Dynamic Consistency and Preference for Randomization

Jürgen Eichberger, Simon Grant and David Kelsey
Heidelberg University,
University of Queensland and Australian National University,
University of Exeter.

University of Exeter

July 2014.

Introduction

- This paper studies the relation between ambiguity and preference for randomization.
- Consider the 2-ball Ellsberg urn:
- The urn contains 100 balls which are either red or black in unknown proportions.
- The subject is given the choice of option a (resp. b) £100 if the ball drawn is red (resp. black).
- Ellsberg (1961) argues that betting on either ball will be viewed as inferior to an objective 50:50 gamble between £100 and £0.

Raiffa

Raiffa (1961) argues that ambiguity can be avoided by flipping a coin and betting on red (resp. black) if it shows heads (resp. tails).

Thus ambiguity-aversion gives rise to a strict preference for randomization.

	R	B
a	100	0
b	0	100
$\frac{1}{2}.a + \frac{1}{2}.b$	$\frac{1}{2}.100 + \frac{1}{2}.0$	$\frac{1}{2}.100 + \frac{1}{2}.0$

Raiffa

Raiffa (1961) argues that ambiguity can be avoided by flipping a coin and betting on red (resp. black) if it shows heads (resp. tails).

Thus ambiguity-aversion gives rise to a strict preference for randomization.

	R	B
a	100	0
b	0	100
$\frac{1}{2}.a + \frac{1}{2}.b$	$\frac{1}{2}.100 + \frac{1}{2}.0$	$\frac{1}{2}.100 + \frac{1}{2}.0$

Consequently a rational individual should not be prepared to pay any positive amount of money to avoid ambiguity.

Raiffa

Raiffa (1961) argues that ambiguity can be avoided by flipping a coin and betting on red (resp. black) if it shows heads (resp. tails).

Thus ambiguity-aversion gives rise to a strict preference for randomization.

	R	B
a	100	0
b	0	100
$\frac{1}{2}.a + \frac{1}{2}.b$	$\frac{1}{2}.100 + \frac{1}{2}.0$	$\frac{1}{2}.100 + \frac{1}{2}.0$

Consequently a rational individual should not be prepared to pay any positive amount of money to avoid ambiguity.

However Raiffa does not explicitly model the randomizing device.

A Savage Framework

- In a Savage framework, the randomizing device must be modelled in the state space.

	RH	RT	BH	BT
a	100	100	0	0
b	0	0	100	100
c	100	0	0	100
d	0	100	100	0

It is no longer obvious that the randomizations c and d should be preferred.

- Eichberger and Kelsey (1996) show that in the convex capacity model, individuals must be indifferent to randomization.

A Savage Framework

- In a Savage framework, the randomizing device must be modelled in the state space.

	RH	RT	BH	BT
a	100	100	0	0
b	0	0	100	100
c	100	0	0	100
d	0	100	100	0

It is no longer obvious that the randomizations c and d should be preferred.

- Eichberger and Kelsey (1996) show that in the convex capacity model, individuals must be indifferent to randomization.
- Eichberger, Grant, and Kelsey (2005) find necessary and sufficient conditions for dynamic consistency in the convex capacity model.

A Savage Framework

- In a Savage framework, the randomizing device must be modelled in the state space.

	RH	RT	BH	BT
a	100	100	0	0
b	0	0	100	100
c	100	0	0	100
d	0	100	100	0

It is no longer obvious that the randomizations c and d should be preferred.

- Eichberger and Kelsey (1996) show that in the convex capacity model, individuals must be indifferent to randomization.
- Eichberger, Grant, and Kelsey (2005) find necessary and sufficient conditions for dynamic consistency in the convex capacity model.
- The conditions for dynamic consistency are the same as those for indifference to randomization.

Outline

- The present paper uses a model free framework.

Outline

- The present paper uses a model free framework.
- We show that dynamic consistency implies that the decision-maker must be indifferent to ex-ante randomization.

Outline

- The present paper uses a model free framework.
- We show that dynamic consistency implies that the decision-maker must be indifferent to ex-ante randomization.
- However it is still possible to have preference for ex-post randomization.

Outline

- The present paper uses a model free framework.
- We show that dynamic consistency implies that the decision-maker must be indifferent to ex-ante randomization.
- However it is still possible to have preference for ex-post randomization.
- We do not assume objective probabilities so the result would apply to an ambiguous randomizing device.

Outline

- The present paper uses a model free framework.
- We show that dynamic consistency implies that the decision-maker must be indifferent to ex-ante randomization.
- However it is still possible to have preference for ex-post randomization.
- We do not assume objective probabilities so the result would apply to an ambiguous randomizing device.
- We show the recent debate between Epstein and KMM on paradoxes for the smooth ambiguity model can be better understood by making a clear distinction between ex-post and ex-ante randomizations.

Set-Up I

- A decision-maker faces uncertainty described by a state space S.
- There is an ex-ante randomizing device. Its output, r, is contained in a set $R = [0, 1]$.
- There is also an ex-post randomizing device. Its output, t, is contained in a set $T = [0, 1]$.
- The randomizing devices are independent and both r and t are uniformly distributed.
- The grand state space, Ω, is a Cartesian product $\Omega = R \times S \times T$.
- An act is a function, $a : \Omega \to X$, where X denotes the space of outcomes.
- Preferences are represented by a binary relation, $\succcurlyeq$, on the set of all acts $A(\Omega)$.
- For any non-empty event $E \subseteq R$, the decision maker has a conditional preference $\succcurlyeq_E$ defined on $A(\Omega)$

Ex-ante Randomization

- The set of acts which do not depend upon the randomizing devices, $A(S) \subseteq A(\Omega)$, is defined by:

$$A(S) = \{a \in A(\Omega) : \forall \hat{r}, \tilde{r} \in R, \forall s \in S, \forall \hat{t}, \tilde{t} \in T, a(\hat{r}, s, \hat{t}) = a(\tilde{r}, s, \tilde{t})\}.$$

We define ex-ante randomization between two acts as follows.

Definition 2.1 *Suppose $f, g \in A(S)$ and $C \subseteq R$, define the ex-ante randomization. $f^C g$ by $f^C g(r, s, t) = f(s)$ if $r \in C$, and $f^C g(r, s, t) = g(s)$ if $r \notin C$.*

Ex-ante Randomization

- The set of acts which do not depend upon the randomizing devices, $A(S) \subseteq A(\Omega)$, is defined by:

$$A(S) = \{a \in A(\Omega) : \forall \hat{r}, \tilde{r} \in R, \forall s \in S, \forall \hat{t}, \tilde{t} \in T, a(\hat{r}, s, \hat{t}) = a(\tilde{r}, s, \tilde{t})\}.$$

We define ex-ante randomization between two acts as follows.

Definition 2.4 *Suppose $f, g \in A(S)$ and $C \subseteq R$, define the ex-ante randomization. $f^C g$ by $f^C g(r, s, t) = f(s)$ if $r \in C$, and $f^C g(r, s, t) = g(s)$ if $r \notin C$.*

Definition 2.5 *We say that an individual has a strict preference for ex ante randomization if there exists non-empty $F \subsetneqq R$ and $a, b \in A(S)$, such that $a \sim b$, $a^F b \succ a$.*

Ex-ante Randomization

- The set of acts which do not depend upon the randomizing devices, $A(S) \subseteq A(\Omega)$, is defined by:

$$A(S) = \{a \in A(\Omega) : \forall \hat{r}, \tilde{r} \in R, \forall s \in S, \forall \hat{t}, \tilde{t} \in T, a(\hat{r}, s, \hat{t}) = a(\tilde{r}, s, \tilde{t})\}.$$

We define ex-ante randomization between two acts as follows.

Definition 2.7 *Suppose $f, g \in A(S)$ and $C \subseteq R$, define the ex-ante randomization. $f^C g$ by $f^C g(r, s, t) = f(s)$ if $r \in C$, and $f^C g(r, s, t) = g(s)$ if $r \notin C$.*

Definition 2.8 *We say that an individual has a strict preference for ex ante randomization if there exists non-empty $F \subsetneqq R$ and $a, b \in A(S)$, such that $a \sim b$, $a^F b \succ a$.*

Definition 2.9 *We say that an individual is indifferent to ex-ante randomization if for all $F \subsetneqq R$ and for all $a, b \in A(S)$, such that $a \sim b$, $a^F b \sim a$.*

Ex Post Randomization

We define ex-post randomization between two acts as follows.

Ex Post Randomization

We define ex-post randomization between two acts as follows.

Definition 2.12 *Suppose* $f, g \in A(S)$ *and* $D \subseteq T$*, define the act* $f_D g$ *by* $f_D g\,(r, s, t) = f\,(s)$ *if* $t \in D$*, and* $f_D g\,(r, s, t) = g\,(s)$ *if* $t \notin D$*.*

Ex Post Randomization

We define ex-post randomization between two acts as follows.

Definition 2.14 *Suppose* $f, g \in A(S)$ *and* $D \subseteq T$*, define the act* $f_D g$ *by* $f_D g\,(r, s, t) = f\,(s)$ *if* $t \in D$*, and* $f_D g\,(r, s, t) = g\,(s)$ *if* $t \notin D$*.*

Definition 2.15 *We say that an individual has a strict preference for ex post randomization if there exists non-empty* $D \subsetneqq T$ *and* $a, b \in A\,(S)$*, such that* $a \sim b$*,* $a_D b \succ a$*.*

Assumptions

Assumption 2.1 (Consequentialism, Conseq) *For any non-null event* $E \subseteq R$*, and any* $a, b, \in A(\Omega)$*,* $a \sim_E a^E b$*.*

Assumption 2.2 (Dynamic Consistency DC) *For any partition* $\{E_k, 1 \leqslant k \leqslant K\}$ *of* R *and any two acts* $a, b \in A(\Omega)$*,* $a \succcurlyeq_{E_k} b$*, for* $1 \leqslant k \leqslant K$*, implies* $a \succcurlyeq b$*.*

- As usual dynamic consistency says that a decision-maker will keep to an ex-ante optimal plan at subsequent events.
- This is a weak dynamic consistency axiom since it only applies after observing an event defined by the output of the ex-ante randomizing device.
- We can get similar results if dynamic consistency is only required for strict preferences.

Independent Randomizations

Assumption 2.3 (Independent Randomizations, IR) *For any event $E \subseteq R$, and any $a, b \in A(S)$, $a \succsim b \implies a \succsim_E b$.*

- The acts $a, b \in A(S)$ do not depend directly on the realization of the randomizing device.
- Moreover the randomizing device does not convey useful information which might improve the decision.
- Hence it seems reasonable that preferences between a and b will not be changed once the outcome of the randomizing device is known.
- Independent randomizations does rule out some kinds of behaviour. For instance if an individual is more willing to take a gamble if his/her lucky number comes up.
- Machina's Mom If a randomizing device is used to allocate an indivisible treat this would cause a violation of IR.

Distribution Invariance

We assume that the decision maker is indifferent between lotteries over *certain* outcomes which have the same distribution whether they are ex-ante or ex-post. If $\ell \in A(R)$ then ℓ is a lottery with known probabilities over utility outcomes. Let F_ℓ denote its cumulative distribution function.

Assumption 2.4 (Distribution Invariance)

1. *For any $\ell \in A(R)$ and any $\lambda \in A(T)$, $F_\ell = F_\lambda \Rightarrow \ell \sim \lambda$;*
2. *if $a, b \in A(\Omega)$ are such that $\forall r \in R$, $\forall s \in S$, $F_{a(r,s,\cdot)} = F_{b(r,s,\cdot)}$ then $a \sim b$.*

Distribution Invariance

We assume that the decision maker is indifferent between lotteries over *certain* outcomes which have the same distribution whether they are ex-ante or ex-post. If $\ell \in A(R)$ then ℓ is a lottery with known probabilities over utility outcomes. Let F_ℓ denote its cumulative distribution function.

Assumption 2.5 (Distribution Invariance)

1. *For any* $\ell \in A(R)$ *and any* $\lambda \in A(T)$, $F_\ell = F_\lambda \Rightarrow \ell \sim \lambda$;
2. *if* $a, b \in A(\Omega)$ *are such that* $\forall r \in R$, $\forall s \in S$, $F_{a(r,s,\cdot)} = F_{b(r,s,\cdot)}$ *then* $a \sim b$.

Hence preferences are independent of the particular randomizing device used.

Thus a gamble which pays £100 if a fair coin shows heads, £30 otherwise; is viewed as equivalent to a gamble which pays £100 if a fair die comes up 4, 5 or 6 and $0 otherwise.

Ellsberg Ambiguity Aversion

Define the subjective mixture of two acts as follows:

Definition 2.16 (Subjective mixture of acts) *For any pair f, $g \in A(S)$ and $F \subseteq S$, the subjective mixture of f and g, is an act $fFg \in A(S)$ defined by $fFg(s) = f(s)$ if $s \in F$ and $fFg(s) = g(s)$ if $s \notin F$.*

The following assumption says that the decision maker displays the usual behaviour in the Ellsberg paradox.

Definition 2.17 (Ellsberg ambiguity-averse) *A decision maker with preferences $\succsim$ on $A(\Omega)$ is Ellsberg Ambiguity-Averse if, for some $E \subset T$, $F \subset S$, and some pair of outcomes $x \succ y$,*

$$x_E y \sim y_E x \succ xFy \sim yFx.$$

Strict Preference for Randomization

Proposition 2.1 *Let* $\{\succcurlyeq, \succcurlyeq_E : E \subseteq R, E \neq \varnothing\}$ *be a family of conditional preference orders that satisfies Consequentialism (2.1) and Dynamic Consistency (2.2). Consider a given event* E*, such that both* E *and* E^c *are non-null and two acts* $a, b \in A(S)$ *such that* $a \sim b$*, if* $a^E b \succ a$ *then* $a \succcurlyeq a^{E^c} b$*.*

Hence the Raiffa preference for randomization $a^E b \succ a$ and $a^{E^c} b \succ a$ is not possible.

This result does not make use of Independent Randomizations.

Indifference to Randomization

Proposition 2.2 *Let $\{\succcurlyeq_E : E \subseteq R\}$ be a family of conditional preference orders which satisfies Consequentialism (2.1), Dynamic Consistency (2.2), and Independent Randomizations (2.3). Then the unconditional preference order $\succcurlyeq$ must be indifferent to randomization.*

Proof. Suppose $a \sim b$.

Indifference to Randomization

Proposition 2.3 *Let* $\{\succcurlyeq_E : E \subseteq R\}$ *be a family of conditional preference orders which satisfies Consequentialism (2.1), Dynamic Consistency (2.2), and Independent Randomizations (2.3). Then the unconditional preference order* $\succcurlyeq$ *must be indifferent to randomization.*

Proof. Suppose $a \sim b$.

Then $a \sim_E b$ and $a \sim_{E^c} b$ by Independent Randomizations.

Indifference to Randomization

Proposition 2.4 *Let* $\{\succcurlyeq_E : E \subseteq R\}$ *be a family of conditional preference orders which satisfies Consequentialism (2.1), Dynamic Consistency (2.2), and Independent Randomizations (2.3). Then the unconditional preference order* $\succcurlyeq$ *must be indifferent to randomization.*

Proof. Suppose $a \sim b$.

Then $a \sim_E b$ and $a \sim_{E^c} b$ by Independent Randomizations.

By Consequentialism $a^E b \sim_E a$ and $a^E b \sim_{E^c} b$.

Indifference to Randomization

Proposition 2.5 *Let $\{\succcurlyeq_E : E \subseteq R\}$ be a family of conditional preference orders which satisfies Consequentialism (2.1), Dynamic Consistency (2.2), and Independent Randomizations (2.3). Then the unconditional preference order $\succcurlyeq$ must be indifferent to randomization.*

Proof. Suppose $a \sim b$.

Then $a \sim_E b$ and $a \sim_{E^c} b$ by Independent Randomizations.

By Consequentialism $a^E b \sim_E a$ and $a^E b \sim_{E^c} b$.

Using $b \sim_{E^c} a$, $a^E b \sim_{E^c} b \Rightarrow a^E b \sim_{E^c} a$

Indifference to Randomization

Proposition 2.6 *Let $\{\succcurlyeq_E : E \subseteq R\}$ be a family of conditional preference orders which satisfies Consequentialism (2.1), Dynamic Consistency (2.2), and Independent Randomizations (2.3). Then the unconditional preference order $\succcurlyeq$ must be indifferent to randomization.*

Proof. Suppose $a \sim b$.

Then $a \sim_E b$ and $a \sim_{E^c} b$ by Independent Randomizations.

By Consequentialism $a^E b \sim_E a$ and $a^E b \sim_{E^c} b$.

Using $b \sim_{E^c} a$, $a^E b \sim_{E^c} b \Rightarrow a^E b \sim_{E^c} a$

By dynamic consistency $a^E b \sim_E a$ and $a^E b \sim_{E^c} a$ implies $a^E b \sim a$. ■

Preference for Ex Post Randomization

Proposition 3.1 *Let* $\succsim$ *be a preference on* $A(\Omega)$ *that satisfies Assumptions 2.4 (distribution invariance) and 2.17 (Ellsberg ambiguity-aversion). Then there exists an event* $D \in \mathcal{R}$, *and a pair of acts* f *and* g *in* $A(S)$, *such that* $f_D g \succ f \sim g$.

Preference for Ex Post Randomization

Proposition 3.2 *Let $\succsim$ be a preference on $A(\Omega)$ that satisfies Assumptions 2.4 (distribution invariance) and 2.17 (Ellsberg ambiguity-aversion). Then there exists an event $D \in \mathcal{R}$, and a pair of acts f and g in $A(S)$, such that $f_D g \succ f \sim g$.*

The main results can be summarised as ambiguity aversion implies indifference to ex-ante randomization and strict preference for ex post randomization.

Preference for Ex Post Randomization

Proposition 3.3 *Let $\succsim$ be a preference on $A(\Omega)$ that satisfies Assumptions 2.4 (distribution invariance) and 2.17 (Ellsberg ambiguity-aversion). Then there exists an event $D \in \mathcal{R}$, and a pair of acts f and g in $A(S)$, such that $f_D g \succ f \sim g$.*

The main results can be summarised as ambiguity aversion implies indifference to ex-ante randomization and strict preference for ex post randomization.

Our axioms are consistent.
They will be satisfied by any set of multiple priors preferences which exhibits a strict preference for ex-post randomization and is rectangular with respect to the ex-ante randomizing device.

Paradoxes for Smooth Ambiguity

There are 2 independent ambiguous urns each containing 50 balls that are either black or white. In addition the decision-maker may flip a fair coin.

Epstein does not specify whether this is an ex-ante or an ex-post randomization. Let the symbol $*$ denote a 50:50 randomization without specifying whether it is ex-post or ex-ante, e.g. $\phi * \psi$ yields ϕ (resp. ψ) if the coin shows heads (resp. tails).

Let $S = \{b_1, w_1\} \times \{b_2, w_2\}$, $R = \{h, t\}$ and $\Omega = R \times S$.
Consider the following acts, where $x \succ y$:

	(b_1, b_2)	(b_1, w_2)	(w_1, b_2)	(w_1, w_2)
$\hat{f}$	x	x	y	y
$\tilde{f}$	x	y	x	y
$\hat{f} * \tilde{f}$	x	$x * y$	$y * x$	y
g	y	y	x	x
$\hat{a}$	$x * y$	$x * y$	$y * x$	$y * x$
$\tilde{a}$	$x * y$	y	x	$y * x$

Epstein

	(b_1, b_2)	(b_1, w_2)	(w_1, b_2)	(w_1, w_2)
$\hat{f}$	x	x	y	y
$\tilde{f}$	x	y	x	y
$\hat{f} * \tilde{f}$	x	$x * y$	$y * x$	y
g	y	y	x	x
$\hat{a}$	$x * y$	$x * y$	$y * x$	$y * x$
$\tilde{a}$	$x * y$	y	x	$y * x$

- Epstein (2010) argues: by symmetry $\hat{f} \sim \tilde{f}$.

Epstein

	(b_1, b_2)	(b_1, w_2)	(w_1, b_2)	(w_1, w_2)
$\hat{f}$	x	x	y	y
$\tilde{f}$	x	y	x	y
$\hat{f} * \tilde{f}$	x	$x * y$	$y * x$	y
g	y	y	x	x
$\hat{a}$	$x * y$	$x * y$	$y * x$	$y * x$
$\tilde{a}$	$x * y$	y	x	$y * x$

- Epstein (2010) argues: by symmetry $\hat{f} \sim \tilde{f}$.
- Also $\hat{f} \sim \hat{f} * \tilde{f}$ since in this case the randomization does not reduce exposure to ambiguity.

Epstein

	(b_1, b_2)	(b_1, w_2)	(w_1, b_2)	(w_1, w_2)
$\hat{f}$	x	x	y	y
$\tilde{f}$	x	y	x	y
$\hat{f} * \tilde{f}$	x	$x * y$	$y * x$	y
g	y	y	x	x
$\hat{a}$	$x * y$	$x * y$	$y * x$	$y * x$
$\tilde{a}$	$x * y$	y	x	$y * x$

- Epstein (2010) argues: by symmetry $\hat{f} \sim \tilde{f}$.
- Also $\hat{f} \sim \hat{f} * \tilde{f}$ since in this case the randomization does not reduce exposure to ambiguity.
- Ambiguity aversion suggests $\hat{a} \succ \tilde{a}$.

Epstein

	(b_1, b_2)	(b_1, w_2)	(w_1, b_2)	(w_1, w_2)
$\hat{f}$	x	x	y	y
$\tilde{f}$	x	y	x	y
$\hat{f} * \tilde{f}$	x	$x * y$	$y * x$	y
g	y	y	x	x
$\hat{a}$	$x * y$	$x * y$	$y * x$	$y * x$
$\tilde{a}$	$x * y$	y	x	$y * x$

- Epstein (2010) argues: by symmetry $\hat{f} \sim \tilde{f}$.
- Also $\hat{f} \sim \hat{f} * \tilde{f}$ since in this case the randomization does not reduce exposure to ambiguity.
- Ambiguity aversion suggests $\hat{a} \succ \tilde{a}$.
- These preferences are not compatible with the smooth ambiguity model.

	(b_1, b_2)	(b_1, w_2)	(w_1, b_2)	(w_1, w_2)
$\hat{f}$	x	x	y	y
$\tilde{f}$	x	y	x	y
$\hat{f} * \tilde{f}$	x	$x * y$	$y * x$	y
g	y	y	x	x
$\hat{a}$	$x * y$	$x * y$	$y * x$	$y * x$
$\tilde{a}$	$x * y$	y	x	$y * x$

- Klibanoff, Marinacci, and Mukerji disagree that $\hat{f} * \tilde{f} \sim \hat{f} \sim \tilde{f}$ is natural.

	(b_1, b_2)	(b_1, w_2)	(w_1, b_2)	(w_1, w_2)
$\hat{f}$	x	x	y	y
$\tilde{f}$	x	y	x	y
$\hat{f} * \tilde{f}$	x	$x * y$	$y * x$	y
g	y	y	x	x
$\hat{a}$	$x * y$	$x * y$	$y * x$	$y * x$
$\tilde{a}$	$x * y$	y	x	$y * x$

- Klibanoff, Marinacci, and Mukerji disagree that $\hat{f} * \tilde{f} \sim \hat{f} \sim \tilde{f}$ is natural.
- "The act $\hat{f} * \tilde{f}$ thus diversifies the individual's exposure across the urns: it provides a hedging of the two independent ambiguities. To an individual who is averse to ambiguity, such diversification is naturally valuable."

- Epstein's argument for indifference between $\hat{f}$ and $\hat{f} * \tilde{f}$ since $\hat{f}$ cannot be improved upon by a randomization between $\hat{f}$ and $\tilde{f}$ because "the compositions of the two urns are unrelated" corresponds to the ex-ante view thus he views $\hat{f} * \tilde{f} = \hat{f}^h \tilde{f}$

- Epstein's argument for indifference between $\hat{f}$ and $\hat{f} * \tilde{f}$ since $\hat{f}$ cannot be improved upon by a randomization between $\hat{f}$ and $\tilde{f}$ because "the compositions of the two urns are unrelated" corresponds to the ex-ante view thus he views $\hat{f} * \tilde{f} = \hat{f}^h \tilde{f}$
- KMM's hedging argument refers to the ex-post equivalence of the 'lottery' outcomes $x * y$ and $y * x$, achieved by the 'randomization' $\hat{f} * \tilde{f}$, hence they seem to be identifying $\hat{f} * \tilde{f}$ with $\hat{f}_h \tilde{f}$.

- Epstein's argument for indifference between $\hat{f}$ and $\hat{f} * \tilde{f}$ since $\hat{f}$ cannot be improved upon by a randomization between $\hat{f}$ and $\tilde{f}$ because "the compositions of the two urns are unrelated" corresponds to the ex-ante view thus he views $\hat{f} * \tilde{f} = \hat{f}^h \tilde{f}$
- KMM's hedging argument refers to the ex-post equivalence of the 'lottery' outcomes $x * y$ and $y * x$, achieved by the 'randomization' $\hat{f} * \tilde{f}$, hence they seem to be identifying $\hat{f} * \tilde{f}$ with $\hat{f}_h \tilde{f}$.
- In combination with our analysis this example shows that the smooth model can accommodate ex-post randomizations, however it appears to have difficulties with ex-ante randomizations.

Conclusion

- Dynamic Consistency is a fundamental normative principle.

Conclusion

- Dynamic Consistency is a fundamental normative principle.
- Our results show that is no normative theory which predicts a strict preference for randomization.

Conclusion

- Dynamic Consistency is a fundamental normative principle.
- Our results show that is no normative theory which predicts a strict preference for randomization.
- Experimental studies find little evidence of dynamic consistency.

Conclusion

- Dynamic Consistency is a fundamental normative principle.
- Our results show that is no normative theory which predicts a strict preference for randomization.
- Experimental studies find little evidence of dynamic consistency.
- This suggests there may be a role for a strict preference for randomization in a descriptive theory.

Conclusion

- Dynamic Consistency is a fundamental normative principle.
- Our results show that is no normative theory which predicts a strict preference for randomization.
- Experimental studies find little evidence of dynamic consistency.
- This suggests there may be a role for a strict preference for randomization in a descriptive theory.
- However the available evidence suggests that subjects do not display a strict preference for randomization, Dominiak and Schnedler (2011).

Conclusion

- Dynamic Consistency is a fundamental normative principle.
- Our results show that is no normative theory which predicts a strict preference for randomization.
- Experimental studies find little evidence of dynamic consistency.
- This suggests there may be a role for a strict preference for randomization in a descriptive theory.
- However the available evidence suggests that subjects do not display a strict preference for randomization, Dominiak and Schnedler (2011).
- Preference for randomization is also relevant for understanding the impact of ambiguity in games.

Conclusion

- Dynamic Consistency is a fundamental normative principle.
- Our results show that is no normative theory which predicts a strict preference for randomization.
- Experimental studies find little evidence of dynamic consistency.
- This suggests there may be a role for a strict preference for randomization in a descriptive theory.
- However the available evidence suggests that subjects do not display a strict preference for randomization, Dominiak and Schnedler (2011).
- Preference for randomization is also relevant for understanding the impact of ambiguity in games.
  - For instance if there is a strict preference for randomization. a mixed equilibrium can be strict.

- We should be careful when using the Anscombe-Aumann framework to study non-expected utility models.
- EUT is linear in probabilities. The order in which different kinds of uncertainty are resolved is unimportant. Hence using the Anscombe-Aumann model is largely without loss of generality in this context.
- Non-EU theories (including ambiguity) are non-linear in the probabilities. In this case the order in which different kinds of uncertainty are resolved matters.
- The preference for randomization argument appears to confound ex-post and ex-ante randomizations. The intuition that randomization reduces exposure to ambiguity derives from ex–post randomizations. However Raiffa et al. seem to want to apply it to ex-ante randomizations as well.
- In a Savage framework these issues do not arise. All uncertainty is resolved in one stage.

Dominiak, A., and W. Schnedler (2011): "Attitudes towards Uncertainty and Randomization: An Experimental Study," *Economic Theory*, 48, 289–312.

Eichberger, J., S. Grant, and D. Kelsey (2005): "CEU Preferences and Dynamic Consistency," *Mathematical Social Sciences*, 49, 143–151.

Eichberger, J., and D. Kelsey (1996): "Uncertainty Aversion and Preference for Randomisation," *Journal of Economic Theory*, 71, 31–43.

Ellsberg, D. (1961): "Risk, Ambiguity and the Savage Axioms," *Quarterly Journal of Economics*, 75, 643–669.

Raiffa, H. (1961): "Risk, Ambiguity and the Savage Axioms: Comment," *Quarterly Journal of Economics*, 75, 690–694.