

## Auctions

- An auction is a mechanism for trading items by means of bidding.
- Dates back to 500 BC where Babylonians auctioned off women as wives.
- Position of Emperor of Rome was auctioned off in 193 ad
- Can have the bidders trying to buy an item: Christie's, ebay, snapnames.
- Can have the bidders trying to sell an item : Procurement, priceline.com
- Spectrum auctions

## Rules to Auctions

- **First-Price Sealed-Bid Auction:** Everyone writes down a bid in secret. The person with the highest bid wins the object and pays what he bids.
- **Second-Price Sealed-Bid (Vickrey) Auction:** Everyone writes down a bid in secret. The person with the highest bid wins the object and pays the **second highest** bid. (used for stamps and by Goethe)
- **English Auction:** The auctioneer starts at a reserve price and increases the price until only one bidder is left.
- **Dutch Auction (Not Demonstrated):** The auctioneer starts at a high price and decreases the price until a bidder accepts the price.

## Auction types

- **All Pay Auction:** Everyone writes down a bid in secret. The person with the highest bid wins. Everyone pays.

## Two types of Settings: Common and Private

- Examples of Common Value Auctions:
  - Spectrum.
  - Oil Drilling
  - Book Example.
- Examples of Private Value Auctions:
  - Consumption items.
  - Memorabilia

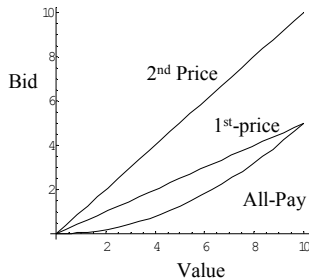
## Private Value Auctions: Example:

- I am auctioning off the right shoe of the star of the Exeter City Football team.
- Al has value £30. Bob has value £40. Chris has value £55.
- What is revenue in the following situations?
  - 2<sup>nd</sup>-price sealed bid.
  - 1<sup>st</sup> price sealed bid. Each bids 2/3 of his value.
  - English, Each bids up to his value
  - Dutch, each bids 2/3 of his value.

## Uniformly distributed values.

- Values are drawn from 0 to £10 with an equal chance of each amount (like in the lab). N is the number of bidders.

## Private values: Equilibrium Bid Functions



## Strategies with Private values: English Auction

- The English: stay in the auction until either
  - you win
  - or the bid goes higher than your value.
  - If not one either makes one lose when it is worthwhile to win or win when it is worthwhile to lose.
- The key to understanding this is to understand that staying in does not affect the price one pays if they win only whether one wins (it does affect others' prices).

## Strategies with Private Values: 2nd Price Auctions

- 2nd price similar logic to English auction.
- It is optimal to bid one's value.
  - One's bid does not affect the price one pays only whether or not one pays.
  - Raising one's bid will cause one to win when it is not worthwhile.
  - Lowering one's bid will cause one to lose when it was worthwhile to win.

## Strategies with Private Values: First Price

- Strategies in the first-price should shade bid below your value
  - This is because one's bid affects one's price.
  - Bidding your value will earn zero surplus.
  - Shading one's bid lowers the probability of winning, but increases the surplus gained when winning.
- There is a natural trade-off between probability of winning and profit if one wins.
  - If bid is  $b$ , value is  $v$ , expected profit is  $\text{Probwin}(b) \cdot (v-b)$
  - Derivative of this w.r.t.  $b$  yields  $\text{Probwin}'(b) \cdot (v-b) - \text{Probwin}(b) = 0$
  - First term is marginal benefit of prob of winning.
  - Second term is marginal cost of the profit if one wins.

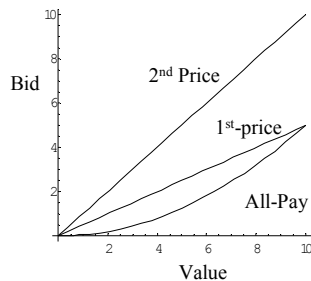
## All-pay auction

- May seem like a strange auction to run/study but...
- It is used in charity auctions and from the lab one can see why. (Losers don't complain so much.)
- Extremely useful modelling tool.
  - Patent Races.
  - Political Campaigns.
  - Technology contests.
  - Procurement contests – Architecture, Next Generation Fighter Jet.
  - Sports contests. (Think of Chelsea, Man U, Arsenal all buying the best players.)

## Strategies with Private Values: All Pay

- In the all-pay auction, you should again shade bid below your value.
- The natural trade-off is now between probability of winning and cost of bidding.
- This cost is incurred whether you win or not.
- It only makes sense to incur a high cost if the probability of winning is fairly high.
- For low values, bids are shaded much more than with first-price auctions.

## Equilibrium Bid Functions



## Strategies with uniform values.

- Values are drawn from 0 to £10 with an equal chance of each amount (like in the lab).  $N$  is the number of bidders.
- 1<sup>st</sup>-price the equilibrium bid  $(N-1)*v/N$  (that is if  $v=£5.50$  and  $N=2$ , bid £2.75).
- Dutch auction is the same as the 1<sup>st</sup>-price.
- 2<sup>nd</sup>-price, optimal to bid value. English optimal to bid up to value.
- All-pay auction, should bid  $(N-1) * (v/10)^N * 10/N$  (looks complicated but only we can see for low values shade bid more than for high values).

## Revenue Equivalence

- For private values, there is *revenue equivalence* among all designs.
- Not only that but all auctions are fully efficient – the buyer who values the object the most winds up buying it.
- If a seller wants to maximize revenue, he can simply use an appropriate minimum bid in any of the designs.
- *Problems happen if:* asymmetry, risk aversion, common values, seller info.

## Revenue Equivalence

One of the major findings of Auction Theory is the celebrated **Revenue Equivalence Theorem**, which states, for private values, that any allocation mechanism/auction in which (i) the bidder with the highest type/signal/value always wins, (ii) the bidder with the lowest possible type/value/signal expects zero surplus, (iii) where all bidders are risk neutral and (iv) drawn from a strictly increasing and atomless distribution will lead to the same revenue for the seller (and player  $i$  of type  $v$  can expect the same surplus across auction types).

## Common Value Auctions

- I am auctioning off the right to sell refreshments during lecture. The value is £200.
- Al thinks it is worth £180
- Bob thinks it is worth £190
- Chris thinks it is worth £200
- Doug thinks it is worth £210
- Eric thinks it is worth £220
- What is the revenue in a 2<sup>nd</sup> price auction where everyone bids their estimated value? What is the average estimate? What should they do to their bids?

## Book pages auction

- **In our auction:** The object is fictitious and worth the number of pages in the book in pence. I will pay the difference between the value and the price:  $v-p$  (if price is above the number of pages, I will receive the difference between the price and value:  $p-v$ ).

## Book Pages Auction

- **First-Price Auction:** The person with the highest bid wins the object and pays what he bids.
- **Second-Price Auction:** The person with the highest bid wins and pays the **second highest** bid.
- **The Prize:** The number of pages in the book in pence. I will pay the difference between this value and the price offered:  $v-p$  (if price is above the value, I will receive the difference between the price and value:  $p-v$ ).

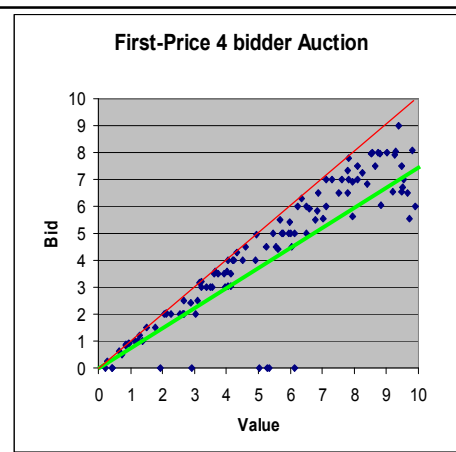
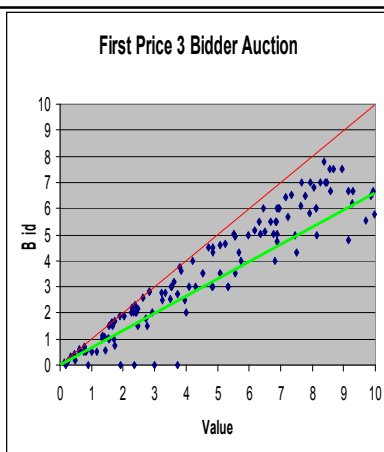
Name: \_\_\_\_\_  
 Estimate the value : \_\_\_\_\_ Pence  
 Bid in First-Price auction : \_\_\_\_\_ Pence  
 Bid in Second-Price auction : \_\_\_\_\_ Pence

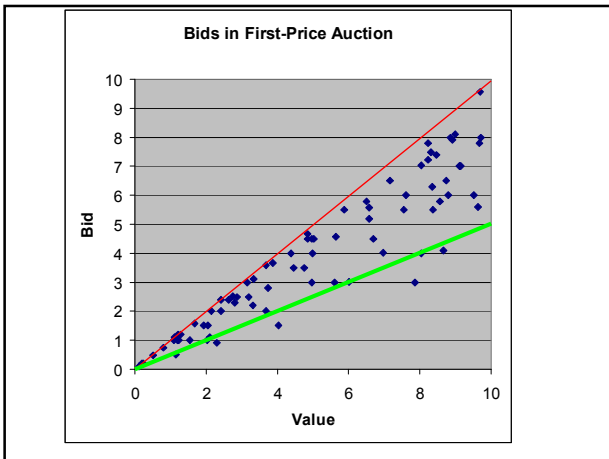
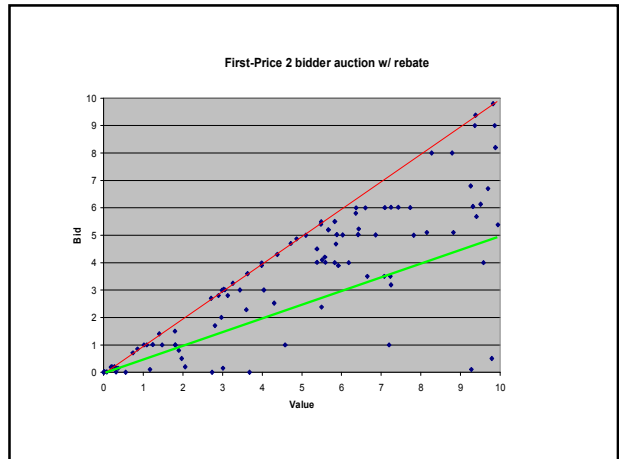
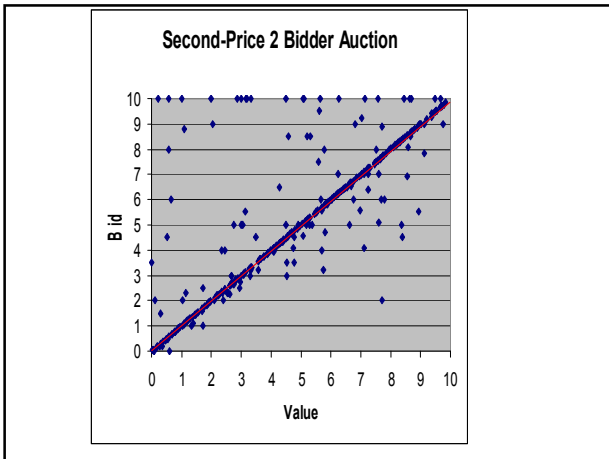
## Results

- Book had approx 450 pages
- Averages were
  - 609 estimate
  - 459 first price
  - 590 second price
- Maxima were
  - 1250 estimate
  - 845 first price
  - 2000 /1250 /900 second price

## Classroom Experiments

- We ran a number of designs in the lab.
  - First-Price with two buyers (random partners)
  - Second-Price with two buyers (random partners)
- The buyers' surplus is their combined profit.
- The seller's surplus is the sale revenue.
- Total surplus is buyers' + seller's.
- Efficiency is  $100 \times \text{total surplus} / \text{potential surplus}$ .





### Classroom Experiment Results

	Bidders' % of pie	Seller's % of pie	Efficiency
FP 2 bidders	23%	74%	97%
SP 2 bidders	40%	54%	94%
AP 2 bidders	10%	84%	95%
Theory 2 bidders	50%	50%	100%
FP 3 bidders	22%		
Theory 3 bidders	25%		
FP 4 bidders	13%		
Theory 4 bidders	20%		
FP rebate	96%	0%	96%
Theory rebate	100%		100%

- ### Classroom Experiment Results
- First Price is best for the seller.
  - Second-Price is best for the buyers.
  - First-Price is best for efficiency.

### Design may matter.

- The airwave 3G auctions \$34 billion in the UK using a design based on the English auction.
- A few months afterwards, mirroring their design Holland raised only \$2.5 billion. On a per capita basis less than 30% of the per capital of the British Auction.
- Why? Klemperer suggests that bids were costly and limited number of buyers.

### Careful design of rules

- In the FCC auction, bidders communicated which area they wanted by ending their bid with the area code. For example, \$1,000,818.
- In Australia auctioned off segments of airways. No punishment for defaulting after auction. Companies placed several bids and defaulted with the high ones. Went for 20<sup>th</sup> highest bid to the same two companies.

### Design Goals

- Efficiency - Want the bidder with the highest value to win.
- Revenue - May want to collect the highest revenue.
- Collusion, communication (FCC). Want to avoid.
- Entry - want to encourage.

### Some tools

- Entry Fee- hurts entry, efficiency, helps revenue.
- Minimum Bid - same.
- Bid Cap – hurts efficiency, helps entry.
- Release information about object. Helps with winner's curse which may help revenue.
- Release information about buyers. Tends to help collusion.
- Binmore: changing institutions changes behavior, non-economists often do not foresee this