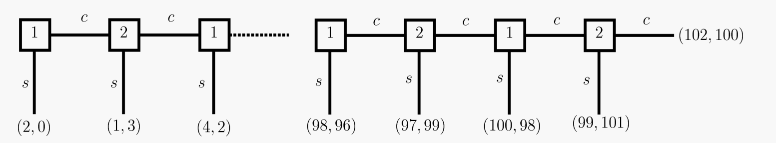
Ambiguity and the Centipede Game

Jürgen Eichberger, Simon Grant and David Kelsey
Heidelberg University,
Australian National University,
University of Exeter.

University of Exeter.

May 2018

The Centipede Game I



- There are two people A and B. Between them is a table which contains $2M$ one-pound coins and a single two-pound coin. They move alternately.

The Centipede Game II

- There are two people A and B. Between them is a table which contains $2M$ one-pound coins and a single two-pound coin. They move alternately.
- At each move there are two actions available.

- There are two people A and B. Between them is a table which contains $2M$ one-pound coins and a single two-pound coin. They move alternately.
- At each move there are two actions available.
  - The player whose move it is may either pick up the two-pound coin in which case the game ends;

The Centipede Game II

- There are two people A and B. Between them is a table which contains $2M$ one-pound coins and a single two-pound coin. They move alternately.
- At each move there are two actions available.
  - The player whose move it is may either pick up the two-pound coin in which case the game ends;
  - or (s)he may pick up 2 one-pound coins keep one and give the other to his/her opponent; in which case the game continues.

The Centipede Game II

- There are two people A and B. Between them is a table which contains $2M$ one-pound coins and a single two-pound coin. They move alternately.
- At each move there are two actions available.
  - The player whose move it is may either pick up the two-pound coin in which case the game ends;
  - or (s)he may pick up 2 one-pound coins keep one and give the other to his/her opponent; in which case the game continues.
- There is a unique iterated dominance equilibrium. At any node the player whose move it is picks up the 2-pound coin and ends the game.

Ambiguity may lead to cooperation

- Intuition suggests that individuals will cooperate to achieve the high pay-offs towards the end of the game.

Ambiguity may lead to cooperation

- Intuition suggests that individuals will cooperate to achieve the high pay-offs towards the end of the game.
- Experimental evidence also shows that subjects manage to cooperate.

Ambiguity may lead to cooperation

- Intuition suggests that individuals will cooperate to achieve the high pay-offs towards the end of the game.
- Experimental evidence also shows that subjects manage to cooperate.
- Ambiguity (unknown probabilities) can cause individuals to overweight bad outcomes. This is know as ambiguity-aversion.

Ambiguity may lead to cooperation

- Intuition suggests that individuals will cooperate to achieve the high pay-offs towards the end of the game.
- Experimental evidence also shows that subjects manage to cooperate.
- Ambiguity (unknown probabilities) can cause individuals to overweight bad outcomes. This is know as ambiguity-aversion.
- Less frequently (but still quite common) ambiguity can cause individuals to over-weight good outcomes. This is know as ambiguity-loving.

Ambiguity may lead to cooperation

- Intuition suggests that individuals will cooperate to achieve the high pay-offs towards the end of the game.
- Experimental evidence also shows that subjects manage to cooperate.
- Ambiguity (unknown probabilities) can cause individuals to overweight bad outcomes. This is know as ambiguity-aversion.
- Less frequently (but still quite common) ambiguity can cause individuals to over-weight good outcomes. This is know as ambiguity-loving.
- Given the high pay-offs towards the end of the centipede it is plausible that ambiguity-loving behaviour may be the reason that people cooperate.

Extensive Form Games with Ambiguity

- There are a number of papers on games with ambiguity.

Extensive Form Games with Ambiguity

- There are a number of papers on games with ambiguity.
- However most of these focus on normal form games.

Extensive Form Games with Ambiguity

- There are a number of papers on games with ambiguity.
- However most of these focus on normal form games.
- Extensive form games pose new problems.

Extensive Form Games with Ambiguity

- There are a number of papers on games with ambiguity.
- However most of these focus on normal form games.
- Extensive form games pose new problems.
- How are beliefs updated as new information is received?

Extensive Form Games with Ambiguity

- There are a number of papers on games with ambiguity.
- However most of these focus on normal form games.
- Extensive form games pose new problems.
- How are beliefs updated as new information is received?
- Does the updating create dynamic consistency (DC) problems?

Ambiguity and Dynamic Consistency

- Ambiguity is compatible with DC if we restrict to a single decision tree and individuals have recursive multiple prior preferences.

Ambiguity and Dynamic Consistency

- Ambiguity is compatible with DC if we restrict to a single decision tree and individuals have recursive multiple prior preferences.
- DC *not* restrictive if we do not require consequentialism.

Ambiguity and Dynamic Consistency

- Ambiguity is compatible with DC if we restrict to a single decision tree and individuals have recursive multiple prior preferences.
- DC *not* restrictive if we do not require consequentialism.
- But non-consequentialist preferences will depend on past decisions.

Ambiguity and Dynamic Consistency

- Ambiguity is compatible with DC if we restrict to a single decision tree and individuals have recursive multiple prior preferences.
- DC *not* restrictive if we do not require consequentialism.
- But non-consequentialist preferences will depend on past decisions.
- We assume consistent planning. Agents take into account that future preferences may be different when making their initial decisions.

Ambiguity and Dynamic Consistency

- Ambiguity is compatible with DC if we restrict to a single decision tree and individuals have recursive multiple prior preferences.
- DC *not* restrictive if we do not require consequentialism.
- But non-consequentialist preferences will depend on past decisions.
- We assume consistent planning. Agents take into account that future preferences may be different when making their initial decisions.
    - People, Places, Things.

Ambiguity and Dynamic Consistency

- Ambiguity is compatible with DC if we restrict to a single decision tree and individuals have recursive multiple prior preferences.
- DC *not* restrictive if we do not require consequentialism.
- But non-consequentialist preferences will depend on past decisions.
- We assume consistent planning. Agents take into account that future preferences may be different when making their initial decisions.
    - People, Places, Things.
- Consistent planning allows decisions to be analysed by backward induction.

Consistent Planning

- At $t = 0$, an individual chooses between:

Consistent Planning

- At $t = 0$, an individual chooses between:
  - a bet, b_W, which pays 1 if the event W obtains in period 2 ;

Consistent Planning

- At $t = 0$, an individual chooses between:
  - a bet, b_W, which pays 1 if the event W obtains in period 2 ;
  - a guaranteed payment, $\bar{b}$ of x .

Consistent Planning

- At $t = 0$, an individual chooses between:
  - a bet, b_W, which pays 1 if the event W obtains in period 2 ;
  - a guaranteed payment, $\bar{b}$ of x .
- At time $t = 1$ (s)he receives a signal which is either good G or bad B.

Consistent Planning

- At $t = 0$, an individual chooses between:
  - a bet, b_W, which pays 1 if the event W obtains in period 2 ;
  - a guaranteed payment, $\bar{b}$ of x .
- At time $t = 1$ (s)he receives a signal which is either good G or bad B.
  - If (s)he chose to bet she now has the option of switching to a certain payment b_G. In effect selling her bet.

Consistent Planning

- At $t = 0$, an individual chooses between:
  - a bet, b_W, which pays 1 if the event W obtains in period 2 ;
  - a guaranteed payment, $\bar{b}$ of x .
- At time $t = 1$ (s)he receives a signal which is either good G or bad B.
  - If (s)he chose to bet she now has the option of switching to a certain payment b_G. In effect selling her bet.
- The pay-offs of the 3 options are:

Options		B	$G \cap L$	$G \cap W$
	$\bar{b}$	x	x	x
	b_G	0	q	q
	b_W	0	0	1

Consistent Planning

- At $t = 0$, an individual chooses between:
  - a bet, b_W, which pays 1 if the event W obtains in period 2 ;
  - a guaranteed payment, $\bar{b}$ of x .
- At time $t = 1$ (s)he receives a signal which is either good G or bad B.
  - If (s)he chose to bet she now has the option of switching to a certain payment b_G. In effect selling her bet.
- The pay-offs of the 3 options are:

		B	$G \cap L$	$G \cap W$
	$\bar{b}$	x	x	x
Options	b_G	0	q	q
	b_W	0	0	1

Consistent Planning

- At $t = 0$, an individual chooses between:
  - a bet, b_W, which pays 1 if the event W obtains in period 2 ;
  - a guaranteed payment, $\bar{b}$ of x .
- At time $t = 1$ (s)he receives a signal which is either good G or bad B.
  - If (s)he chose to bet she now has the option of switching to a certain payment b_G. In effect selling her bet.
- The pay-offs of the 3 options are:

Options		B	$G \cap L$	$G \cap W$
	$\bar{b}$	x	x	x
	b_G	0	q	q
	b_W	0	0	1

- Suppose $b_W \succ \bar{b} \succ b_G$ but $b_G \succ_G b_W$.

Consistent Planning

- At $t = 0$, an individual chooses between:
  - a bet, b_W, which pays 1 if the event W obtains in period 2 ;
  - a guaranteed payment, $\bar{b}$ of x .
- At time $t = 1$ (s)he receives a signal which is either good G or bad B.
  - If (s)he chose to bet she now has the option of switching to a certain payment b_G. In effect selling her bet.
- The pay-offs of the 3 options are:

Options		B	$G \cap L$	$G \cap W$
	$\bar{b}$	x	x	x
	b_G	0	q	q
	b_W	0	0	1

- Suppose $b_W \succ \bar{b} \succ b_G$ but $b_G \succ_G b_W$.
- If DM *anticipates* she will prefer b_G in period 1 when told the signal's realization is G , then may decide to choose $\bar{b}$ at $t = 0$.

Leicester City 2016-17 Football Season

- At the beginning of the 2016-17 season the betting odds were 5,000-1 against Leicester winning the premier league title.

- At the beginning of the 2016-17 season the betting odds were 5,000-1 against Leicester winning the premier league title.
- Despite this Leicester were top of the premier league.

- At the beginning of the 2016-17 season the betting odds were 5,000-1 against Leicester winning the premier league title.
- Despite this Leicester were top of the premier league.
- Any fan who had bet on Leicester stood to make a fortune.

Leicester City 2016-17 Football Season

- At the beginning of the 2016-17 season the betting odds were 5,000-1 against Leicester winning the premier league title.
- Despite this Leicester were top of the premier league.
- Any fan who had bet on Leicester stood to make a fortune.
- Halfway through the season it was clear that Leicester were doing much better than expected.

Leicester City 2016-17 Football Season

- At the beginning of the 2016-17 season the betting odds were 5,000-1 against Leicester winning the premier league title.
- Despite this Leicester were top of the premier league.
- Any fan who had bet on Leicester stood to make a fortune.
- Halfway through the season it was clear that Leicester were doing much better than expected.
- A good signal was received.

- At the beginning of the 2016-17 season the betting odds were 5,000-1 against Leicester winning the premier league title.
- Despite this Leicester were top of the premier league.
- Any fan who had bet on Leicester stood to make a fortune.
- Halfway through the season it was clear that Leicester were doing much better than expected.
- A good signal was received.
- Many fans cashed out, i.e. sold their bets back to the bookmaker.

The Neo-additive Model of Ambiguity

We use neo-additive preferences, which are represented by:

$$\int u(a)\, d\nu = \alpha\delta m(a) + \delta(1-\alpha) M(a) + (1-\delta)\mathbf{E}_{\pi} u(a). \tag{1}$$

- ν is a neo-additive capacity,

The Neo-additive Model of Ambiguity

We use neo-additive preferences, which are represented by:

$$\int u(a)\, d\nu = \alpha\delta m(a) + \delta(1-\alpha) M(a) + (1-\delta)\mathbf{E}_\pi u(a). \tag{1}$$

- ν is a neo-additive capacity,
  - δ is a measure of perceived ambiguity;

The Neo-additive Model of Ambiguity

We use neo-additive preferences, which are represented by:

$$\int u(a)\, d\nu = \alpha\delta m(a) + \delta(1-\alpha) M(a) + (1-\delta)\mathbf{E}_\pi u(a). \qquad (1)$$

- ν is a neo-additive capacity,
  - δ is a measure of perceived ambiguity;
  - α measures ambiguity-attitude, $\alpha = 1$ (resp. $\alpha = 0$) corresponding to pure optimism (resp. pessimism).

The Neo-additive Model of Ambiguity

We use neo-additive preferences, which are represented by:

$$\int u(a)\, d\nu = \alpha\delta m(a) + \delta(1-\alpha) M(a) + (1-\delta)\mathbf{E}_{\pi} u(a). \qquad (1)$$

- ν is a neo-additive capacity,
  - δ is a measure of perceived ambiguity;
  - α measures ambiguity-attitude, $\alpha = 1$ (resp. $\alpha = 0$) corresponding to pure optimism (resp. pessimism).
- $M(a)$ denotes the maximum utility of act a ,

The Neo-additive Model of Ambiguity

We use neo-additive preferences, which are represented by:

$$\int u(a)\,d\nu = \alpha\delta m(a) + \delta(1-\alpha) M(a) + (1-\delta)\mathbf{E}_{\pi} u(a). \qquad (1)$$

- ν is a neo-additive capacity,
  - δ is a measure of perceived ambiguity;
  - α measures ambiguity-attitude, $\alpha = 1$ (resp. $\alpha = 0$) corresponding to pure optimism (resp. pessimism).
- $M(a)$ denotes the maximum utility of act a ,
- $m(a)$ denotes the minimum utility of act a ,

The Neo-additive Model of Ambiguity

We use neo-additive preferences, which are represented by:

$$\int u(a)\, d\nu = \alpha\delta m(a) + \delta(1-\alpha) M(a) + (1-\delta)\mathbf{E}_{\pi} u(a) . \qquad (1)$$

- ν is a neo-additive capacity,
  - δ is a measure of perceived ambiguity;
  - α measures ambiguity-attitude, $\alpha = 1$ (resp. $\alpha = 0$) corresponding to pure optimism (resp. pessimism).
- $M(a)$ denotes the maximum utility of act a ,
- $m(a)$ denotes the minimum utility of act a ,
- $\mathbf{E}_{\pi} u(a)$ denotes the expected utility of act a.

The Neo-additive Model of Ambiguity

We use neo-additive preferences, which are represented by:

$$\int u(a)\, d\nu = \alpha\delta m(a) + \delta(1-\alpha) M(a) + (1-\delta)\mathbf{E}_{\pi} u(a). \qquad (1)$$

- ν is a neo-additive capacity,
  - δ is a measure of perceived ambiguity;
  - α measures ambiguity-attitude, $\alpha = 1$ (resp. $\alpha = 0$) corresponding to pure optimism (resp. pessimism).
- $M(a)$ denotes the maximum utility of act a ,
- $m(a)$ denotes the minimum utility of act a ,
- $\mathbf{E}_{\pi} u(a)$ denotes the expected utility of act a.

The Neo-additive Model of Ambiguity

We use neo-additive preferences, which are represented by:

$$\int u(a)\, d\nu = \alpha\delta m(a) + \delta(1-\alpha) M(a) + (1-\delta)\mathbf{E}_\pi u(a) \,. \qquad (1)$$

- ν is a neo-additive capacity,
  - δ is a measure of perceived ambiguity;
  - α measures ambiguity-attitude, $\alpha = 1$ (resp. $\alpha = 0$) corresponding to pure optimism (resp. pessimism).
- $M(a)$ denotes the maximum utility of act a ,
- $m(a)$ denotes the minimum utility of act a ,
- $\mathbf{E}_\pi u(a)$ denotes the expected utility of act a.
- Only 2 additional parameters needed compared to expected utility.

Updating Ambiguous Beliefs I

Suppose that event $E \subseteq S$ is observed.

Let ν_E denote the updated capacity then:

$$a \succsim_E b \Longleftrightarrow \int u(a(s))d\nu_E(s) \succsim \int u(a(s))d\nu_E(s).$$

Generalized Bayesian Updating (GBU)

$$\nu_E(A) = \frac{\nu(A \cap E)}{\nu(A \cap E) + 1 - \nu(E^c \cup A)} = \frac{\nu(A \cap E)}{\nu(A \cap E) + \bar{\nu}(A^c \cap E)},$$

where $\bar{\nu}(E) = 1 - \nu(E^c)$.

Updating Ambiguous Beliefs

Suppose that event $E \subseteq S$ is observed.

Let ν_E denote the updated capacity then:

$$a \succsim_E b \Longleftrightarrow \int u(a(s))d\nu_E(s) \succsim \int u(a(s))d\nu_E(s).$$

Generalized Bayesian Updating (GBU)

$$\nu_E(A) = \frac{\nu(A \cap E)}{\nu(A \cap E) + 1 - \nu(E^c \cup A)} = \frac{\nu(A \cap E)}{\nu(A \cap E) + \bar{\nu}(A^c \cap E)},$$

where $\bar{\nu}(E) = 1 - \nu(E^c)$.

Note that $\nu_E(A)$ is defined even if $\nu(E) = 0$. Thus it is possible to update beliefs off the equilibrium path.

Multi-Stage Games

Definition

A multi-stage game Γ *is a triple* $\langle \{1,2\}, \bar{H}, (u_i, \delta_i, \alpha_i)_{i=1,2} \rangle$ *, where* $\bar{H} = H \cup Z$ *is the set of histories, and* $(u_i, \delta_i, \alpha_i)$ *,* $i = 1, 2$ *characterizes the players' neo-expected pay-offs.*

Definition

A (pure)strategy of a player $i = 1, 2$ *is a function* s_i *which assigns to each history* $h \in H$ *an action* $a_i \in A_i^h$ *.*

Multi-Stage Games

Definition

A multi-stage game Γ is a triple $\langle \{1,2\}, \bar{H}, (u_i, \delta_i, \alpha_i)_{i=1,2} \rangle$, where $\bar{H} = H \cup Z$ is the set of histories, and $(u_i, \delta_i, \alpha_i)$, $i = 1, 2$ characterizes the players' neo-expected pay-offs.

Definition

A (pure)strategy of a player $i = 1, 2$ is a function s_i which assigns to each history $h \in H$ an action $a_i \in A_i^h$.

- S_i (resp., S_{-i}) denotes i 's (resp., her opponent's) strategy set.

Multi-Stage Games

Definition

A multi-stage game Γ *is a triple* $\langle \{1,2\}, \bar{H}, (u_i, \delta_i, \alpha_i)_{i=1,2} \rangle$ *, where* $\bar{H} = H \cup Z$ *is the set of histories, and* $(u_i, \delta_i, \alpha_i)$ *,* $i = 1, 2$ *characterizes the players' neo-expected pay-offs.*

Definition

A (pure)strategy of a player $i = 1, 2$ *is a function* s_i *which assigns to each history* $h \in H$ *an action* $a_i \in A_i^h$ *.*

- S_i (resp., S_{-i}) denotes i 's (resp., her opponent's) strategy set.
- $S = S_i \times S_{-i}$ denotes the set of strategy profiles.

Multi-Stage Games

Definition

A multi-stage game Γ *is a triple* $\langle \{1,2\}, \bar{H}, (u_i, \delta_i, \alpha_i)_{i=1,2} \rangle$ *, where* $\bar{H} = H \cup Z$ *is the set of histories, and* $(u_i, \delta_i, \alpha_i)$ *,* $i = 1, 2$ *characterizes the players' neo-expected pay-offs.*

Definition

A (pure)strategy of a player $i = 1, 2$ *is a function* s_i *which assigns to each history* $h \in H$ *an action* $a_i \in A_i^h$ *.*

- S_i (resp., S_{-i}) denotes i 's (resp., her opponent's) strategy set.
- $S = S_i \times S_{-i}$ denotes the set of strategy profiles.
- For each non-terminal history $h \in H$:

Multi-Stage Games

Definition

A multi-stage game Γ *is a triple* $\langle \{1,2\}, \bar{H}, (u_i, \delta_i, \alpha_i)_{i=1,2} \rangle$ *, where* $\bar{H} = H \cup Z$ *is the set of histories, and* $(u_i, \delta_i, \alpha_i)$ *,* $i = 1,2$ *characterizes the players' neo-expected pay-offs.*

Definition

A (pure)strategy of a player $i = 1,2$ *is a function* s_i *which assigns to each history* $h \in H$ *an action* $a_i \in A_i^h$ *.*

- S_i (resp., S_{-i}) denotes i 's (resp., her opponent's) strategy set.
- $S = S_i \times S_{-i}$ denotes the set of strategy profiles.
- For each non-terminal history $h \in H$:
  - $S(h) \subset S$ denotes the set of strategy profiles that lead to the play of history h with respective marginals $S_i(h)$ and $S_{-i}(h)$.

Multi-Stage Games

Definition

A multi-stage game Γ *is a triple* $\langle \{1,2\}, \bar{H}, (u_i, \delta_i, \alpha_i)_{i=1,2} \rangle$ *, where* $\bar{H} = H \cup Z$ *is the set of histories, and* $(u_i, \delta_i, \alpha_i)$ *,* $i = 1, 2$ *characterizes the players' neo-expected pay-offs.*

Definition

A (pure)strategy of a player $i = 1, 2$ *is a function* s_i *which assigns to each history* $h \in H$ *an action* $a_i \in A_i^h$ *.*

- S_i (resp., S_{-i}) denotes i 's (resp., her opponent's) strategy set.
- $S = S_i \times S_{-i}$ denotes the set of strategy profiles.
- For each non-terminal history $h \in H$:
    - $S(h) \subset S$ denotes the set of strategy profiles that lead to the play of history h with respective marginals $S_i(h)$ and $S_{-i}(h)$.
    - S_i^h , S_{-i}^h & S^h are the corresponding continuation sets.

Generalised Bayesian Updating (GBU)

Axiomatised by, Eichberger, Grant and Kelsey, JME, 2007.

If Player i 's initial beliefs are given by the probability π_i on S_{-i} ; and history h is observed then his/her updated preferences are given by:

$$V_i^h\left(s_i^h|\pi_i\right) = \left(1-\delta_i^h\right)\mathbb{E}_{\pi_i^h} u_i\left(s_i^h,\cdot\right)$$

$$+\delta_i^h\left[\alpha_i \min_{s_{-i}^h \in S_{-i}^h} u_i^h\left(s_i^h, s_{-i}^h\right) + (1-\alpha_i) \max_{s_{-i}^h \in S_{-i}^h} u_i^h\left(s_i^h, s_{-i}^h\right)\right],$$

where $\delta_i^h = \frac{\delta_i}{\delta_i + (1-\delta_i)\pi_i(S_{-i}(h))}$ and π_i^h is the Bayesian update of π_i .

Consistent Planning Equilibrium

- Players have neo-additive preferences and use the GBU rule.

Consistent Planning Equilibrium

- Players have neo-additive preferences and use the GBU rule.
- In the continuation after any history, they choose a best action.

Consistent Planning Equilibrium

- Players have neo-additive preferences and use the GBU rule.
- In the continuation after any history, they choose a best action.
- They anticipate how information they may receive in the future will change their preferences. (consistent planning)

Consistent Planning Equilibrium

- Players have neo-additive preferences and use the GBU rule.
- In the continuation after any history, they choose a best action.
- They anticipate how information they may receive in the future will change their preferences. (consistent planning)

Consistent Planning Equilibrium

- Players have neo-additive preferences and use the GBU rule.
- In the continuation after any history, they choose a best action.
- They anticipate how information they may receive in the future will change their preferences. (consistent planning)

Definition (*Consistent Planning Equilibrium*)

CP-EUA is a profile of beliefs $\langle \nu_1, \nu_2 \rangle$ *such that for each player* $i = 1, 2,$

$$\forall s_i \in \operatorname{supp} \nu_i \Rightarrow V_i^h \left(s_i^h | \pi_i \right) \geqslant V_i^h \left(\left(a_i, s_i^h (-t) \right) | \pi_i \right),$$

$$\forall a_i \in A_i^h, \text{ every } h \in H^{t-1}, \text{ and every } t = 1, \ldots, T.$$

Existence of Equilibrium

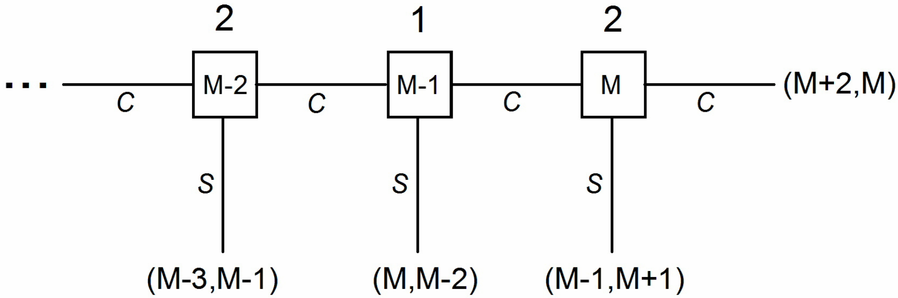
Equilibrium exists under the usual assumptions as the following result shows.

Theorem

Let Γ *be a multi-stage game with* 2 *players. Then* Γ *has at least one CP-EUA for any given parameters* $\alpha_1, \alpha_2, \delta_1, \delta_2$*, where* $1 \leqslant \alpha_i \leqslant 0, 0 < \delta_i \leqslant 1$*, for* $i = 1, 2$.

Since α and δ are exogenous, this result allows us to study the comparative statics of ambiguity and ambiguity-attitude.

Returning to the Centipede Game



Suppose $\delta_1 = \delta_2 = \delta$ and $\alpha_1 = \alpha_2 = \alpha$.
Recall δ measures ambiguity, α represents ambiguity-attitude.

Pure-strategy equilibria

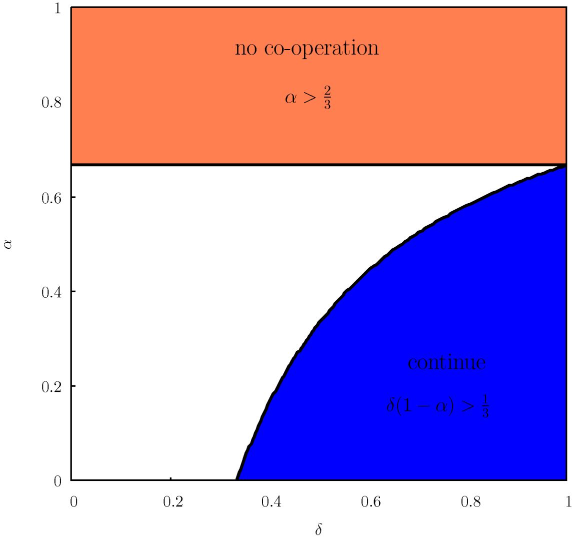
1. **Cooperation**
   If there is sufficient ambiguity and players are sufficiently ambiguity-loving (that is, provided $\delta(1-\alpha) \geqslant \frac{1}{3}$), then equilibrium involves playing "continue" until the final node.

   - At final node player 2 chooses "stop" since it is a dominant strategy for player 2 at that point.

2. **Non-cooperation**
   With high levels of ambiguity-aversion (that is, provided $\alpha \geqslant \frac{2}{3}$) the only equilibrium is playing "stop" at every node.

   - Similar to Nash equilibrium.

Cooperation versus Non-Cooperation



Mixed Equilibria I

Proposition

Let Γ *be a* $2M$ *stage centipede game, where* $M \geqslant 2$*, then* Γ *does not have a pure strategy equilibrium when* $\alpha < \frac{2}{3}$ *and* $\delta(1-\alpha) < \frac{1}{3}$.

- Kilka and Weber, (2001) experimentally estimate neo-additive preferences and find on average $\alpha = \delta = \frac{1}{2}$.

Mixed Equilibria I

Proposition

Let Γ be a $2M$ stage centipede game, where $M \geqslant 2$, then Γ does not have a pure strategy equilibrium when $\alpha < \frac{2}{3}$ and $\delta(1-\alpha) < \frac{1}{3}$.

- Kilka and Weber, (2001) experimentally estimate neo-additive preferences and find on average $\alpha = \delta = \frac{1}{2}$.
- Under these assumptions there is a mixed strategy equilibrium. The two players randomly choose "stop" at one of the final two nodes at which they have to play.

Mixed Equilibria I

Proposition

Let Γ *be a* $2M$ *stage centipede game, where* $M \geqslant 2$, *then* Γ *does not have a pure strategy equilibrium when* $\alpha < \frac{2}{3}$ *and* $\delta(1-\alpha) < \frac{1}{3}$.

- Kilka and Weber, (2001) experimentally estimate neo-additive preferences and find on average $\alpha = \delta = \frac{1}{2}$.
- Under these assumptions there is a mixed strategy equilibrium. The two players randomly choose "stop" at one of the final two nodes at which they have to play.
- Thus the game may end at one of the last three stages, $M-2$, $M-1$ or M.

Mixed Equilibria II

Proposition

If $\alpha < \frac{2}{3}$ and $\delta(1-\alpha) < \frac{1}{3}$, there is a (CP-EUA) $\langle (s_1^*, \nu_1^*), (s_2^*, \nu_2^*) \rangle$ in which:

- *player 1 believes with degree of ambiguity δ that player 2 will choose his strategies with probability μ_1, $\mu_1(s_2) = p$, for $s_2 = s_2^{2M}$; $1-p$ for $s_2 = s_2^{2M-2}$; where*

$$p = \frac{\delta(2-3\alpha)}{(1-\delta)}.$$

- *player 2 believes that player 1 will choose her strategies with probability μ_2; $\mu_2(s_1) = q$, for $s_1 = s_1^{2M+1}$; $\mu_2(s_1) = 1-q$, for $s_1 = s_1^{2M-1}$, where*

$$q = \frac{1-3\delta(1-\alpha)}{3(1-\delta)}.$$

The game will end either at $M-2$, $M-1$or M.

Non-Cooperative Bargaining

- Alternating Offers Bargaining (Rubinstein-Stahl)

Non-Cooperative Bargaining

- Alternating Offers Bargaining (Rubinstein-Stahl)
  - Sub-game perfection predicts that bargaining is efficient.

Non-Cooperative Bargaining

- Alternating Offers Bargaining (Rubinstein-Stahl)
  - Sub-game perfection predicts that bargaining is efficient.
  - In practice delays, strikes etc. are common.

Non-Cooperative Bargaining

- Alternating Offers Bargaining (Rubinstein-Stahl)
  - Sub-game perfection predicts that bargaining is efficient.
  - In practice delays, strikes etc. are common.
- We show that ambiguity loving behaviour can cause inefficiency.

Non-Cooperative Bargaining

- Alternating Offers Bargaining (Rubinstein-Stahl)
  - Sub-game perfection predicts that bargaining is efficient.
  - In practice delays, strikes etc. are common.
- We show that ambiguity loving behaviour can cause inefficiency.
- Ambiguity-loving behaviour may cause individual to make demands above Nash equilibrium levels

Non-Cooperative Bargaining

- Alternating Offers Bargaining (Rubinstein-Stahl)
  - Sub-game perfection predicts that bargaining is efficient.
  - In practice delays, strikes etc. are common.
- We show that ambiguity loving behaviour can cause inefficiency.
- Ambiguity-loving behaviour may cause individual to make demands above Nash equilibrium levels
- This leads to delays in bargaining.

Pure Equilibria

- Finite games of complete and perfect information, always have a Nash equilibrium in pure strategies.

Pure Equilibria

- Finite games of complete and perfect information, always have a Nash equilibrium in pure strategies.
- This is found by backward induction.

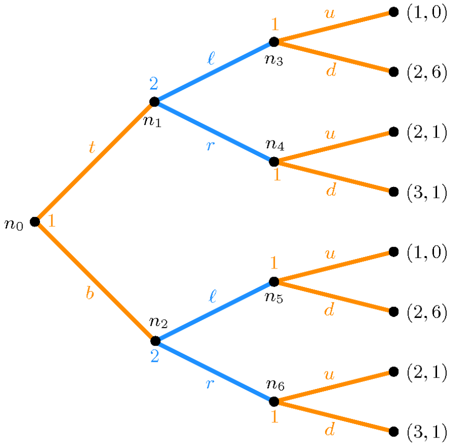
Pure Equilibria

- Finite games of complete and perfect information, always have a Nash equilibrium in pure strategies.
- This is found by backward induction.
- We show that a finite game of complete and perfect information may not have a pure strategy equilibrium if there is ambiguity.

Pure Equilibria

- Finite games of complete and perfect information, always have a Nash equilibrium in pure strategies.
- This is found by backward induction.
- We show that a finite game of complete and perfect information may not have a pure strategy equilibrium if there is ambiguity.
- Thus a well-known property of Nash equilibria may no longer apply when there is ambiguity.

Pure Equilibria



Game A

Equilibria with Ambiguity III

Proposition

Provided $\alpha > \frac{5}{6} \geqslant \alpha\delta$, *Game A has no pure strategy equilibrium with ambiguity.*

- The parameter restrictions, $\alpha > \frac{5}{6} \geqslant \alpha\delta$, imply that players perceive positive degrees of ambiguity and have high levels of ambiguity-aversion.

Equilibria with Ambiguity III

Proposition

Provided $\alpha > \frac{5}{6} \geqslant \alpha\delta$, Game A has no pure strategy equilibrium with ambiguity.

- The parameter restrictions, $\alpha > \frac{5}{6} \geqslant \alpha\delta$, imply that players perceive positive degrees of ambiguity and have high levels of ambiguity-aversion.
- Player 2 has different ordinal preferences at node n_2 depending whether it is on or off the equilibrium path.

Equilibria with Ambiguity III

Proposition

Provided $\alpha > \frac{5}{6} \geqslant \alpha\delta$, Game A has no pure strategy equilibrium with ambiguity.

- The parameter restrictions, $\alpha > \frac{5}{6} \geqslant \alpha\delta$, imply that players perceive positive degrees of ambiguity and have high levels of ambiguity-aversion.
- Player 2 has different ordinal preferences at node n_2 depending whether it is on or off the equilibrium path.
- Ambiguity-aversion causes Player 2 to choose the relatively safe action r off the equilibrium path.

Extensions/Work in Progress

- Similar results apply in the centipede game if utility is concave or convex.

Extensions/Work in Progress

- Similar results apply in the centipede game if utility is concave or convex.
- Games with complete and perfect information may not have a pure strategy equilibrium with ambiguity.

Extensions/Work in Progress

- Similar results apply in the centipede game if utility is concave or convex.
- Games with complete and perfect information may not have a pure strategy equilibrium with ambiguity.
- Extend to a larger class of games e.g. multi-player games and/or incomplete information.

Extensions/Work in Progress

- Similar results apply in the centipede game if utility is concave or convex.
- Games with complete and perfect information may not have a pure strategy equilibrium with ambiguity.
- Extend to a larger class of games e.g. multi-player games and/or incomplete information.
- Repeated Games

Extensions/Work in Progress

- Similar results apply in the centipede game if utility is concave or convex.
- Games with complete and perfect information may not have a pure strategy equilibrium with ambiguity.
- Extend to a larger class of games e.g. multi-player games and/or incomplete information.
- Repeated Games
- Herding and Bubbles in financial markets.

Experiments

- Joint with Han Bleichrodt and Chen Li (Erasmus, Rotterdam), Jürgen Eichberger and Simon Grant).

Experiments

- Joint with Han Bleichrodt and Chen Li (Erasmus, Rotterdam), Jürgen Eichberger and Simon Grant).
- Testing centipede paper.

Experiments

- Joint with Han Bleichrodt and Chen Li (Erasmus, Rotterdam), Jürgen Eichberger and Simon Grant).
- Testing centipede paper.
- How do individuals make decisions over time?

Experiments

- Joint with Han Bleichrodt and Chen Li (Erasmus, Rotterdam), Jürgen Eichberger and Simon Grant).
- Testing centipede paper.
- How do individuals make decisions over time?
- Myopic choice, dynamic consistency or consistent planning.

Experiments

- Joint with Han Bleichrodt and Chen Li (Erasmus, Rotterdam), Jürgen Eichberger and Simon Grant).
- Testing centipede paper.
- How do individuals make decisions over time?
- Myopic choice, dynamic consistency or consistent planning.
  - People, places, things.

Experiments II

- A box contains 200 cards numbered from 1 to 200. The cards numbered 1 to 66 are red. The next $2N$ cards are blue and the rest are yellow.

	RED	BLUE	YELLOW
ODD	33	N	$67 - N$
EVEN	33	N	$67 - N$

- A box contains 200 cards numbered from 1 to 200. The cards numbered 1 to 66 are red. The next $2N$ cards are blue and the rest are yellow.

	RED	BLUE	YELLOW
ODD	33	N	$67 - N$
EVEN	33	N	$67 - N$

- Unambiguous options:

- A box contains 200 cards numbered from 1 to 200. The cards numbered 1 to 66 are red. The next $2N$ cards are blue and the rest are yellow.

	RED	BLUE	YELLOW
ODD	33	N	$67 - N$
EVEN	33	N	$67 - N$

- Unambiguous options:
  - bet on red 66 chances to win,

Experiments II

- A box contains 200 cards numbered from 1 to 200. The cards numbered 1 to 66 are red. The next $2N$ cards are blue and the rest are yellow.

	RED	BLUE	YELLOW
ODD	33	N	$67 - N$
EVEN	33	N	$67 - N$

- Unambiguous options:
  - bet on red 66 chances to win,
  - bet on blue if odd yellow if even 67 chances to win.

Experiments II

- A box contains 200 cards numbered from 1 to 200. The cards numbered 1 to 66 are red. The next $2N$ cards are blue and the rest are yellow.

	RED	BLUE	YELLOW
ODD	33	N	$67 - N$
EVEN	33	N	$67 - N$

- Unambiguous options:
  - bet on red 66 chances to win,
  - bet on blue if odd yellow if even 67 chances to win.
- Ambiguous options bet on blue or yellow.

Experiments III

	RED	BLUE	YELLOW
ODD	33	N	$67 - N$
EVEN	33	N	$67 - N$

- Now assume the experimenter announces whether the card is odd or even but not its colour.

Experiments III

	RED	BLUE	YELLOW
ODD	33	N	$67 - N$
EVEN	33	N	$67 - N$

- Now assume the experimenter announces whether the card is odd or even but not its colour.
- Blue if odd yellow if even is now ambiguous.

Experiments III

	RED	BLUE	YELLOW
ODD	33	N	$67 - N$
EVEN	33	N	$67 - N$

- Now assume the experimenter announces whether the card is odd or even but not its colour.
- Blue if odd yellow if even is now ambiguous.
- Do subjects switch to red?

Experiments III

	RED	BLUE	YELLOW
ODD	33	N	$67 - N$
EVEN	33	N	$67 - N$

- Now assume the experimenter announces whether the card is odd or even but not its colour.
- Blue if odd yellow if even is now ambiguous.
- Do subjects switch to red?
- Most subjects satisfy consequentialism.

Experiments III

	RED	BLUE	YELLOW
ODD	33	N	$67 - N$
EVEN	33	N	$67 - N$

- Now assume the experimenter announces whether the card is odd or even but not its colour.
- Blue if odd yellow if even is now ambiguous.
- Do subjects switch to red?
- Most subjects satisfy consequentialism.
- Violations of dynamic consistency are common.

	RED	BLUE	YELLOW
ODD	33	N	$67 - N$
EVEN	33	N	$67 - N$

- Now assume the experimenter announces whether the card is odd or even but not its colour.
- Blue if odd yellow if even is now ambiguous.
- Do subjects switch to red?
- Most subjects satisfy consequentialism.
- Violations of dynamic consistency are common.
- After the experimenter announces whether the card is odd or even.

	RED	BLUE	YELLOW
ODD	33	N	$67 - N$
EVEN	33	N	$67 - N$

- Now assume the experimenter announces whether the card is odd or even but not its colour.
- Blue if odd yellow if even is now ambiguous.
- Do subjects switch to red?
- Most subjects satisfy consequentialism.
- Violations of dynamic consistency are common.
- After the experimenter announces whether the card is odd or even.
- The most common reaction is to switch from "Blue if odd yellow if even" to Red.