Ambiguity and Uncertainty

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Risk versus Ambiguity

But why did big shareholders not move to stop over-leveraging before it reached dangerous levels. Why did legislators not demand regulatory intervention? The answer I believe is that they had no sense of Knightian uncertainty. So they had no sense of the possibility of a huge break in housing prices and no sense of the fundamental inapplicability of the risk management models used in the banks.

Risk came to mean volatility over the recent past.

Edmund Phelps, Financial Times 12th May 2007

Uncertainty: Ambiguity versus Risk

- Risk refers to uncertainty with known probabilities, e.g. flipping a fair coin.
- Ambiguity refers to uncertainty with unknown probabilities e.g.
  - will we find intelligent life on other planets?
  - will nuclear fusion provide a cheap and safe source of energy?
  - will leaving the EU improve the UK economy?

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  - will we find intelligent life on other planets?
  - will nuclear fusion provide a cheap and safe source of energy?
  - will leaving the EU improve the UK economy?
- Reasons why it may be difficult to assign probabilities.
  - Unique events, for which it is not possible to estimate probabilities from relative frequencies.
  - Complexity The outcome may depend on new and advanced technology, e.g. the BP oil spill.
  - The resolution of uncertainty depends on human behaviour.

Policy Issues

- Ambiguity plays an important role in a number of current policy issues.
- How to respond to new diseases, SARS, Ebola etc.?
- What should be done about threats from terrorism and rouge states?
- How do we redesign the financial system to make it less vulnerable to crises?
- What action should be taken in response to environmental dangers?
- Can the private sector provide insurance for extreme events such as earthquakes and hurricanes?

Plan of Talk

- The Ellsberg Paradox
- Modelling Ambiguity
- Environmental Application
- Ambiguity and Strategic Complementarity
- Banking and Finance
- Experimental Research
- Ambiguity and Law
- Conclusion

The Ellsberg Paradox

An urn contains 90 balls, 30 of which are red, R. The remainder are either blue B or yellow Y. The proportion of blue and yellow balls is unknown.

		30	60		
		R	B	Y	
Choice 1	a	100	0	0	✓
	b	0	100	0	
Choice 2	c	100	0	100	
	d	0	100	100	✓

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- Most people prefer a to b and d to c.
- This is not compatible with the standard model of choice under uncertainty, expected utility theory.
- Nor is it compatible with any other theory, which represents beliefs as probabilities.

Conclusions from the Ellsberg Paradox

1. People behave differently in situations where probabilities are unknown.
   - Decision problems with unknown probabilities are said to be *ambiguous*.
2. Ambiguity usually causes people to behave cautiously.
   - This referred to as ambiguity-aversion.

Modelling Ambiguity I

- A common way to model ambiguity is to represent an individual's beliefs by a *set*, C, of probabilities. For instance the probability that David Cameron will win the next election is between 0.2 and 0.5.
- When there is ambiguity individuals consider a number of probability distributions to be possible.
- Acts are evaluated by the least favourable probability distribution.

Ambiguity-aversion has the effect of over-weighting bad outcomes.
Act a is preferred to act b if:

$$\min_{p \in C} \mathbf{E}_p u(a) > \min_{p \in C} \mathbf{E}_p u(b).$$

$\mathbf{E}_p u$ denotes the usual expected value of u with respect to probability distribution p.

Modelling Ambiguity II

$$\min_{p \in C} \mathbf{E}_p u(a) > \min_{p \in C} \mathbf{E}_p u(b),$$

Intuition If an individual does not know the true probabilities (s)he considers a number of probability distributions to be possible and behaves cautiously.

ADVANTAGES

- Expected utility is a special case;
- Relatively easy to apply in the social sciences;
- Corporate Planning Managers are encouraged to plan for a number of alternative scenarios.

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An alternative interpretation is that people do assign subjective probabilities to events. However they do not have the same mathematical properties as the probabilities used in the physical sciences.

Ambiguity and Environmental Economics

- There are n communities, $1 \leqslant i \leqslant n$, which need to cooperate to protect the environment.
- Let e_i denote the effort which community i exerts to prevent pollution.
- The quality, Q, of the environment is determined by the lowest effort level,

$$Q = \text{minimum}\left\{e_1, ..., e_n\right\}.$$

- This could describe number of neighbouring districts building a sea-wall.

Two Communities

For simplicity assume there are just two communities and effort levels can be either high, H, or low, L. This is summarised in the following table:

		Player 2	
		H	L
Player 1	H	$2,2$	$0,1$
	L	$1,0$	$1,1$

There are two equilibria:

- a "good" equilibrium, $\langle 2,2 \rangle$, in which all build their section of the wall;
- a "bad" equilibrium, $\langle 1,1 \rangle$, in which no community builds its section.

We assume that a high (resp. low) quality of the environment gives a benefit of 4 (resp. 2). The cost of high effort is 2 and the cost of low effort is 1.

The Effect of Ambiguity

		Player 2	
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The more ambiguity the more likely is the bad equilibrium.

With high levels of ambiguity there is only one equilibrium which is $\langle 1, 1 \rangle$.

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Ambiguity may be measured by a parameter γ which takes values between 0 and 1. Larger values of γ correspond to more ambiguity. The pay-off of a typical player is given by,

$$V(H) = 2 \cdot (1 - \gamma) \text{ and } V(L) = 1.$$

High effort will only be chosen if $\gamma \leqslant \frac{1}{2}$. Thus the more ambiguity there is the more likely is the bad equilibrium.

Many Communities

With n communities

$$V(H) = 2 \cdot (1-\gamma)^n \text{ and } V(L) = 1.$$

Note $0 < \gamma < 1$, hence as $n \to \infty$, $(1-\gamma)^n \to 0$.

The bad equilibrium is more likely

- when there is more ambiguity;
- the larger the number of agents who have to cooperate.

These predictions have been confirmed experimentally by van Huyck, Battalio and Beil, *American Economic Review 1990.*

Environmental Coordination Problems

1. If a fishery is used by a number of countries its health will depend on the weakest of the regulations in the surrounding countries.
2. If an industrial zone crosses an international border and firms can easily locate, the level of pollution will depend on the weakest emission controls.
3. Global warming depends on the total amount of carbon dioxide released. If industry can move to other counties, then the level of pollution will depend on the weakest controls.

In each of these cases their are "good" equilibria in which all act to protect the environment and "bad" equilibria in which nobody does so.

Strategic Complementarity

- Strategic complementarity means that when one individual chooses a greater action this gives others an incentive to do likewise.
- One firm investing makes it more attractive for another firm to invest.
- Strategic complementarity can give rise to multiple equilibria.
  - one where all invest,
  - one where nobody invests.
- Macroeconomic coordination (Cooper and John)
  - In the presence of strategic complementarity the economy can have two equilibria one with a high level of activity and one with a low level of activity.
- Strategic Substitutes describes the opposite situation if one agent taking a greater action induces his/her opponent to choose a lower action.

Strategic Complementarity and Ambiguity

- Ambiguity causes individuals to overweight bad outcomes.
- In the example a bad outcome would be the other firm making a low investment.
- Given strategic complementarity the other firm making a low investment will reduce your own incentive to invest.
- Thus an increase in ambiguity decreases investment in equilibrium.
- There is a predictable comparative static effect of ambiguity when there is strategic complementarity.
- Moreover if there is enough ambiguity equilibrium will be unique.
- In the macroeconomic coordination problem, an increase in ambiguity can cause the economy to move to the low equilibrium.

Banking

- In theory you can withdraw your money from the bank at any time.
- However banks only hold a small fraction of total deposits as cash.
- Consequently everybody cannot withdraw simultaneously.
- The more people you see withdrawing money from the bank the greater your incentive to withdraw.
- Bank runs are an example of strategic complementarity.
- There are two equilibria one where nobody tries to withdraw and one where everybody tries to withdraw. (The Northern Rock Equilibrium).
- Ambiguity can make banks less stable.

Finance

- The current financial crisis was triggered when it was realised that there was considerable ambiguity concerning complex derivatives written on the US housing market.
- The financial system is also subject to shocks due to ambiguous risks elsewhere.
- Home Bias
  - In the USA (UK) investors allocate 94% (82%) of their portfolio to domestic equities.
  - Normative portfolio models suggest investors should *short* their national stock market because of its correlation with their labour income.
  - Investors have less foreign securities in their portfolios than is needed for diversification.
  - This can be explained if investors view foreign stocks as being more ambiguous.

Experiments

- We have clear predictions of the effect of ambiguity in economic models and games which exhibit strategic complements or substitutes.
- These results are suitable for testing in experiments.
- The theory predicts that ambiguity has the opposite effect in situations of strategic complements and strategic substitutes.
- Consider two games which are as similar as possible except one is a game of strategic complements and the other is a game of strategic substitutes.
- If ambiguity is introduced, the theory predicts the equilibrium will change in the opposite direction in the two cases.

The Granny Experiment

- How can we introduce ambiguity into an experiment without lying to the subjects?
- The identity of the opponent may induce subjects to expect more/less ambiguous behaviour.
- Subjects played other subjects, a Professor of Game Theory or the experimenter's grandmother.
- The results showed that subjects did indeed find the granny's behaviour more ambiguous.
- Ambiguity had the predicted effect both in games of strategic complements and strategic substitutes.
- "Granny Versus Game Theorist: An Experimental Test of Ambiguity in Games" (with Jürgen Eichberger and Burkhard Schipper), *Theory and Decision*, 64, 333-362 2008.

Law and Ambiguity I

- Most research on ambiguity is in economics.
- However we believe ambiguity has important implications in social science more generally.
- We plan to study the implications of ambiguity for law.
- Ambiguity is likely in legal cases since:
  - Apart from lawyers individuals have limited contact with law,
  - legal cases can be very complex,
  - legal cases usually depend on the behaviour of other people which is intrinsically difficult to assign probabilities to.

Accident Law

- Accidents are the cause of many environmental problems, e.g. BP, Deepwater Horizon; Bhopal (1984).
- Accidents are ambiguous since many accidents are rare events. Also accidents can be very complex e.g. the recent BP oil spill.
- In an accident there is an externality. One party takes actions which risk damaging the property or livelihood of another.
- Tort law aims to ensure that individuals take into account social welfare when taking decisions.

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- negligence is more robust to ambiguity than strict liability,
- with ambiguity it is possible to get agents to take efficient actions even when these actions are not observable.

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Liability Rules: Strict Liability versus Negligence

Strict Liability

- The injurer pays the full cost of the damages in all circumstances.
- The injurer will supply the optimal level of care, since the externality has been internalised.
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Negligence

- The injurer pays the full cost of the damages if the level of care is below a specified level x^*. Otherwise the victim bears the full cost.
- If x^* is set correctly the injurer will supply the optimal level of care.
- Given this the victim bears the full cost of an accident.
- Since the externality is internalised, the victim will supply the efficient level of care.

Acts of God

- Many contracts allow for non-performance if an extreme event occurs.
- e.g. the "Act of God Clause" in insurance contracts.
- Building contracts have similar clauses which allow non-performance after an unexpected event.
- The injurer can use the occurrence of an unexpected event as a defence in tort cases.
- We find that ambiguity can explain the anomalous features of such contracts. Specifically risk sharing can be difficult if different individual have different perceptions of ambiguity
- "Sharing Ambiguous Risks", (with Surajeet Chakravarty) forthcoming in *Journal of Mathematical Economics.*

Unawareness

- Unawareness When choosing an action a decision-maker may not know the probability of any given consequence. In addition (s)he may not know all of the consequences.
- A situation sometimes called "unknown unknowns".
- Unawareness is relevant for law:
  - an individual may not be aware that an act (s)he is undertaking is considered to be a crime.
  - an injurer may not be aware that a particular action may cause a loss to somebody else,
  - medical malpractice: a doctor may be unaware of potential harmful consequences of a new treatment.

We are studying how unawareness affects liability rules. Under negligence, when the court announces stipulated care levels, unawareness may be reduced.

Future Directions

- Unawareness When choosing an action a decision-maker may not know the probability of any given consequence. In addition (s)he may not know all of the consequences.
- A situation sometimes called "unknown unknowns".
- Ambiguity in Law
- Medical Decision Making.
  - Treating rare diseases. This is an ambiguous decision if the doctor has not previously seen a similar case.
- Environmental Science Many environmental risks are ambiguous.
- War and Conflict
- These slides and a number of related papers can be found on my web-page: http://people.exeter.ac.uk/dk210/

Entry Deterrence I

A new firm (the entrant) is considering whether to enter an industry dominated by a monopolist.

The entrant has two choices to enter the industry, e, or not to enter, ne. The incumbent can either accommodate entry, a, or fight a price war, f.

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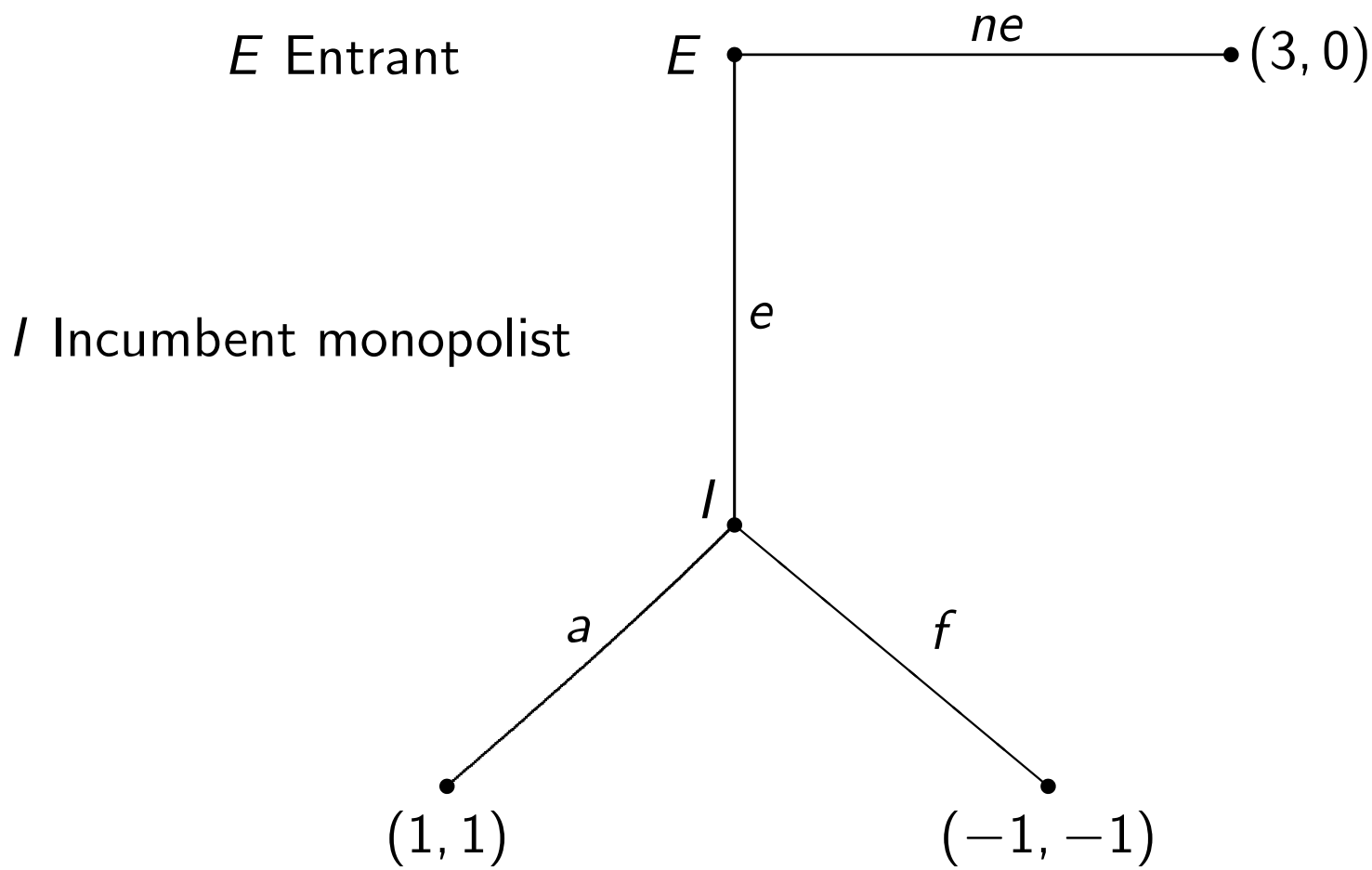
- If the monopolist accommodates entry both firms receive profits $= 1$.
- If the monopolist fights entry, both firms receive profits $= -1$.
- Without entry the monopolist's profits are 3.
- The interaction may be represented by the following diagram.

Entry Deterrence II



Entry Deterrence III

The same interaction may be represented in matrix form:

		Entrant	
		e	ne
Incumbent	a	$1, 1$	$3, 0$
	f	$-1, -1$	$3, 0$

This game has two Nash equilibria, $\langle a, e \rangle$ and $\langle f, ne \rangle$.

Entry Deterrence and Ambiguity

		Entrant	
		e	ne
Incumbent	a	$1, 1$	$3, 0$
	f	$-1, -1$	$3, 0$

Accommodate is always a best response for the monopolist.
The entrant's pay-offs are:

- $1 - 2\gamma$ from entering;
- 0 from staying out.

If $\gamma > \frac{1}{2}$, in equilibrium the entrant stays out and the monopolist would accommodate if entry occurs.

Expected Utility

There is a set of states of nature, $S = \{s_1, ..., s_n\}$.
An act a assigns an outcome (e.g. money) to each state, i.e. $a(s_i)$ is the outcome in state s_i.
The decision-makes utility or benefit from an outcome is given by a function u.