Political Economy and the Firm[1]

Preliminary and Incomplete

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Abstract

This paper argues that there are a number of common features between voting, lobbying, and bargaining. We present a general model which combines the main elements of these phenomena. In this model, agents with more extreme preferences have the greatest incentive to lobby and/or vote. We demonstrate existence of equilibrium and provide a characterization of a focal equilibrium. Then comparative statics of equilibrium are studied. We show that small differences in the cost of voting can have a large effect on the outcome of elections. An application to the theory of the firm is considered.

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1 INTRODUCTION

This paper argues that voting, bargaining and lobbying have a number of factors in common. It presents a model which combines their main features.

1.1 Median Voter Rule

Bulkley, Myles, and Pearson (2001) (henceforth BMP) present a model of voting in committees. Individuals have to pay a cost either in time or money to join a committee, which makes a decision concerning a public good. The committee's decision is made by the median voter rule restricted to the set of individuals who choose to join the committee. They show that it is the individuals with the more extreme preferences who have the greatest incentive to join.[1]

The intuition is that when an individual with extreme preferences joins the committee, the median shifts by more than it would if an individual with moderate preferences joined. Consequently it is those with ideal points furthest from the median who have the greatest incentive to join. Despite this, the actual decision will typically not be extreme. The committee will contain equal numbers from the two extremes in equilibrium. As a result the two sides balance and the overall decision is centrist.

In one respect their model is very specific. They assume that the committee

[1] See also related results in Bulkley and Myles (2001).

makes decisions by majority voting hence the decision is equal to the median voters' preferred point. However much of the intuition of there analysis applies more generally. e.g. to Nash bargaining, menu auction lobbying etc. In all cases one would expect that it is the extremes who have the most incentive to participate in the political process.

In the present paper we set up an abstract model of which median voting is a special case. We show that similar results hold in this more general model.

1.2 Menu-Auction Lobbying

Redoano (2010) uses the menu-auction model of lobbying (see Bernheim and Whinston (1986) and Dixit, Grossman, and Helpman (1997)) to study whether a political union increases or decreases lobbying. This model is similar to BMP since the more extreme groups have the greatest incentive to lobby.[2]

She finds that the effect of political unions is ambiguous. Lobbying may either increase or decrease after union.

Redoano (2010) argues that lobbying may be reduced by entering into a political union with a country with opposite preferences.

[2] However note that Redoano (2010) assumes that extreme groups have the greatest incentive to lobby, see page xy.

1.3 The Political Process

This paper starts with the observation that there are a number of common features between voting, lobbying, bargaining and some other situations. Assume that we are considering a one-dimensional decision, e.g. how much of a public good to provide. What voting and lobbying have in common is that there are two types of individuals. Actions of an individual of the same type as yourself are strategic substitutes for your own actions. If another person on your own side increases their effort this reduces your incentive to supply effort. This is a free riding effect. In contrast actions of the other type of individuals are strategic complements for your own actions. Team sports are similar. It is desirable to increase one's own effort when the other team increases their effort. However increased effort by a member of one's own team can induce a free-riding effect and reduce the incentive to supply effort.

In this paper we consider an abstract model, where a finite number of agents have to make a collective decision such as how much of a public good to provide. These can decide whether or not to participate in the political process. Participation is costly but gives influence over the decision. We demonstrate existence of equilibrium and provide a characterization of a focal equilibrium. We show that, in equilibrium, it is only agents with more extreme preferences which choose to participate. However both extremes are equally represented, hence the outcome is centrist. Comparative static analysis shows that a reduction in the cost, will increase the number of individuals participating in the political process, (after allowing for potential non-uniqueness of equilibrium).

1.4 Applications

There are a number of potential applications.

- The two types of individuals could be men and women. Suppose they have to pay a cost to meet, e.g. by paying for a dating agency/website or by paying to go to the disco. There is another of strategic complementarity since the more women there are at the disco the greater is the incentive for men to attend. However this would reduce the incentives for women to attend. Hence there is also an element of strategic substitutes. Then the k men, most keen to find a partner, go to the dating agency or disco, (similarly for the women).

- Groups lobbying the government for favours. It is the more extreme groups who will spend the most time and money lobbying.

- Volunteer armies. Consider community conflict as in Northern Ireland. Then this is a similar situation. Paramilitary action by the opposing community will increase the incentive for an individual to engage in armed conflict, while if ones own community is strong there is a temptation to free ride on the efforts of others. The model implies that it is the extremists on both sides who join the paramilitaries.

- Oligopoly. Consider a number of firms which compete in price Bertrand style. They produce differentiated goods which can be put into two groups. Call them L and R. Goods in group L are substitutes (in consumption) for other goods in group L and are complements for goods in group R. Vice versa for goods in group R. There are a number of examples e.g. alcoholic drinks and mixers. It is the most profitable drinks and the most profitable mixers which enter the market.

1.5 Differences in Voting Costs

Consider a society which needs to vote to determine a one-dimensional issues such as the level of provision of a public good. Society contains two or more identifiable groups of individuals. Suppose group A may vote without cost, while group B faces a small positive cost of voting. In Section 4 we show that, under some assumptions, in equilibrium only group A will vote. If the outcome is determined by the median voter rule then the outcome of voting will coincide with the median preference in group A. In contrast if both groups faced the same cost of voting (positive or zero) the outcome would be the median of the whole population, which will typically be quite different. Thus it is possible that a small difference in the cost of voting can have a large effect.

An implication is that small biases in election rules can be important. To some extent there will always be differential costs of voting, if only because people live different distances from the polling station. The government should ensure that there is no group which faces an asymmetric cost of voting. Thus all areas should be equally well provided with polling stations. Their opening hours should be chosen as far as possible not to exclude people for instance those who have to work early/late.

Two major determinants of the cost of voting are the time it takes to travel to and from the polling station and the opportunity cost of the time taken in voting. On-line voting would tend to equalise these costs. Thus our analysis may provide a reason for replacing voting systems which require physical attendance at a polling station with on-line voting.

In the case of corporate democracy one could argue that corporate insiders get greater benefit from voting as well as possibly facing larger costs of voting. Our results may explain how it is possible to keep control of a company despite owning a relatively small proportion of the equity.

In the area of corporate governance, governments should try to equalise costs of voting between insiders and outsiders. In the past SEC rules have made it harder for outsiders to communicate. This is perverse and helps to entrench incumbent management.

1.6 Rent-Seeking within Organisations

Rent seeking was originally proposed to describe aspects of the relations between private firms and government. For instance, Posner (1975) considers a situation where a firm is lobbying government for the right to be the sole provider of a good or service. He argues that the firm will be prepared to spend the entire monopoly profits on lobbying. In this case, the social cost of monopoly is much greater than the usual deadweight loss.

Meyer, Milgrom, and Roberts (1992) (henceforth MMR) argue that rent seeking can take place within private sector firms as well as between firms and the government. They use this to build a theory of divestitures. The evidence they cite shows that usually when a divestiture occurs it is the least profitable division which is divested. The buyer is either a management buyout or a firm in a related line of business. Less frequently the most profitable division is divested. This is because weaker divisions may lobby the head office. Effectively such lobbying aims for the division to be subsidised by the rest of the firm. Examples would be requests for equal investment and equal promotion compared to more successful divisions. Such lobbying is undesirable because it risks not allocating investment where it has the highest marginal benefit. If management does redistribute resources the highest cost is borne by the more productive divisions. This is supported by the evidence in **Find References.** Moreover the resources used in lobbying are wasted and finally lobbying wastes time in the head office.

As in the public sector this rent-seeking is dissipative. Thus it is desirable that the organization of firms is chosen to minimise internal rent seeking. MMR argue that divestitures occur to prevent costly rent-seeking within the firm. The least profitable division has the greatest incentive to lobby for transfer payments. This may be prevented by divesting the declining division as a stand-alone entity. This fits well with the evidence described above. Less frequently, the most profitable division is divested. It may be more successful alone as it would not be constrained to cross subsidise the rest of the group. Conversely internal rent-seeking may provide an explanation of why a large proportion of mergers fail.

Consider a conglomerate firm which is divided into a number of divisions. Then, if internal rent seeking does take place, one would expect the various divisions to lobby the head office for transfers. Rent-seeking will depend on institutional details which vary from firm to firm. Consequently we shall adopt a general model which abstracts from specific institutional details. As the results in section 3.2 show, in equilibrium, it is the extreme divisions which lobby the centre for transfers. Thus, as in MMR, the costs of lobbying may be reduced by divesting them.

In political economy, it is argued that when countries merge there is less likely to be lobbying due to the *preference dilution* effect. A special interest lobby is likely to be a smaller minority in a larger country than in a small country and hence will be less effective. For example the whisky industry may be able to form a powerful lobby which could influence the Scottish government. However its influence over a European government is unlikely to be as great. Thus we appear to get the paradoxical result that in the political context rent-seeking is less in larger units while in the context of the firm, rent seeking is less when production is organised in smaller units.

Organisation of the Paper Our framework and definition are set out in Section 2. Existence of equilibrium is demonstrated in Section 3, which then proceeds to investigate the properties of equilibrium. In particular the comparative statics of changing the cost of voting. The case where the costs of voting are different for different groups of individuals is studied in Section 4 and Section 5 concludes.

2 Model

We aim to build an abstract model which covers voting, lobbying and some other cases.

2.1 Set-Up

Society needs to make a one dimensional policy decision, for instance to determine the level of provision of a public good. The policy x is chosen from a policy space X, which for convenience we shall identify with the real line $\mathbb{R}$.

Society consists of a set I of $2n$ individuals, divided into two equal sized groups, high agents $H = \{H_1, ..., H_n\}$ and low agents $L = \{L_1, ..., L_n\}$.[3] For instance if we are considering political voting, the high agents could be those on the left and the low agents could be those on the right. The division into 2 groups is for expositional convenience. The crucial assumption is that preferences differ in one dimension and are single-peaked.

Individual i has a strictly concave utility function $u_i : X \to \mathbb{R}$. Hence (s)he has a unique optimal provision of the public good, $x_i^* \in X$, which reflects his/her trade off between the benefit of the public good and paying his/her share of the cost of provision. Voters from the H-group (resp. L-group) want public good provision above (resp. below) the default level. Higher numbered individuals from the H-group (resp. L-group) are assumed to want higher (resp. lower) levels of public good.

Each individual makes a discrete decision whether or not to participate. We

[3] In practice the groups can usually be determined endogenously. For instance if the decision is made by the median voter rule the H-group (resp. L-group) would be the set of individuals with ideal points above (resp. below) the median.

consider a game where each player can choose between two strategies accept, a, become a member or reject, r, not became a member. The cost of joining a lobby (or voting) is c_i. Except in Section 4, we shall assume a common cost of voting, i.e. $c_i = c$ for all $i \in I$.

In such a game a strategy profile can be described by the set Λ of individuals who have joined, i.e. play strategy a. We shall refer to Λ as a *lobby* using terminology from the menu-auction model. However other interpretations are possible. It may also denote those individuals who choose to join a decision-making committee or those citizens who choose to vote when voting is costly.

Let Λ denote the set of individuals who choose to participate in the political process and let $x(\Lambda)$ denote the provision of the public good. Define $v_i(\Lambda) = u_i(x(\Lambda))$, for $i \in I$. Then if $i \in \Lambda$, individual i's marginal benefit of participating is given by $\Delta_i(\Lambda) = v_i(\Lambda) - v_i(\Lambda \backslash i)$. Individual i will choose to participate if $\Delta_i(\Lambda) > c_i$.

Higher numbered individual have greater total and marginal benefits from the public good, hence if $i > j$, $v_{H_i}(z) > v_{H_j}(z)$ and $v'_{H_i}(z) > v'_{H_j}(z)$. Similarly $i > j$, $v_{L_i}(z) > v_{L_j}(z)$ and $v'_{L_i}(z) > v'_{L_j}(z)$. Thus

$$x^*_{H_n} > x^*_{H_{n-1}} > ... > x^*_{H_1} > x^*_{L_1} > ... > x^*_{L_n}.$$

Each individual, i, decides whether or not to participate at cost c_i. (The cost of voting can be either in money or in time.)

There is strategic complementarity between groups. However there is free riding within a group, which means this is also partly a game of strategic substitutes.

For analytic tractability we shall confine attention to lobbying games which are symmetric in the following sense.

Definition 2.1 *Say that a lobby* Λ *is* symmetric *if* $H_k \in \Lambda \Leftrightarrow L_k \in \Lambda, 1 \leqslant k \leqslant n.$ *A lobbying game* Γ *is* symmetric *if when* $H_k \in \Lambda, L_k \in \Lambda, \Delta_{H_k}(\Lambda) = \Delta_{L_k}(\Lambda'),$ *where* $\Lambda' = \{H_k : L_k \in \Lambda, L_k : H_k \in \Lambda\}$ *and* $1 \leqslant k \leqslant n.$

This says that the model is symmetric in the sense that H_k and L_k have equal and opposite influence on decision-making. We do not believe it crucially affects the results and that similar analysis would apply to situations which are not too far from symmetric.

2.2 Assumptions

This section states the main assumptions we shall be using. The first assumption says that the marginal benefit of lobbying is increasing in k.

Assumption 1 *Marginal benefit is increasing in* k, *i.e.*

1. *If* $H_j, H_\ell \in \Lambda,$ *with* $j > \ell$ *then* $\Delta_{H_j}(\Lambda) > \Delta_{H_\ell}(\Lambda).$

2. *If* $L_j, L_\ell \in \Lambda,$ *with* $j > \ell$ *then* $\Delta_{L_j}(\Lambda) > \Delta_{L_\ell}(\Lambda).$

There are two motivations for this assumption. Firstly higher numbered individuals get higher marginal benefit from the public good. Secondly by joining a lobby a higher numbered individual may be more able to shift the outcome more in his/her favour.[4]

The following assumption states that the marginal benefit of joining is increasing in the number of members of the opposing group which are in the lobby. When more opponents join, this will move the outcome further from H_k's ideal point. Assuming the utility loss is a convex function of the distance from the ideal point, this will increase the marginal benefit of joining.

Assumption 2 *Marginal benefit is greater the more opponents are lobbying. Similarly marginal benefit is higher the fewer people who are lobbying on your own side.*

1. *if* $L_j \notin \Lambda, H_k \in \Lambda, \Delta_{H_k}(\Lambda \cup L_j) \geqslant \Delta_{H_k}(\Lambda)$;

2. *if* $H_j \notin \Lambda, L_k \in \Lambda, \Delta_{L_k}(\Lambda \cup H_j) \geqslant \Delta_{L_k}(\Lambda)$;

3. *if* $H_j \notin \Lambda, H_k \in \Lambda, \Delta_{H_k}(\Lambda \cup H_j) \leqslant \Delta_{H_k}(\Lambda)$;

4. *if* $L_j \notin \Lambda, L_k \in \Lambda, \Delta_{L_k}(\Lambda \cup L_j) \leqslant \Delta_{L_k}(\Lambda)$.

The following assumption captures the effect of having a more influential opponent.

[4] This would even occur with the median voter rule if there were currently an even number of voters and the new member's ideal point was adjacent to the median. This effect would be stronger in the context of lobbying or bargaining.

Assumption 3 (Higher Rank) *If* $\hat{k} > \tilde{k}$ *and* $H_{\tilde{k}} \in \Lambda$, *and* $H_{\hat{k}} \notin \Lambda$, *and* $H_k, L_k \in \Lambda$, *then* $\Delta_{H_k}(\Lambda) \geqslant \Delta_{H_k}\left((\Lambda \backslash H_{\tilde{k}}) \cup H_{\hat{k}}\right)$ *and* $\Delta_{L_k}(\Lambda) \leqslant \Delta_{L_k}\left((\Lambda \backslash H_{\tilde{k}}) \cup H_{\hat{k}}\right).$

The motivation for this assumption is similar to that for Assumption 2, which says that more opponents will increase the marginal benefit of participation. This assumption says that, ceteris paribus, having a more influential opponent also increases the marginal benefit of participation. In menu-auction lobbying a higher ranked individual would have a more benefit from public goods and so would lobby harder. Similarly under the median voter rule a higher ranked individual joining may cause the median to shift by more.[5]

The following assumption says that adding two groups with equal and opposite preferences does not affect the marginal benefit of a third party.

Assumption 4 (Matched Pairs) *For any symmetric lobby* Λ, *if* $H_{\tilde{k}} \notin \Lambda, L_{\tilde{k}} \notin \Lambda, k \neq \tilde{k}$ *then* $\Delta_{H_k}(\Lambda) = \Delta_{H_k}\left(\Lambda \cup H_{\tilde{k}} \cup L_{\tilde{k}}\right)$, *for all* $k \neq \tilde{k}$.

The motivation for this is that in a symmetric lobby the H and L-groups have equal and opposite influence. Adding a further two groups with equal and opposite influence to a symmetric lobby will not change the political equilibrium. Hence $\Delta_{H_k}\left(\Lambda \cup H_{\tilde{k}} \cup L_{\tilde{k}}\right)$ will only depend on H_k's tastes which have not changed. This clearly applies to the median voter rule. It also applies to menu-auction lobbying,

[5] This occurs in the event of a tie. In any event it never helps to have a more influential opponent.

since lobbying by two equal and opposite groups will not change the outcome. We need to restrict this axiom to symmetric lobbies otherwise $H_{\tilde{k}}$ and $L_{\tilde{k}}$ may both be on the same side of the outcome $x(\Lambda)$.

Lemma 2.1 *Assumptions 1 and 4 imply that the marginal benefit* $\Delta_{H_k}(\Lambda_k)$ *is a strictly increasing function of* k, *where* $\Lambda_k = \{L_k, ..., L_n, H_k, ..., H_n\}$.

Proof. By Assumption 4, $\Delta_{H_k}(\Lambda_k) = \Delta_{H_k}(\Lambda_{k-1})$, (note Λ_k is symmetric) By Assumption 1, $\Delta_{H_k}(\Lambda_{k-1}) > \Delta_{H_{k-1}}(\Lambda_{k-1})$. ■

This result may be motivated as follows. The marginal benefit $\Delta_{H_k}(\Lambda_k)$ depends both on how the addition of H_k impacts the political equilibrium and on H_k's taste for the public good. Recall that the H_k's have been numbered so that if $\hat{k} > \tilde{k}$, $H_{\hat{k}}$ has higher total and marginal benefit from the public good than $H_{\tilde{k}}$. Thus for a given impact on the political equilibrium, $H_{\hat{k}}$ would get greater marginal benefit from the increase in the public good. Hence $\Delta_{H_k}(\Lambda_k)$ will be increasing, unless the impact of H_k on the political equilibrium is decreasing in k.

Since for all k, for every group in Λ_k there is a group with the equal and opposite preference a case can be made that the marginal impact on the political equilibrium of adding H_k would be constant. This would be the case if the decision is made by the median voter rule. Alternatively it could be argued that the marginal impact of adding H_k to Λ_k would be increasing. For higher values of k, Λ_k is a smaller lobby thus one might expect the impact of an additional group to be greater. This would be the case if the outcome is determined by menu-auction lobbying. In neither case does a decreasing marginal impact on the political equilibrium seem plausible.

3 Equilibrium

The political process is a game with finite set of players and finite strategy sets. The solution concept we shall use is Nash equilibrium in pure strategies. In this section we prove existence by explicitly constructing an equilibrium lobby. We show that any equilibrium lobby must be balanced in the sense that it contains an equal number of L and H individuals. Finally we study the comparative static properties of equilibrium.

3.1 Existence and Properties of Equilibrium

3.1.1 Existence

The next result is our main existence result.

Theorem 3.1 *Consider a symmetric lobbying game* Γ, *which satisfies Assumptions 2, and 4. Then* Γ *has a symmetric equilibrium.*

Proof. Let $k^* = \min\{k : \Delta_{H_k}(\Lambda_k) \geqslant c\}$. We shall show that Λ_{k^*} is a symmetric equilibrium lobby. By Assumption 2, if $k' < k^*$, $\Delta_{H_{k'}}(\Lambda_{k^*} \cup H_{k'}) <$

$\Delta_{H_{k'}}(\Lambda_{k^*} \cup H_{k'} \cup L_{k'})$.

By Assumption 4, $\Delta_{H_{k'}}(\Lambda_{k^*} \cup H_{k'} \cup L_{k'}) = \Delta_{H_{k'}}(\Lambda_{k'}) < c$ by definition of k^*, (note $\Lambda_{k'}$ is symmetric). Thus $H_{k'}$ has no incentive to join Λ_{k^*}.

If $k'' > k^*$, then by Assumption 2, $\Delta_{H_{k''}}(\Lambda_{k^*}) > \Delta_{H_{k^*}}(\Lambda_{k^*}) \geqslant c$. Thus $H_{k''}$ does not have an incentive to leave the lobby. A similar argument applies to the L-groups, which establishes that $\Lambda_{H_{k^*}}$ is a symmetric equilibrium lobby. ■.

The diagram is illustrated for the case where $k^* = n-2$ in figure 1. In the situation shown two individuals on either extreme participate H_n, H_{n-1}, L_n, and L_{n-1}. The remainder abstain.

Existence of equilibrium follows from Nash's theorem. However this result goes further since it establishes existence of equilibrium in pure strategies and constructs a focal equilibrium.

3.1.2 Uniqueness

In general, we would not expect the equilibria to be unique due to strategic complementarity. Assumption 2 (MB opponents) implies $\Delta_{H_n}(\varnothing) < \Delta_{H_n}(H_n, L_n)$. Thus if the cost of voting, c, is such that

$$\Delta_{H_n}(H_n) < c < \Delta_{H_n}(H_n, L_n)$$

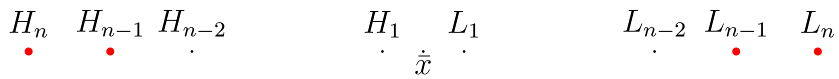

Figure 1:

there will be two equilibria one where nobody votes and one where both H_n and L_n vote. For similar reasons there is non-uniqueness of equilibrium when

$$\Delta_{H_k}\left(\Lambda_{k+1}\cup H_k\right) < c < \Delta_{H_k}\left(\Lambda_{k+1}\right).$$

Thus non-uniqueness occurs at many points in the parameter space.[6]

[6]Bulkley and Myles (2001) claim to have shown uniqueness of equilibrium in a related model. However in fact what they show is that any equilibrium must be balanced and connected (in their

3.1.3 Balanced

The following result demonstrates that any equilibrium lobby must contain an equal number of L and H groups.

Definition 3.1 *A lobby Λ is* balanced *if it contains an equal number of L and H groups.*

Proposition 3.1 *Under Assumption 2, an equilibrium lobby must be balanced.*

Proof. We shall prove the result by contradiction. Suppose if possible that there exists an equilibrium lobby Λ, which is not balanced.

Let $\mathcal{H} = \{H_i : H_i \in \Lambda, L_i \notin \Lambda\}$ and $\mathcal{L} = \{L_i : L_i \notin \Lambda, H_i \in \Lambda\}$. Without loss of generality we may assume that Λ contains more H groups than L groups, which implies that both $\mathcal{H}$ and $\mathcal{L}$ are non-empty. Then since Λ is an equilibrium lobby if $H_i \in \mathcal{H}$, $\Delta_{H_i}(\Lambda) \geqslant 0$. By symmetry, $\Delta_{L_i}(\Lambda \cup \mathcal{L} \backslash \mathcal{H}) \geqslant 0$. By Assumption 2, $\Delta_{L_i}(\Lambda \cup \mathcal{L}) > 0$. The fact that marginal benefit is decreasing in the number of groups lobbying on the same side implies $\Delta_{L_i}(\Lambda \cup L_i) > 0$. However this latter inequality implies that Λ is not an equilibrium lobby, since L_i has an incentive to join. The result follows. ■

The intuition is that if H_i has an incentive to join lobby Λ then so does L_i.[7]

terminology). They do not show that there cannot be multiple balanced and connected equilibria. Our argument shows that there will typically be parameters values at which equilibrium is not unique.

[7] See BMP p. 127, for an example of an equilibrium lobby which is balanced but not symmetric.

3.2 Comparative Statics

In this section, we continue the characterization of equilibrium by investigating its comparative static properties. We find that a cost reduction will increase the size of the largest and smallest equilibrium lobbies.

Consider the following thought experiment. Imagine the cost of lobbying is initially very high and is gradually lowered to zero. Then initially no group will lobby. As the cost is lowered we shall get to a point where it is just worthwhile for H_n to lobby provided that L_n does as well. In this range, there are two equilibria. One where no group lobbies and one where the two most extreme groups, H_n and L_n lobby. If cost is lowered further then there will come a point where it is worthwhile for H_n to lobby even if L_n does not. In this case there is a unique equilibrium where both H_n and L_n lobby.

This process will repeat it self as c is lowered there will come a critical point where H_{n-1} and L_{n-1} lobby. As c is progressively lowered there will be series of critical points, at each of these, the two most extreme groups on either side which are not already lobbying, will join the lobby. This process will continue until $c = 0$ when all groups lobby.[8]

Taking into account the potential non-uniqueness of equilibrium the comparative static result can be stated as a reduction in the cost of lobbying will increase

[8] This comparative static argument is similar to the pseudo-dynamic discussed in Bulkley and Myles (2001).

the size of the largest and smallest equilibrium lobbies. To prove this we need an extra assumption in addition to our maintained hypotheses.

Assumption 5 (Replacement) *Assume that* $\hat{k} > \tilde{k}$. *If* Λ *is a balanced lobby such that* $H_{\hat{k}} \notin \Lambda$ *and* $H_{\tilde{k}} \in \Lambda$ *then* $\Delta_{H_{\hat{k}}}((\Lambda \backslash H_{\tilde{k}}) \cup H_{\hat{k}}) > \Delta_{H_{\tilde{k}}}(\Lambda)$.

The intuition is that the addition of $H_{\hat{k}}$ will shift the political equilibrium more in the H's favour than the addition of $H_{\tilde{k}}$. Or at least the addition of $H_{\hat{k}}$ will not have a less favourable effect on the political equilibrium. Since $H_{\hat{k}}$ has a higher marginal benefit of the public good it follows that $\Delta_{H_{\hat{k}}}((\Lambda \backslash H_{\tilde{k}}) \cup H_{\hat{k}}) > \Delta_{H_{\tilde{k}}}(\Lambda)$.

Next is our main result on the comparative statics of the cost of lobbying which shows that an increase in the cost of lobbying reduces the size of the largest and smallest equilibrium lobbies.

Theorem 3.2 *Consider a symmetric lobbying game* Γ, *which satisfies Assumptions 1, 2, 3, 4, and 5. Then the size of largest and smallest equilibrium lobbies is a decreasing function of the cost of lobbying* c.

Proof. Suppose that $\bar{\Lambda}$ is the largest equilibrium lobby when the cost of lobbying is $\tilde{c}$. Consider $\hat{c} > \tilde{c}$. We need to show that when the cost is $\hat{c}$ all equilibrium lobbies are no larger than $\bar{\Lambda}$. If $\bar{\Lambda} = \mathcal{I}$ the result is trivial, so we may assume $\bar{\Lambda} \subsetneqq \mathcal{I}$.

Suppose that Λ' is strictly larger than $\bar{\Lambda}$. Then Λ' is not an equilibrium lobby. Thus there exists an individual, j, in Λ' who wishes to leave, i.e. $\Delta_j(\Lambda') < \tilde{c}$..

This implies $\Delta_j(\Lambda') < \hat{c}$, thus Λ' is not an equilibrium lobby when the cost is $\hat{c}$ either. Hence the largest lobby when the cost is $\hat{c}$ is no greater than $\bar{\Lambda}$. A similar argument applies to the smallest equilibrium lobby. ■

4 Differential Costs

In this section we study how differences in the cost of voting can affect the outcome of elections. Our model considers population where the cost of voting varies between different groups of individuals. We show that small differences in the cost of voting can have large effects on the outcome of elections.

Assume first that all individuals face zero cost of voting. Then everybody will vote and the outcome will be the median of the whole community, $\bar{x}$. Now assume that both groups face the same positive cost of voting $c_H = c_L = c$. Then in equilibrium there exists k^*, such that $H_n, ..., H_{k^*}$ and $L_n, ..., L_{k^*}$ vote and the rest do not, see Section 3. As the distribution of voters is symmetric the outcome of voting is still the community median, $\bar{x}$. Thus a cost of voting will not affect the outcome if it applies to all individuals equally, even if the cost is large. In contrast we show that when the costs of voting differ between individuals, they can have a large effect.

4.1 Analysis

The model is the same as in previous sections however we restrict attention to the median voter rule.

Consider a situation in which all members of group H vote and no members of group L vote. Let m_H denote the median of $\left\{x^*_{H_1}, ..., x^*_{H_n}\right\}$. Then the entry of any of the L's will shift the equilibrium from m_H to $x^*_{H_\nu}$, where $n = 2\rho + 1$ if odd, $= 2\rho$ if even.

The next result considers a situation where one group can vote free, while the other has to pay a small positive cost for voting. It shows that if the cost exceeds a certain threshold nobody from the group which faces a positive cost will vote. We shall argue that this threshold can be quite low. The outcome is accordingly the median preference of the group which can vote free.

Proposition 4.1 *Assume that H's face zero cost of voting. Provided the cost of voting for the L's,* $c_L \geqslant \tilde{c} = v_{L_n}\left(x^*_{H_\rho} - x_{L_n}\right) - v_{L_n}\left(m_H - x_{L_n}\right)$ *there exists an equilibrium, in which none of the L-group votes, all of the H-group votes and the outcome is the median of the H-group,* m_H.

Proof. All of the H-group will vote. Voting always has a positive, if small, benefit as it shifts the equilibrium, at least marginally, in one's favour. Since the H-group face zero cost of voting all of them will vote. If none of the L's vote then the outcome will be the median of the H's i.e. m_H.

Consider individual L_n. If L_n votes, his utility is $u^V_{L_n} = v_{L_n}\left(x^*_{H_\rho} - x_{L_n}\right) + \bar{y}_{L_n} - c_L$. If L_n does not vote, his utility is $u^{NV}_{L_n} = v_{L_n}\left(m_H - x_{L_n}\right) + \bar{y}_{L_n}$.

The difference is: $u^V_{L_n} - ^{NV}_{L_n} = v_{L_n}\left(x^*_{H_\rho} - x_{L_n}\right) - v_{L_n}\left(m_H - x_{L_n}\right) - c_L$. Thus L_n will vote if the marginal benefit of voting $v_{L_n}\left(x^*_{H_\rho} - x_{L_n}\right) - v_{L_n}\left(m_H - x_{L_n}\right)$ exceeds the marginal cost of voting c_L. Thus under the stated assumptions L_n will not vote. Since L_n has the greatest marginal and total benefit of voting, none of the L's will vote. This establishes that the given profile of preferences is indeed an equilibrium. ■

The result is illustrated in figure 2. The entry of L_n only produces a small shift in the median of the set of voters and thus can be deterred by a relatively small cost. Specifically the entry of any L-individual shifts the outcome from the community-wide median to H_m's ideal point.

The key point is that the entry of any of the L's only shifts the outcome from the median of the H's to the next lower ideal point of a member of the H-group. The benefit which any member of the L-group gets from this is small. Thus a relatively small cost differential may have a large effect. There is an externality problem here. Members of the L-group do not take into account the benefits they give to other L's when deciding whether or not to vote.

The critical cost of voting $\tilde{c}$ need not be large. Recall $\tilde{c}$ is defined by $\tilde{c} = v_{L_n}\left(x^*_{H_\rho} - x_{L_n}\right) - v_{L_n}\left(m_H - x_{L_n}\right)$. This is just the difference between the benefit

Figure 2:

to L_n of m_H and the benefit to L_n of the next lower ideal point from the H-group. If there are a lot of H's this difference could be quite small. If instead of equal spacing, the ideal points of the H's were clustered more closely to the median, this would reduce the incentive for the L's to vote still further. The effect of this would be to reduce the amount by which an L could shift the policy outcome in his/her favour by entering. Thus for a given cost, $L_n > 0$, if there is an H sufficiently close to the median then there will exist an equilibrium in which none of the L's vote.

This result is perhaps easiest to understand in the extreme case where there are a continuum of H-individuals. In this case if a single L-individual votes then (s)he will only have an infinitesimal effect on the outcome. Hence if any positive cost of voting, no matter how small, is imposed on the L- group none of them will vote.

We believe the equilibrium described in Proposition 4.1 is reasonably unique. Given a zero cost of voting all of the H-group will vote. This makes the marginal benefit of voting for the L-group low.

4.2 Both Groups Face Positive Costs

Suppose both types of voter have to pay a cost of voting but one type faces a lower cost than the other. Then we get a similar result however the effect is less stark.

To see this assume that the H-group also has to pay a cost of voting $c_H < c_L$. Define ν by $\nu = \max\left\{k : x^*_{H_k} \leqslant m_H, v_{H_k}\left(x^*_{H_k}\right) - v_{H_k}\left(m_H\right) \geqslant c_H\right\}$. Thus H_ν is the marginal person who just thinks it worthwhile to vote. (In fact there are two such marginal voters. Of the two, H_ν is the one with the lower ideal point.)

Define $\bar{c} = v_{L_n}\left(m_H - x^*_{L_n}\right) - v_{L_n}\left(x^*_{H_\nu} - x^*_{L_n}\right)$.

The following result shows the situation is similar if both groups face a positive cost of voting. If the cost differential is above a threshold the L-group will not vote. However only a subset of the H-group will vote. The political debate only takes place within the H-group. In particular only the two extremes of the H-group vote.

Proposition 4.2 *Assume the H-group face a cost of voting c_H and the L-group face a cost of voting c_L. If $c_L > c_H$ and $c_L \geqslant \bar{c} = v_{L_n}\left(x^*_{H_\nu} - x_{L_n}\right) - v_{L_n}\left(m_H - x_{L_n}\right)$ there exists an equilibrium, in which individuals $H_1, ..., H_\nu$ and $H_{n-\nu}, ..., H_n$ from the H-group and none of the L-group votes. The outcome is the median ideal point of the H-group, m_H.*

Proof. We need to demonstrate that the stated voting strategies constitute a Nash equilibrium. Assume first that only members of the H-group vote. Then we may apply the analysis of Sections 2 and 3 to the H-group. The fact that the given strategies constitute a Nash equilibrium is follows from the proof of Theorem 3.1. This result also shows that under these assumptions the outcome will be m_H.

If none of the L's vote then the outcome will be the median of the H's i.e. m_H. If L_n votes his utility is $u^V_{L_n} = v_{L_n}\left(x^*_{H_\nu} - x_{L_n}\right) + \bar{y}_{L_n} - c_L$. If L_n does not vote his utility is $u^{NV}_{L_n} = v_{L_n}\left(m_H - x_{L_n}\right) + \bar{y}_{L_n}$. The difference is: $u^V_{L_n} - u^{NV}_{L_n} = v_{L_n}\left(x^*_{H_\nu} - x_{L_n}\right) - v_{L_n}\left(m_H - x_{L_n}\right) - c_L$. Thus L_n will vote if the marginal benefit of voting $v_{L_n}\left(x^*_{H_\nu} - x_{L_n}\right) - v_{L_n}\left(m_H - x_{L_n}\right)$ exceeds the marginal cost of voting c_L. Thus under the stated assumptions L_n will not vote. Since L_n has the greatest marginal and total benefit of voting none of the L's will vote. This demonstrates that the stated set of strategies constitutes a Nash equilibrium. ■

The smaller is c_H the more H's will vote and the smaller is the difference between m_H and $x^*_{H_\nu}$. (Recall ν depends on c_H.) If a member of the L-group enters this will shift the equilibrium from m_H to $x^*_{H_\nu}$. If this difference between these two points is small then a small cost of voting will deter all members from voting.

4.3 Relative Cost of Voting

We have shown above that small absolute differences in the cost of voting can give rise to situation where all the individuals who actually vote come from the same group.

A natural question is whether small *relative* differences in the cost of voting can have a similar effect. The graph below represents an example in which the answer is negative, unless one group faces zero cost of voting. In the graph utility takes a simple quadratic form. The cost of voting of for group H, c_H is denoted by x. For a given x, y denotes the lowest cost of voting for the L's compatible with an equilibrium in which none of the L's vote.

The key thing to note about the graph is that it has infinite slope at the origin. Hence it is not possible to get an equilibrium in which no member of group L votes with c_L, c_H small and $\frac{c_L}{c_H}$ close to one.

4.4 Examples

In this section we consider some examples where small biases in election rules appear to have had a large impact on outcomes.

Joh Bjelke-Petersen was state premier of Queensland from 1968-1987. His party, the right-wing National party held power from 1957 to 1989.[9] However Queensland is not intrinsically right-wing. The left-wing Labor party was regularly in power both before and after this period. Bjelke-Petersen's dominance of power was achieved by biasing the electoral rules in his favour. The system favoured rural voters who tended to vote for the National party. However any bias was not great. Queensland had a constitution which closely followed the Westminster system of parliamentary democracy. Australia was viewed as being a full member of the group of western democracies during this time.

Another issue is the redrawing of constituency boundaries. This should be done carefully to avoid introducing biases into the electoral process. Queensland is an example where the boundaries did create bias. Another implication is that irregular shaped constituencies seen, for instance, in many US states may have a significant influence on the outcome of elections.

More generally one can show that differences in the *benefit* of voting can also have a disproportionate effect on the outcomes of elections. Indeed perhaps Bjelke-

[9] The National party changed name during this period. It was previously called the Country party. The change of name was intended to attract more votes in urban areas.

Petersen's Queensland is better seen as an example of differences in benefits of voting. The benefit of voting in a small rural constituency is greater than that of voting in a large urban constituency because the probability of being pivotal is higher.

These issues may also be relevant to corporate democracy. The costs of voting are lower for corporate insiders than for outsiders. As a result in shareholder votes, management nominees have a very significant advantage. This implies that insiders may keep control with only a relatively small minority of shares.

It is sometime alleged that older people have disproportionate influence in politics. Retired people may well have a lower opportunity cost of time and thus a lower cost of voting. If so, this would be compatible with our result.

5 Conclusion

This paper has provided a unified framework which covers both the menu-auction and median voter models of collective decision-making. It shows that when there is a cost to voting/lobbying it is the more extreme groups on both sides which have the greatest incentive to vote/lobby. We have suggested applications in the area of corporate governance.

This model can be interpreted as a game of incomplete information. In this interpretation there are two players H and L. Each has n types depending on his/her strength of preference. Players can influence the collective decision by voting/lobbying at cost c. In equilibrium it is the types k^* to n of each individual who choose to vote.

5.1 Related Literature

Two closely related papers are BMP and Bulkley and Myles (2001). Their results are in some respects more and in other respects less general than ours. They are more general since some of their results do not require symmetry. However they confine attention to the median voter rule and they do not investigate comparative statics or the implications of differential costs of voting.

5.2 Welfare Effects

Assume that transfers between individuals are not possible. Then any level of public good provision between the ideal point of H_n and that of L_n would be Pareto optimal. Thus the political process would choose a Pareto optimal level of the public good. The process as a whole is not Pareto optimal since the cost of voting is wasted.

If transfers between individuals are allowed then at the optimum the usual Samuelson condition for provision of public goods will be satisfied. Bowen (1943) shows that median voter rule satisfies this condition if the median and mean voter have the same ideal point. If this (restrictive) assumption is satisfied the Pareto optimal level of public good will be produced.

In this model lobbying is costly, yet in equilibrium it does not change the decision. Thus lobbying is wasteful. There are two cases where the outcome is efficient. If $c = 0$ then all groups lobby. However they do not change the decision and there are no dissipative. costs to this lobbying. There is also no deadweight loss from lobbying if c is sufficiently large that no group chooses to lobby.

Recall that for a given c it is the extreme divisions on both sides which lobby. If we take c as given, then dissipative cost may be reduced by reducing the number of extreme divisions. As in MMR it would be desirable to spin off the extreme divisions as stand alone firms.

This would be an argument for devolution if the devolved regions were more homogenous and hence had less extremists.

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