choosing his strategy is given by the probability measure π_i on S_{-i};
Then the probability of reaching stage t via history h is given by

$$\pi_i \left(S_{-i}(h)\right)$$

and his updated theory π_i^h on S_{-i}^h is the Bayesian update of π_i .

Hence her evaluation of the *conditional neo-expected payoff* of her continuation strategy s_i^h is given by:

$$V_i^h\left(s_i^h|\pi_i\right) = \left(1-\delta_i^h\right)\mathbb{E}_{\pi_i^h} u_i\left(s_i^h, \cdot\right)$$

$$+\delta_i^h\left[\alpha_i \min_{s_{-i}^h \in S_{-i}^h} u_i^h\left(s_i^h, s_{-i}^h\right) + (1-\alpha_i)\max_{s_{-i}^h \in S_{-i}^h} u_i^h\left(s_i^h, s_{-i}^h\right)\right],$$

$$\text{where } \delta_i^h = \frac{\delta_i}{\delta_i + (1-\delta_i)\,\pi_i\left(S_{-i}(h)\right)}.$$

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Definition (*Consistent Planning Equilibrium Under Ambiguity*)

CP-EUA is a profile of beliefs $\langle \nu_1, \nu_2 \rangle$ *such that for each player* $i = 1, 2$,

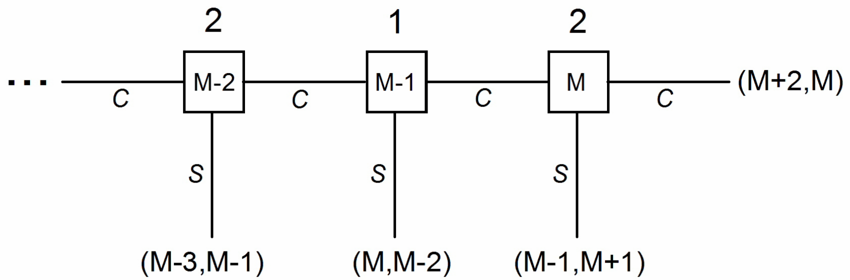
$$\forall s_i \in \operatorname{supp} \nu_i \Rightarrow V_i^h \left(s_i^h | \pi_i \right) \geqslant V_i^h \left(\left(a_i, s_i^h \left(-t \right) \right) | \pi_i \right)$$

$$\forall a_i \in A_i^h, \textit{ every } h \in H^{t-1}, \textit{ and every } t = 1, \ldots, T.$$

Theorem

Let Γ *be a multi-stage game with 2 players. Then* Γ *has at least one CP-EUA for any given parameters* $\alpha_1, \alpha_2, \delta_1, \delta_2$, *where* $1 \leqslant \alpha_i \leqslant 0, 0 < \delta_i \leqslant 1$, *for* $i = 1, 2$.

Returning to the Centipede Game

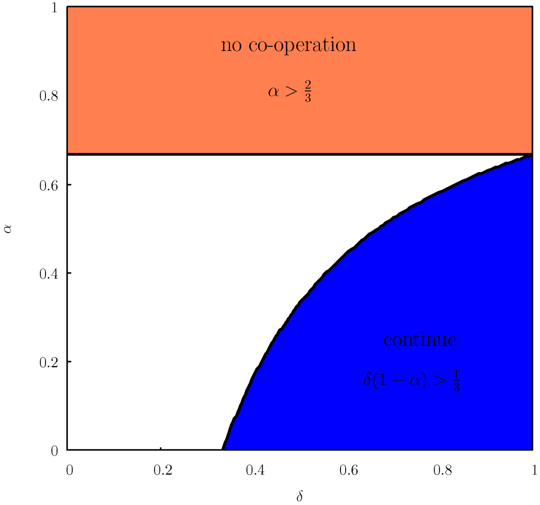


Suppose $\delta_1 = \delta_2 = \delta$ and $\alpha_1 = \alpha_2 = \alpha$.
Recall δ measures ambiguity, α represents ambiguity-attitude.

Pure-strategy equilibria

1. **Cooperation**
   If there is sufficient ambiguity and players are sufficiently ambiguity-loving (that is, provided $\delta(1-\alpha) \geqslant \frac{1}{3}$), then equilibrium involves playing "continue" until the final node.
   - At final node player 2 chooses stop since it is a dominant strategy for player 2 at that point.
2. **Non-cooperation**
   With high levels of ambiguity-aversion (that is, provided $\alpha \geqslant \frac{2}{3}$) the only equilibrium is playing stop at every node.
   - Similar to Nash equilibrium.

Cooperation versus Non-Cooperation



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Proposition

Let Γ be a $2M$ stage centipede game, where $M \geqslant 2$, then Γ does not have a pure strategy equilibrium when $\alpha < \frac{2}{3}$ and $\delta(1-\alpha) < \frac{1}{3}$.

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- Under these assumptions there is a mixed strategy equilibrium. The two players randomly choose stop at one of the final two nodes at which they have to play.

Mixed Equilibria I

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- Under these assumptions there is a mixed strategy equilibrium. The two players randomly choose stop at one of the final two nodes at which they have to play.
- Thus the game may end at one of the last three stages, $2M-2, 2M-1$ or $2M$.

Mixed Equilibria II

Proposition

Given GB-updating, $\delta(p) := \frac{\delta}{\delta+(1-\delta)p}$, *there is a (CP-EUA)* $\langle (s_1^*, \nu_1(\mu_1^*, \alpha, \delta)), (s_2^*, \nu_2(\mu_2^*, \alpha, \delta)) \rangle$ *in which:*

- *player 1 believes with degree of ambiguity* δ *that player 2 will choose his strategies with probability* μ_1, $\mu_1(s_2) = p$, *for* $s_2 = s_2^{2M}; 1-p$ *for* $s_2 = s_2^{2M-2}; 0$ *otherwise, where* $p = \frac{\delta(2-3\alpha)}{(1-\delta)}$.
- *player 2 believes that player 1 will choose her strategies with probability* μ_2; $\mu_2(s_1) = q$, *for* $s_1 = s_1^{2M+1}; \mu_2(s_1) = 1-q$, *for* $s_1 = s_1^{2M-1}; \mu_2(s_1) = 0$ *otherwise, where* $q = \frac{1-3\delta(1-\alpha)}{3(1-\delta)}$.

The game will end either at $2M-2$ *or* $2M$ *with player 2 exiting, or* $2M-1$ *with player 1 exiting.*

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