Microeconomics, 2nd Edition

David Besanko and Ronald Braeutigam

Chapter 16: General Equilibrium Theory

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Outline

- Trade involves more than one market
- General equilibrium model replaced labour theory of value, Debreu: Theory of value
- Main results in general equilibrium theory:
  - Conditions for the existence of a competitive market equilibrium, i.e., a price system where all markets clear if all agents produce, sell and buy such that they maximize their utility or profits while taking all prices as given.
  - First welfare theorem: Market equilibria are Pareto efficient in the sense that now one can be made better off by a different plan of production and a different allocation of goods without making anybody else worse off.
  - Second welfare theorem: conversely, every Pareto optimum is a market equilibrium for a suitable system of prices and a well-chosen initial allocation of goods

- Assumptions needed for the theorems to hold:
  - Homo oeconomicus (individuals must be rational + selfish in the sense their utility only depends on their own consumption)
  - Survival assumptions. Roughly speaking If individuals do not own enough by themselves to survive, a market equilibrium might not exist.
  - Convexity assumptions
- First: Pure barter economy, two goods, two people (e.g. Robinson and Freitag)
  - Edgeworth box
  - Description of barter
  - Notion of Pareto-efficiency
  - The contract curve
  - Market equilibrium
  - First and second welfare theorem
- Then: Adding production

Efficiency in a Pure Exchange Economy

1. For simplicity there are only two individuals and two goods in the economy. Otherwise there could not be any exchange.

2. There is a fixed, total amount of each good in the economy. For now there is no production. Let w^1 be the total amount available of commodity one and let w^2 be the total amount available of commodity 2.

3. Once these goods are allocated to the two agents, property rights are assigned and we obtain a **barter** economy if people are allowed to trade freely (or if a black markets can form).

- In such a barter economy the agents can voluntarily trade given their initial allocations. Let the initial endowment (or allocation) of Ann (Bob) be w_A^1 (w_B^1) units of good 1 and w_A^2 (w_B^2) units of good 2. Let (x_A^1, x_A^2) and (x_B^1, x_B^2) denote the amounts Ann and Bob end up with.
- A feasible allocation is PARETO EFFICIENT if there is no trade from which the two can gain, more precisely, there is no other feasible allocation that makes at least one individual better off and no individual worth off.

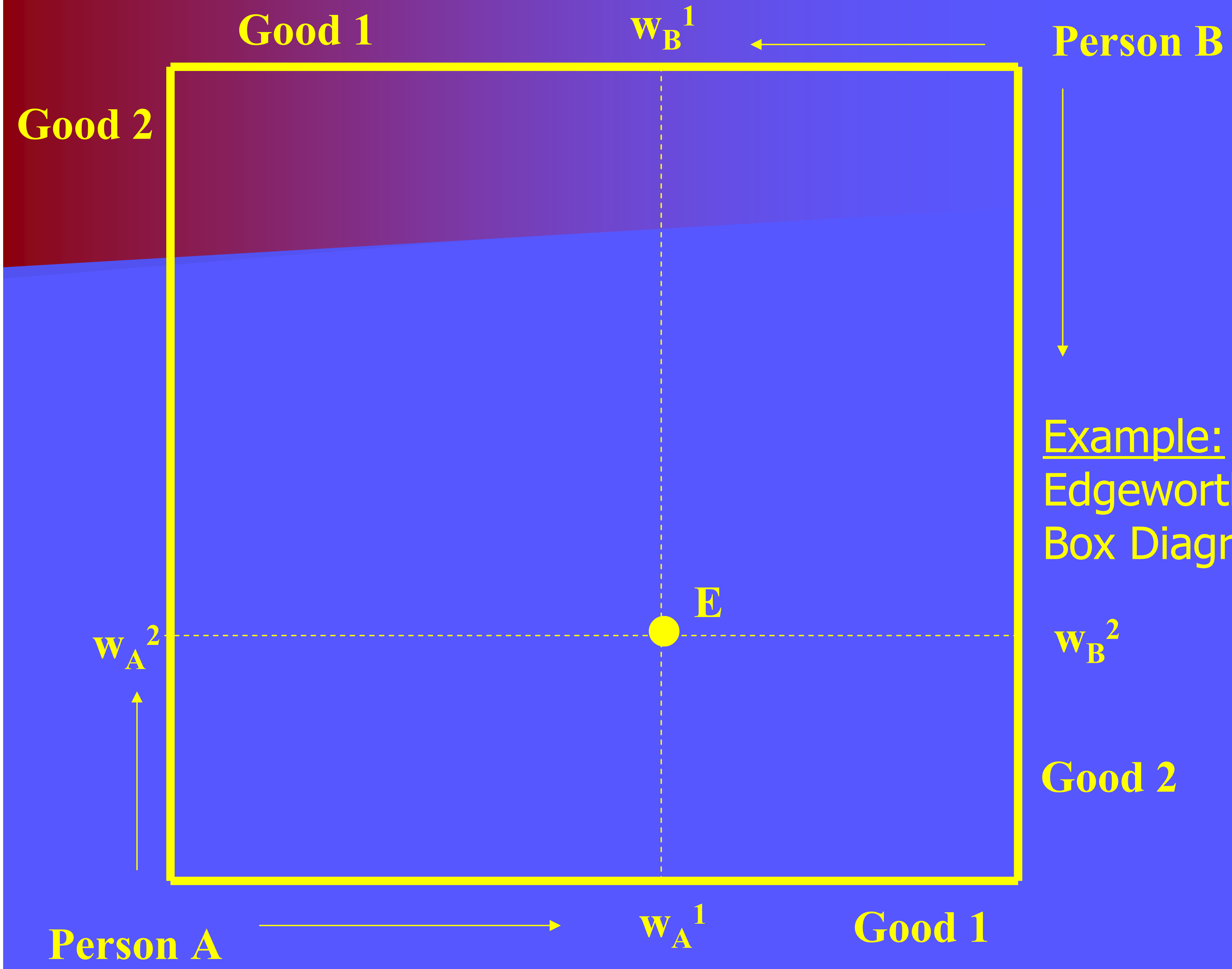
- *"Feasible" means hereby*

"final demand" ≤ "initial supply"

$$x_A^1+x_B^1 \leq w^1 = w_A^1 + w_B^1$$
$$x_A^2+x_B^2 \leq w^2 = w_A^2 + w_B^2$$

- *The notion of Pareto-efficiency is defined for any allocation of goods. Strictly speaking, it does not require markets (that just helps with the intuition) and certainly not prices. Superficial criticisms often ignore that. Agents are required to have clear preferences. They may have altruistic preferences, although this creates "externalities" and therefore the welfare theorems may not hold.*

- Assuming selfish preferences and that no commodities are thrown away, we can use the Edgeworth Box to see whether an allocation is efficient or not. We also assume the usual shape of the indifference curves.
- Any point in the Edgeworth box represents an allocation of the available goods where noting is thrown away, as for instance the initial allocation E in the following graph.



<u>Example:</u> An Edgeworth Box Diagram

In an Edgeworth Box…

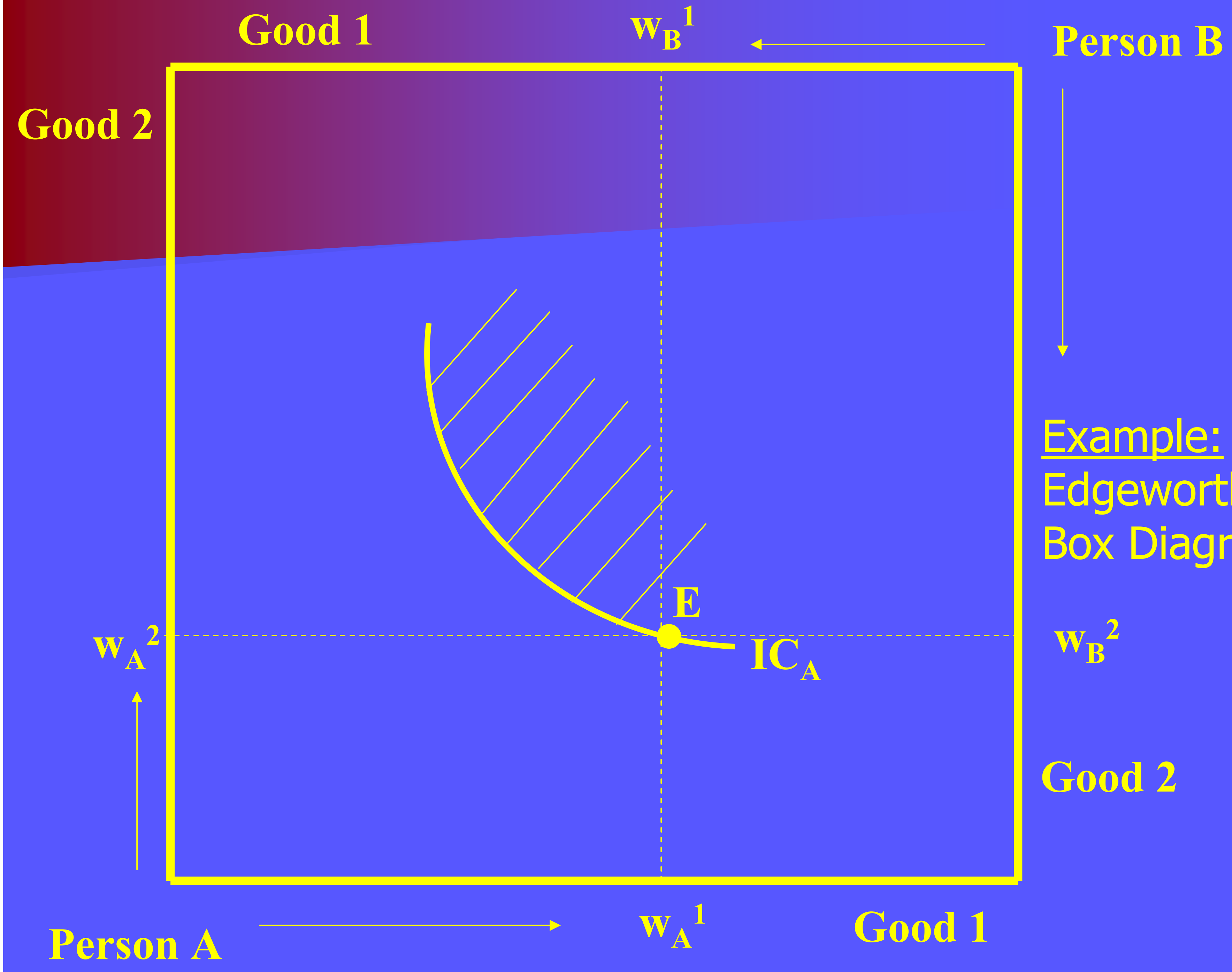
1. The length of the side of the box measures the total amount of the good available.

2. Person A's consumption choices are measured from the lower left hand corner, Person B's consumption choices are measured from the upper right hand corner.

3. We can represent an initial endowment, (w_A^1, w_A^2), (w_B^1, w_B^2) as a point in the box. This is the allocation that consumers have before any exchange occurs.

Ann’s preferred allocations

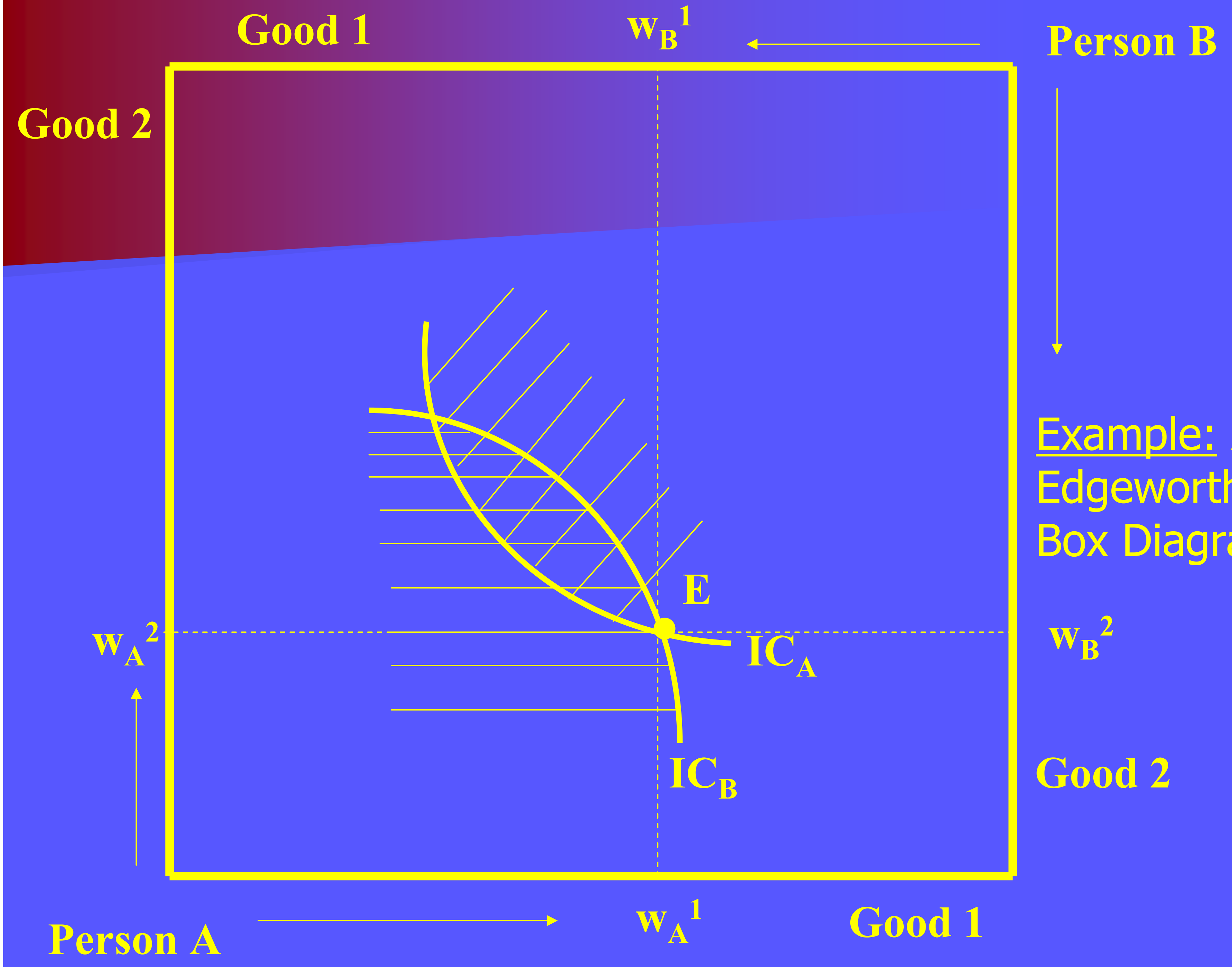
- Ann prefers any allocation that is on or above her indifference curve through E.



Example: An Edgeworth Box Diagram

Bob's preferred allocation

- For Bob things are reversed. The top right corner is the point where he gets nothing. Moving down and left. Bob prefers any allocation that is on or below his indifference curve through E.

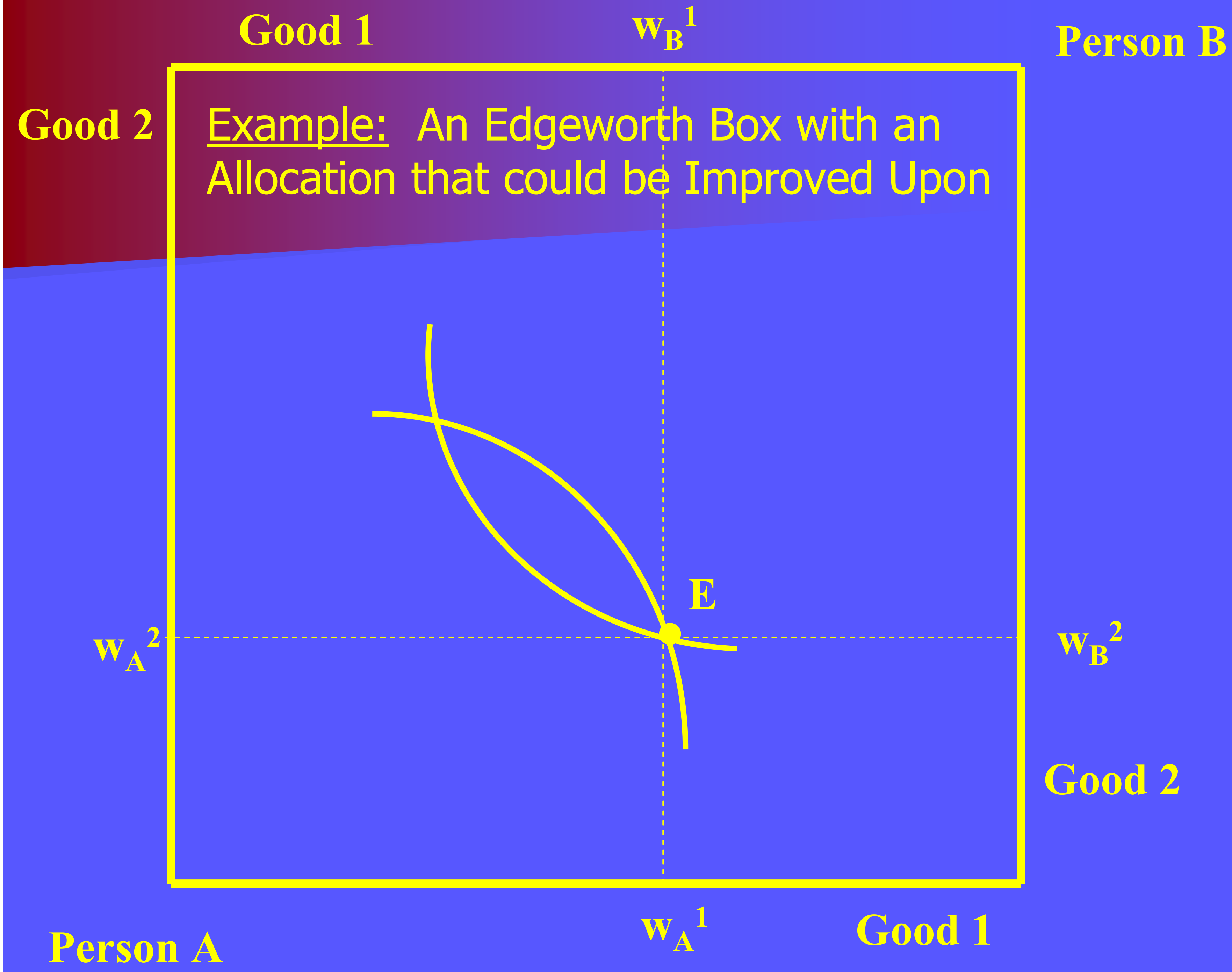


Example: An Edgeworth Box Diagram

Gains from trade

- The lens that is above Ann’s and below Bob’s indifference curve shows the feasible trades that would make both agents better off. In such a trade Ann would give away apples in exchange for oranges. The initial allocation is not Pareto-efficient.

Example: An Edgeworth Box with an Allocation that could be Improved Upon



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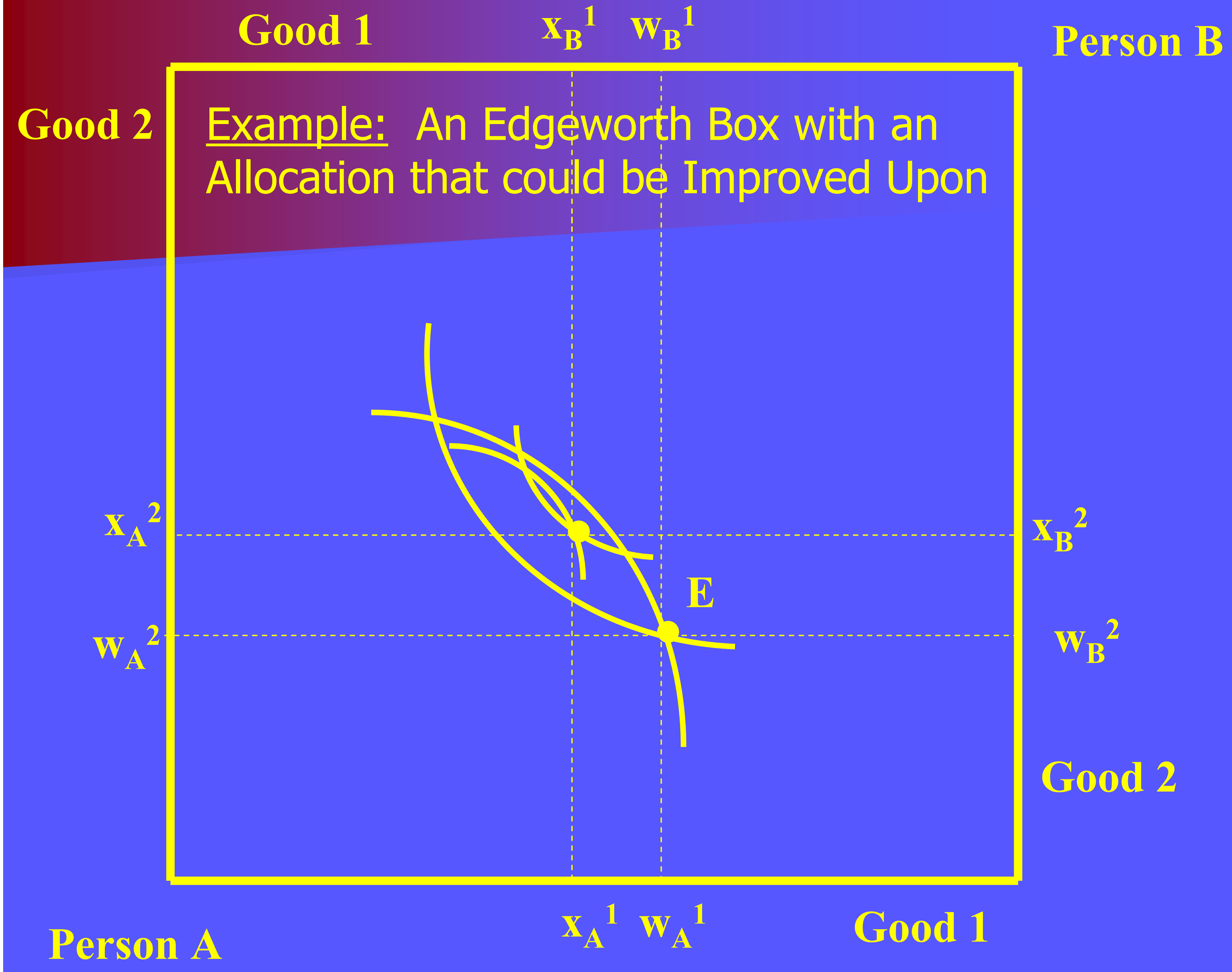
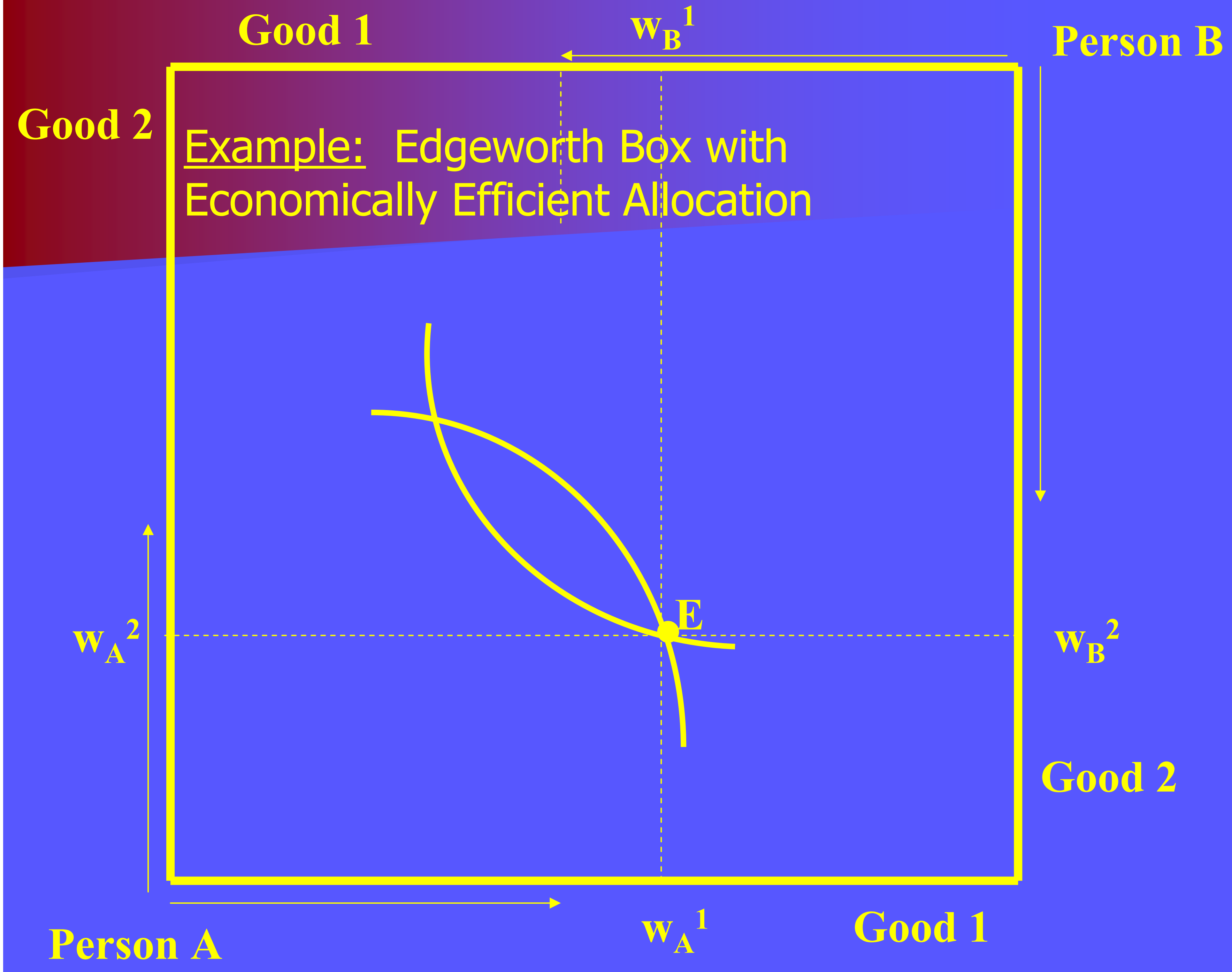


Example: Edgeworth Box with Economically Efficient Allocation



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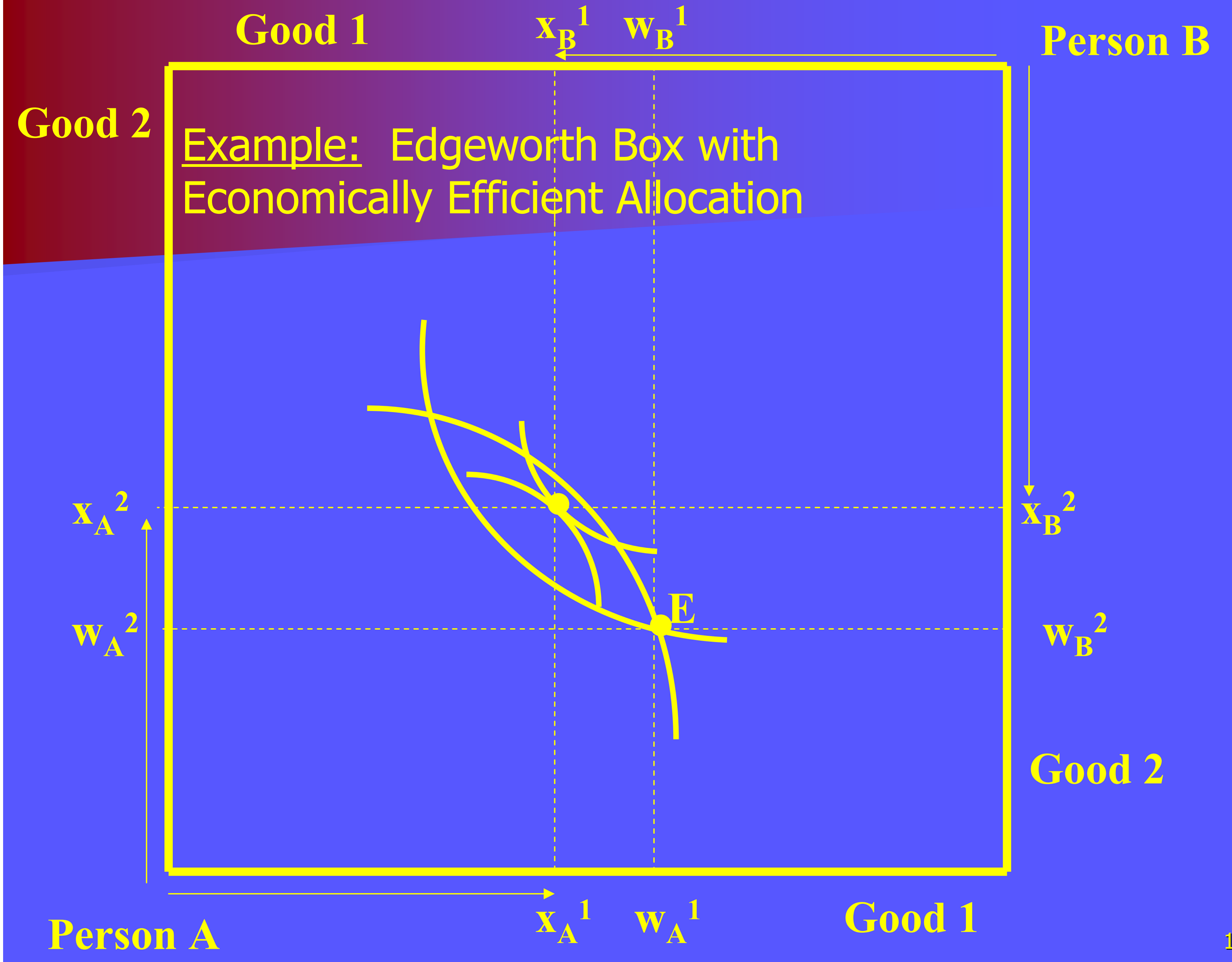

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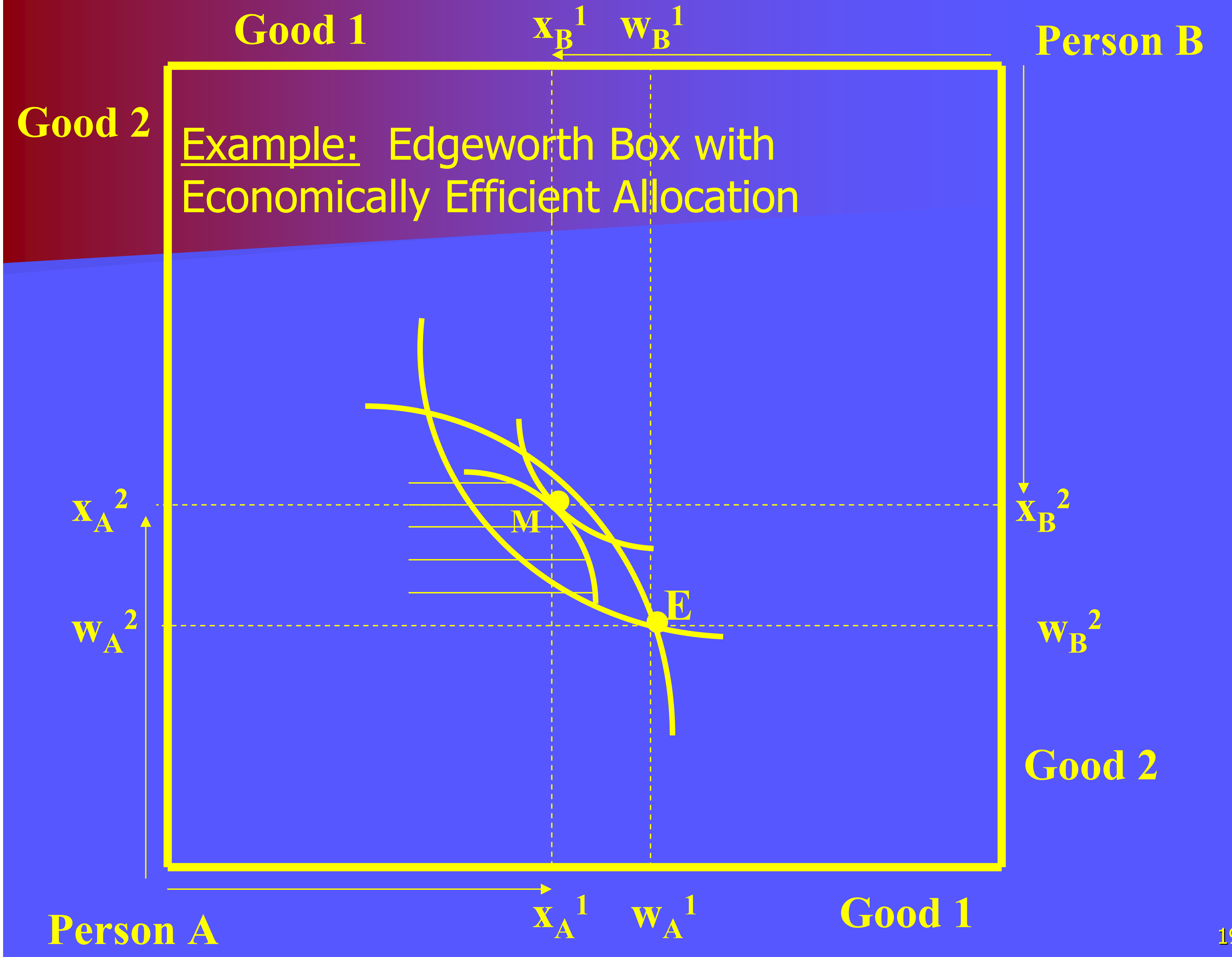

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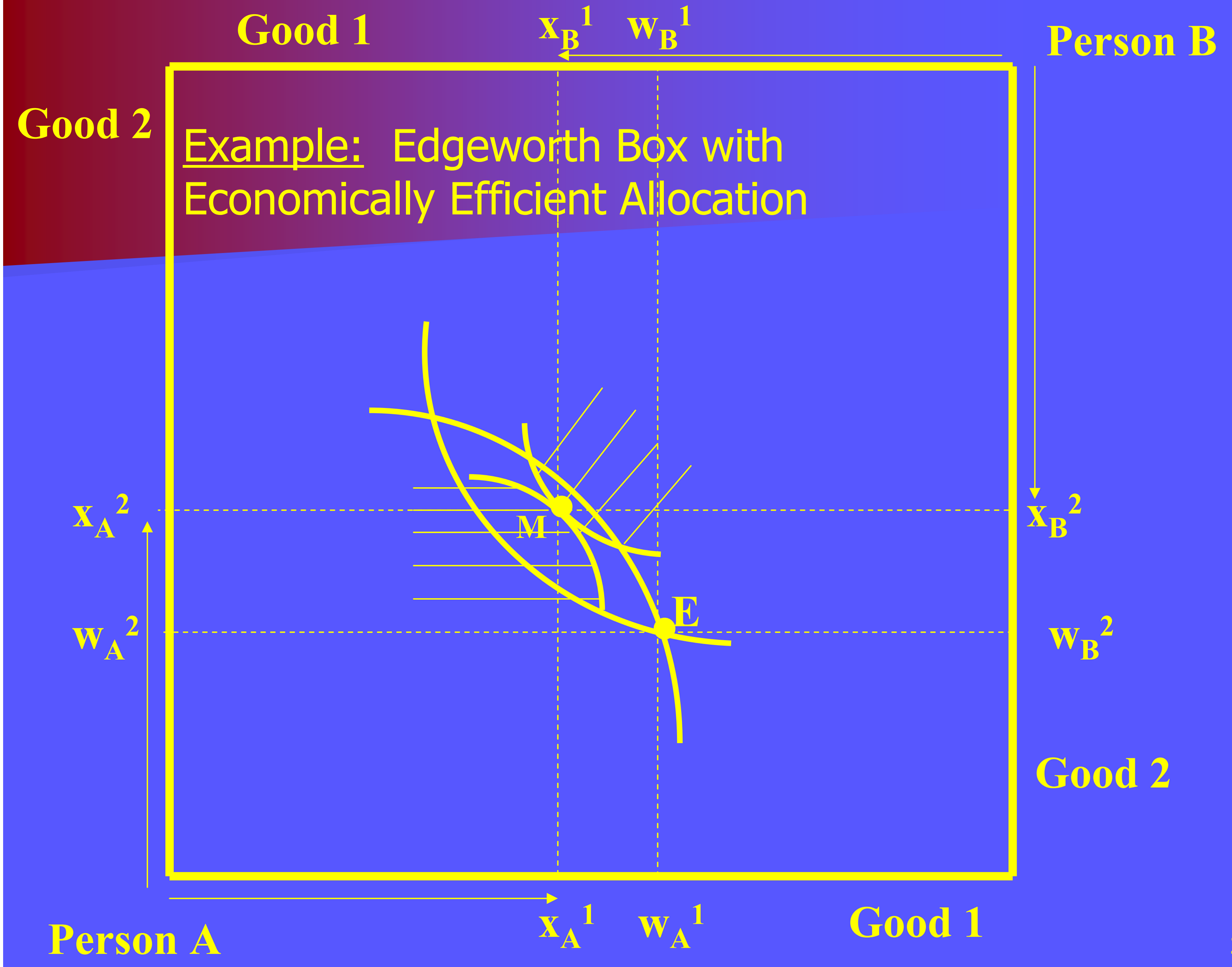


A Pareto-efficient allocation

- Repeat:
- Is a distribution of the commodities (i.e., an allocation) of the available commodities in such a way that is not possible to change the allocation such that at least one individual is made better off an no individual is made worth off.
- In other words, there is no reason to trade

1. M is Pareto efficient implies in the interior:

2. M is at a tangency point of the two individuals' indifference curves

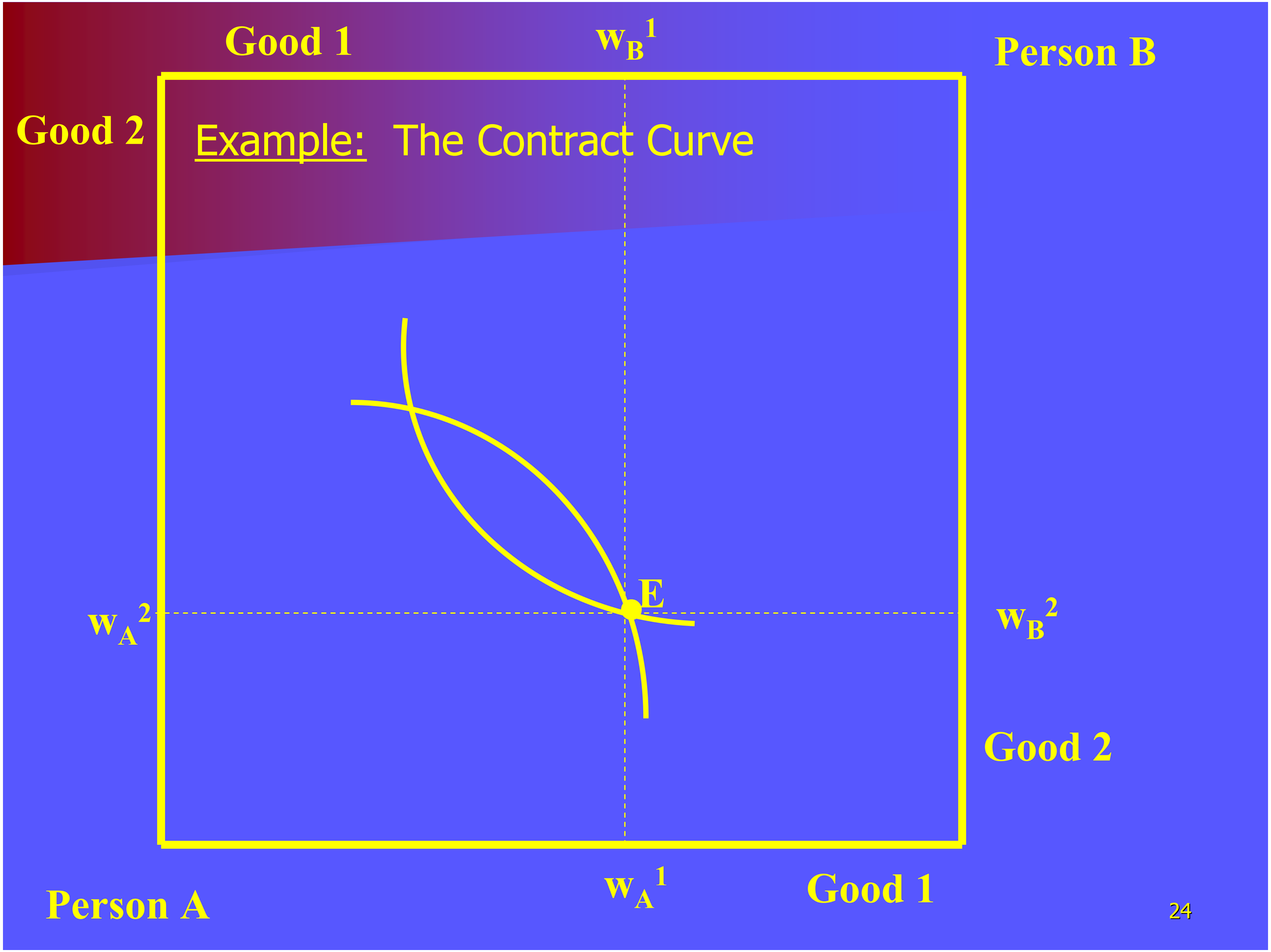
$$MRS^A_{1,2} = MRS^B_{1,2}$$

3. Definition: The set of all Pareto efficient points in the Edgeworth box is known as the **Pareto set** or the **Contract Curve**.

This set typically will stretch from one corner to the other of the box…(M not unique)

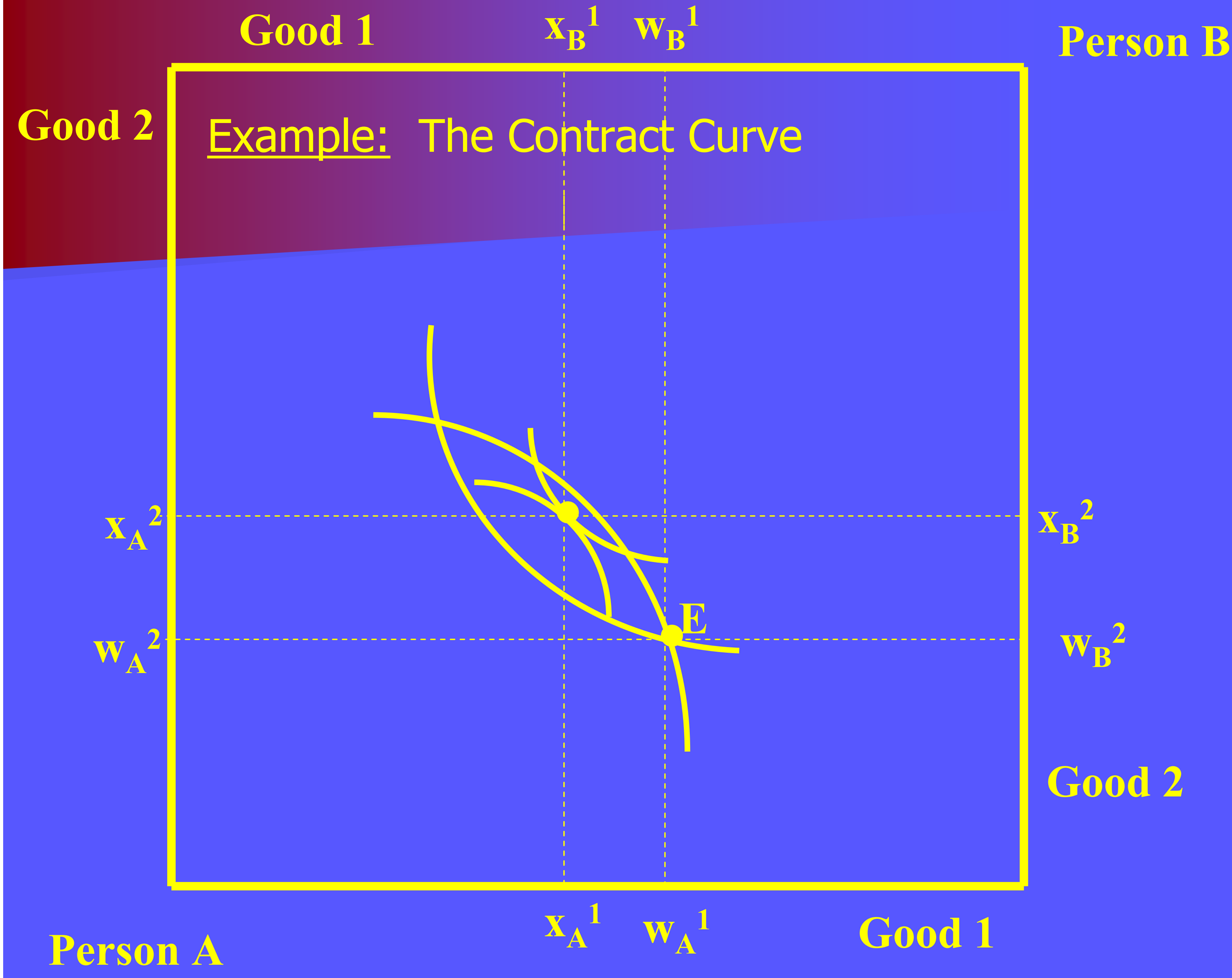
A subset of this set will contain the points that are Pareto efficient with respect to the initial endowment.

What is the set of Pareto efficient allocations if there is only one commodity?

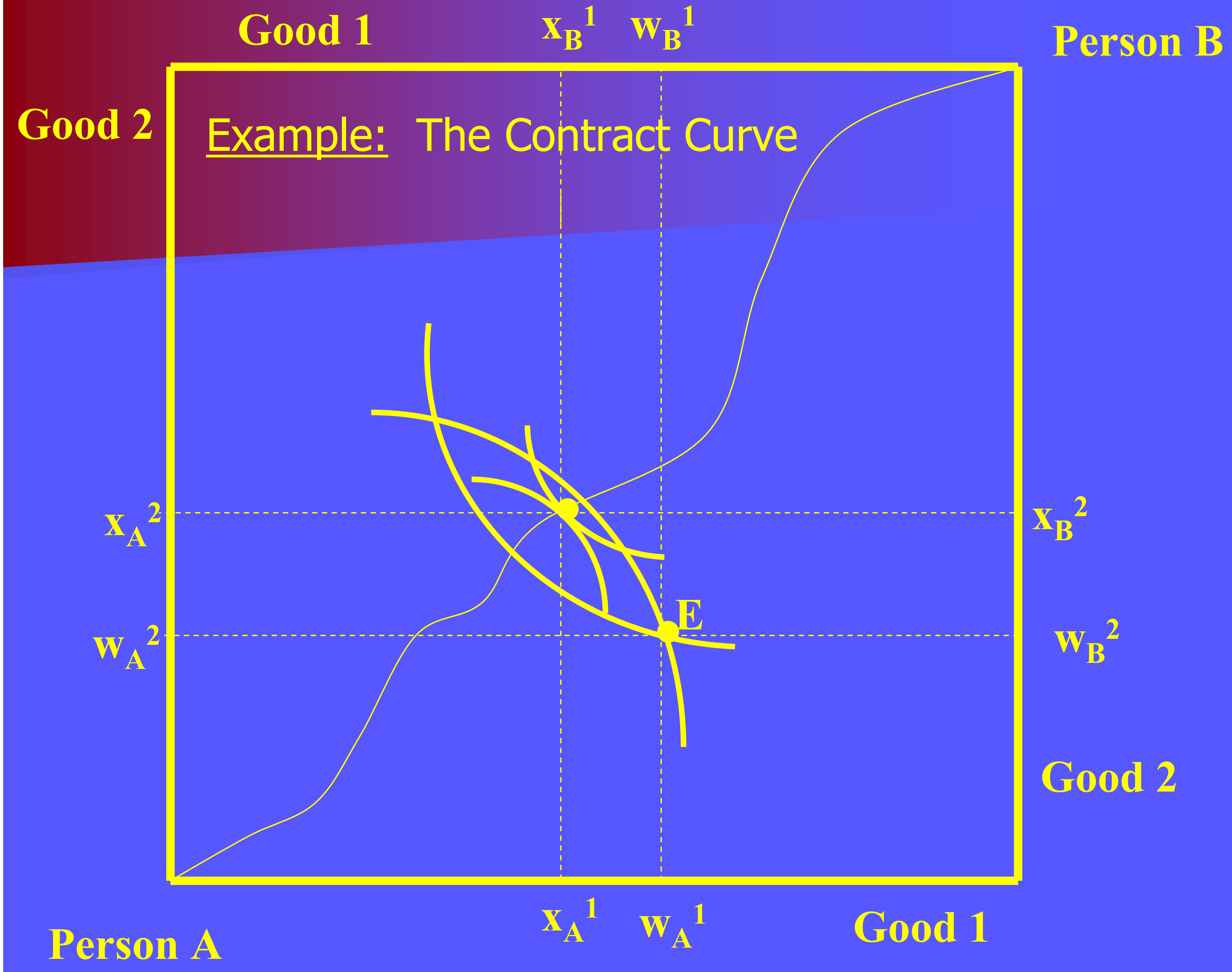


Good 1
w_B^1
Person B
Good 2
Example: The Contract Curve
E
w_A^2
w_B^2
Good 2
w_A^1
Good 1
Person A

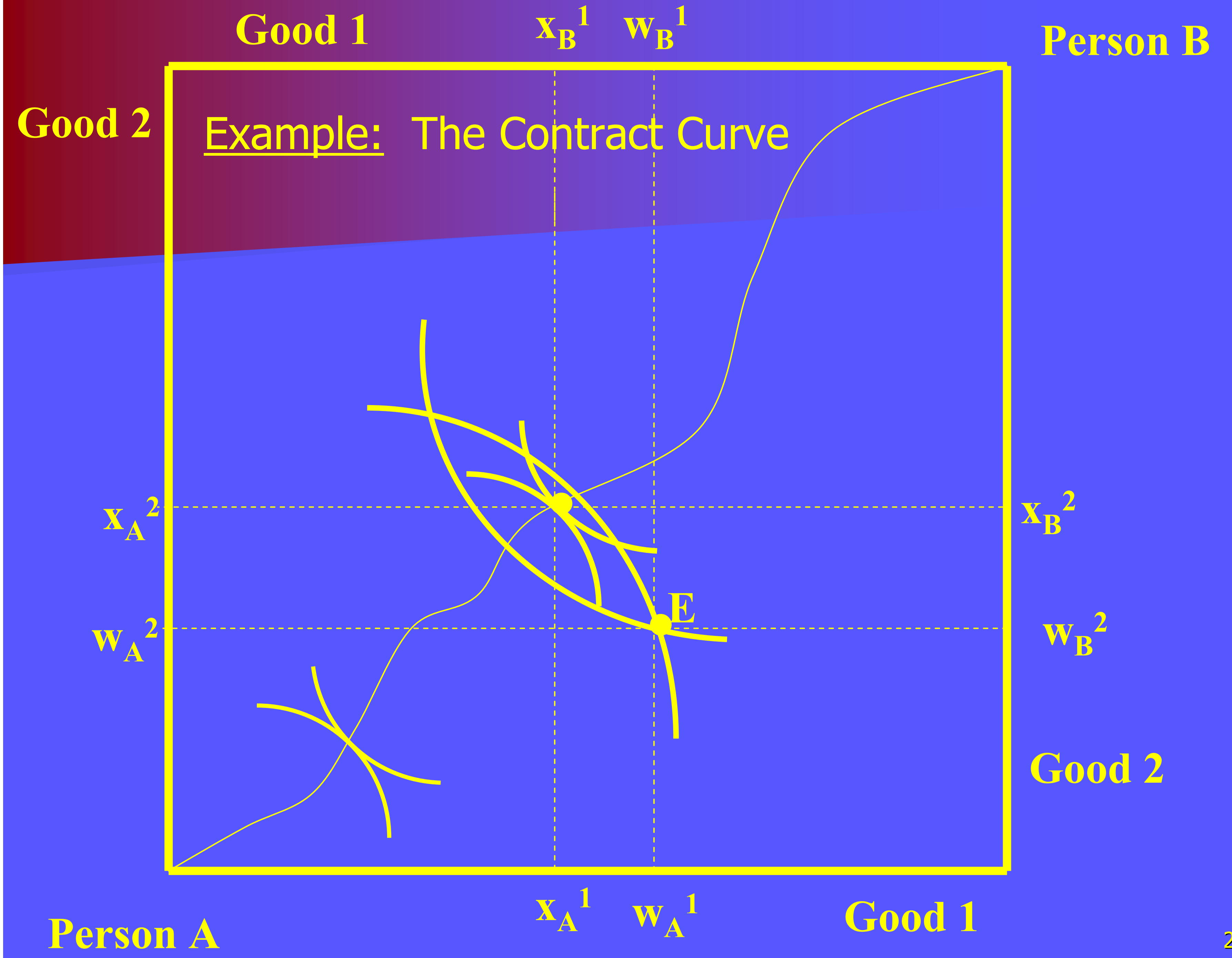
Example: The Contract Curve



Example: The Contract Curve



Example: The Contract Curve



Terminology and findings

- The contract curve consists of the set of all Pareto efficient allocations. At Pareto-efficient allocations inside the Edgeworth box the agents have identical marginal rates of substitution.
- An initial endowment E defines a "lens" of mutually beneficially trades. The intersection of this lens with the contract curve was called the CORE by Edgeworth. This is were efficient trade would end.

Example: Calculating a Contract Curve

2 individuals, A and B with "Cobb-Douglas" utility functions over 2 goods, X and Y.

$$U_A = (X_A)^{\alpha}(Y_A)^{1-\alpha}$$
$$U_B = (X_B)^{\beta}(Y_B)^{1-\beta}$$

$$MU_Y^A = (1-\alpha)X_A^{\alpha}Y^{-\alpha}$$
$$MU_X^B = \beta X_A^{\beta-1}Y^{1-\alpha\beta}$$
$$MU_Y^B = (1-\beta)X_A^{\beta}Y^{-\beta}$$

$$X_A + X_B = 100$$
$$Y_A + Y_B = 200$$

...This gives the size of the Edgeworth Box

Therefore,

$MRS_{X,Y}^{A} = MU_{X}^{A}/MU_{Y}^{A} = [\alpha/(1-\alpha)][Y_A/X_A]$
$MRS_{X,Y}^{B} = MU_{X}^{B}/MU_{Y}^{B} = [\beta/(1-\beta)][Y_B/X_B]$

And

$X_A = 100 - X_B$ *...feasibility constraints...*
$Y_A = 200 - Y_B$

$MRS_{X,Y}^A = MRS_{X,Y}^B$ *...tangency condition for contract curve...*

$[\alpha/(1-\alpha)][(200 - Y_B)/(100 - X_B)]$
$= [\beta/(1-\beta)][Y_B/X_B]$

or

$$(\beta-\alpha)Y_B X_B - (1-\alpha)\beta(100Y_B) + \alpha(1-\beta)200X_B = 0$$

or equivalently…

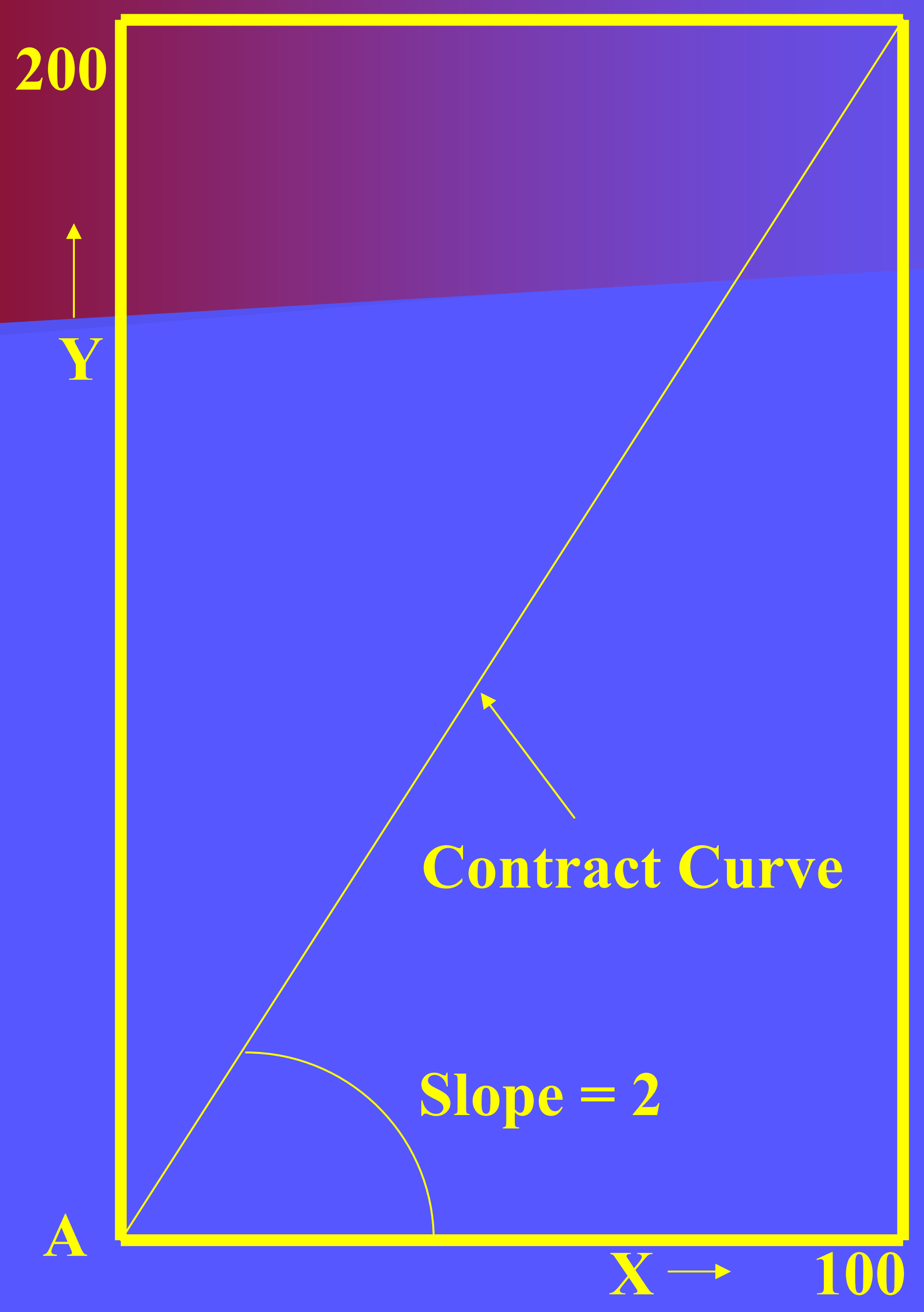
$$(\beta-\alpha)Y_A X_A + \alpha(1-\beta)(100Y_A) - (1-\alpha)\beta 200X_A = 0$$

B. *Draw the contract curve for* $\alpha = \beta = \frac{1}{2}$

The equations for the contract curves simplify to:

$Y_A = 2X_A$ and $Y_B = 2X_B$

Example: A Contract Curve



The market equilibrium

- Suppose both agents are price takers. (This is very artificial and would be satisfactory only for many agents.)
- Suppose there are prices p^1 and p^2.
- Buy selling his initial endowments each agent can guarantee the income

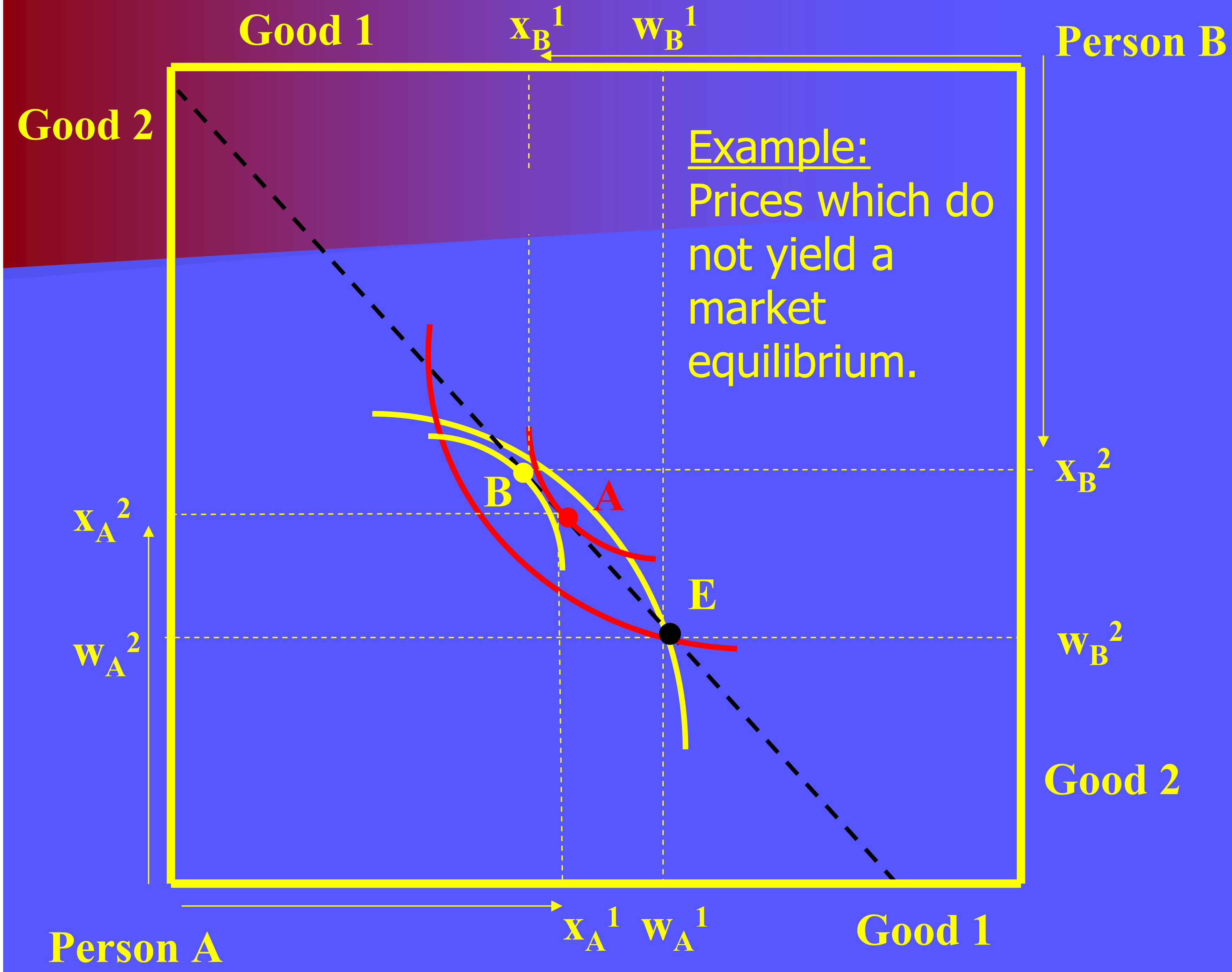
$$y_A = p^1 w_A^1 + p^2 w_A^2 \quad \text{or} \quad y_B = p^1 w_B^1 + p^2 w_B^2$$

- This money can then be used to buy the commodity bundle which maximizes utility.

- If markets then clear, we have a market equilibrium
- If Ann's optimal demand is (x_A^1, x_A^2) and (x_B^1, x_B^2) denote the commodity bundles that Ann and Bob will buy at the given prices. Then markets clear if the optimal demands define a feasible allocation:

"final demand" $\leq$ *"initial supply"*

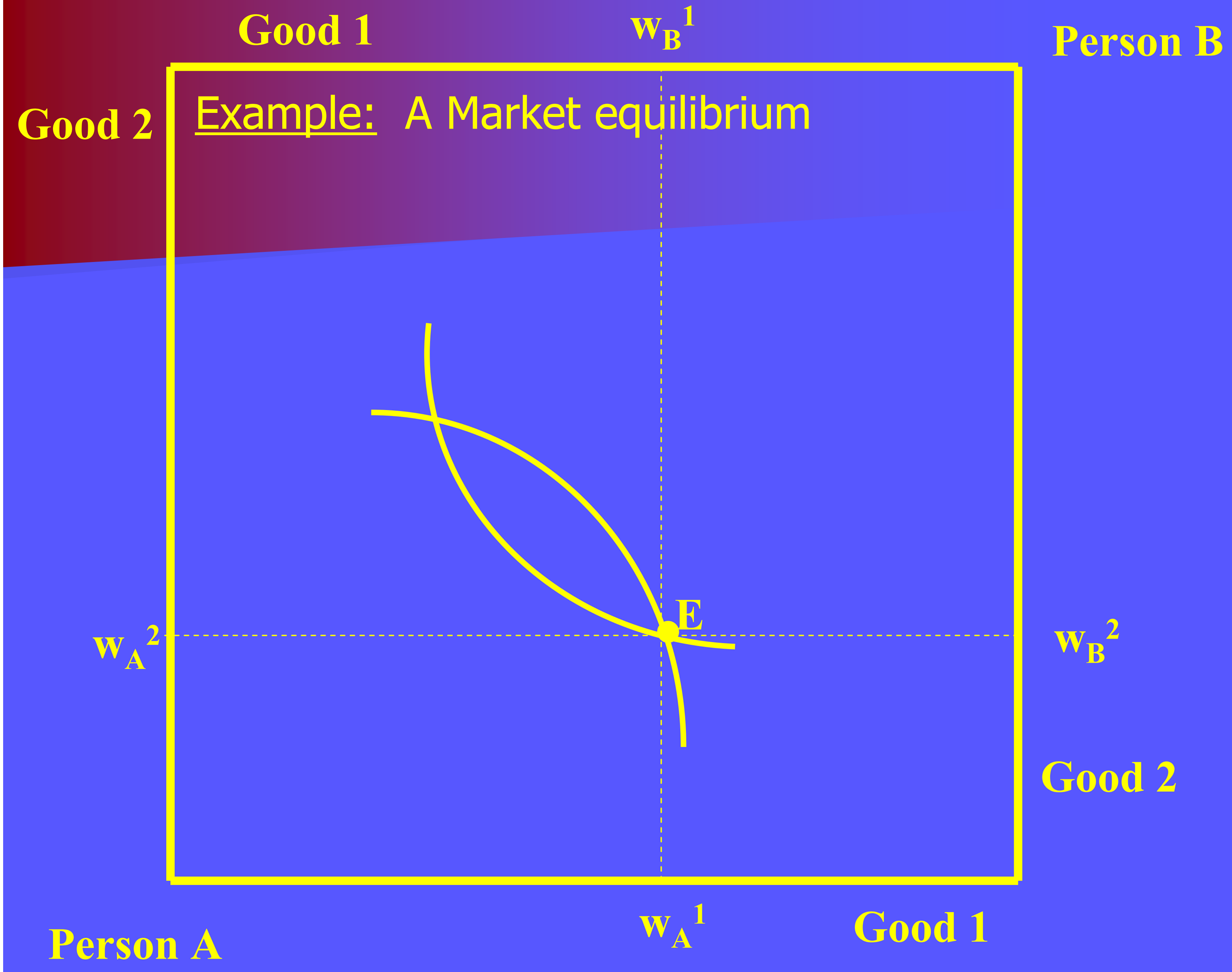
$$x_A^1 + x_B^1 \leq w^1 = w_A^1 + w_B^1$$
$$x_A^2 + x_B^2 \leq w^2 = w_A^2 + w_B^2$$



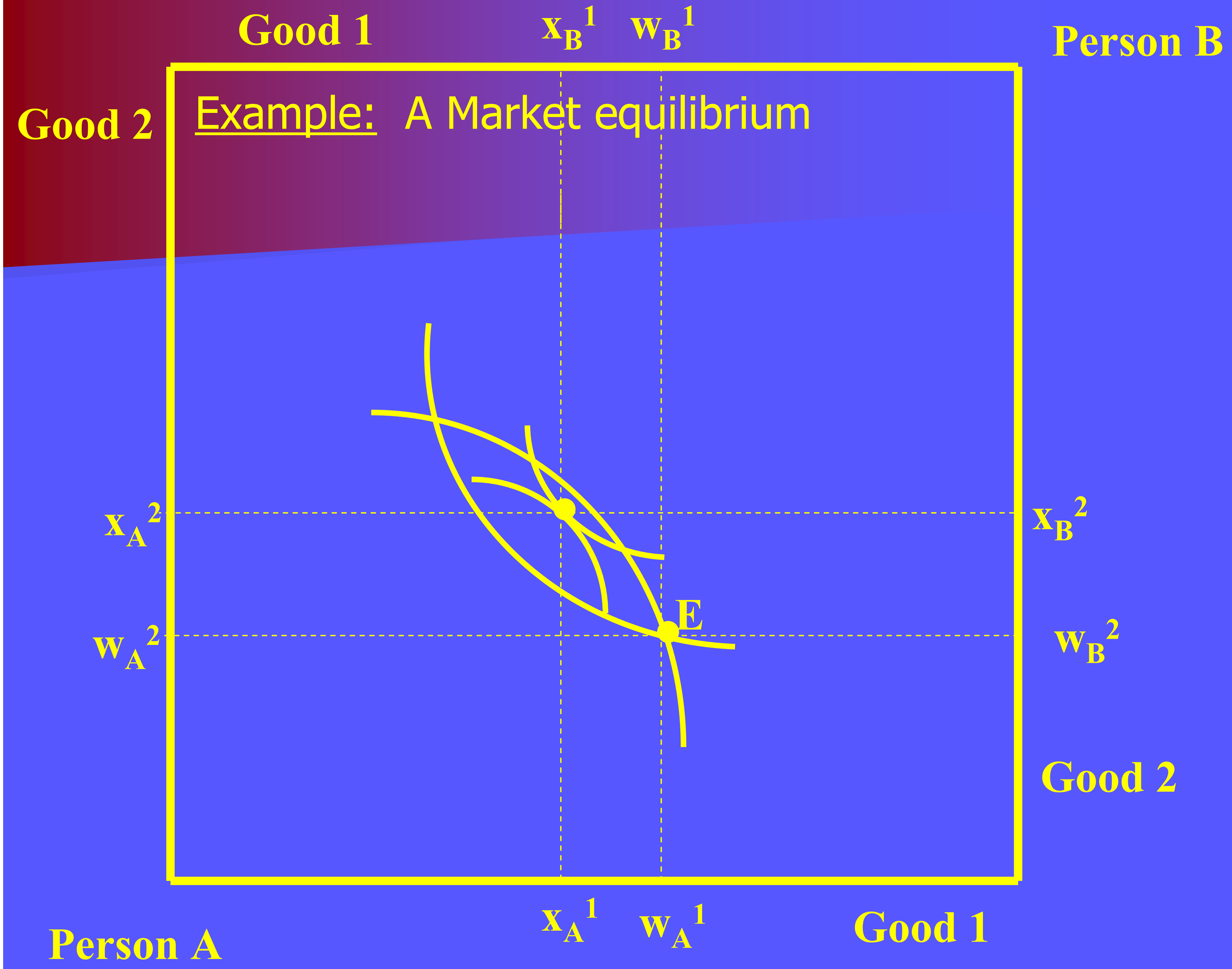
Example:

Prices which do not yield a market equilibrium.

Example: A Market equilibrium



Example: A Market equilibrium



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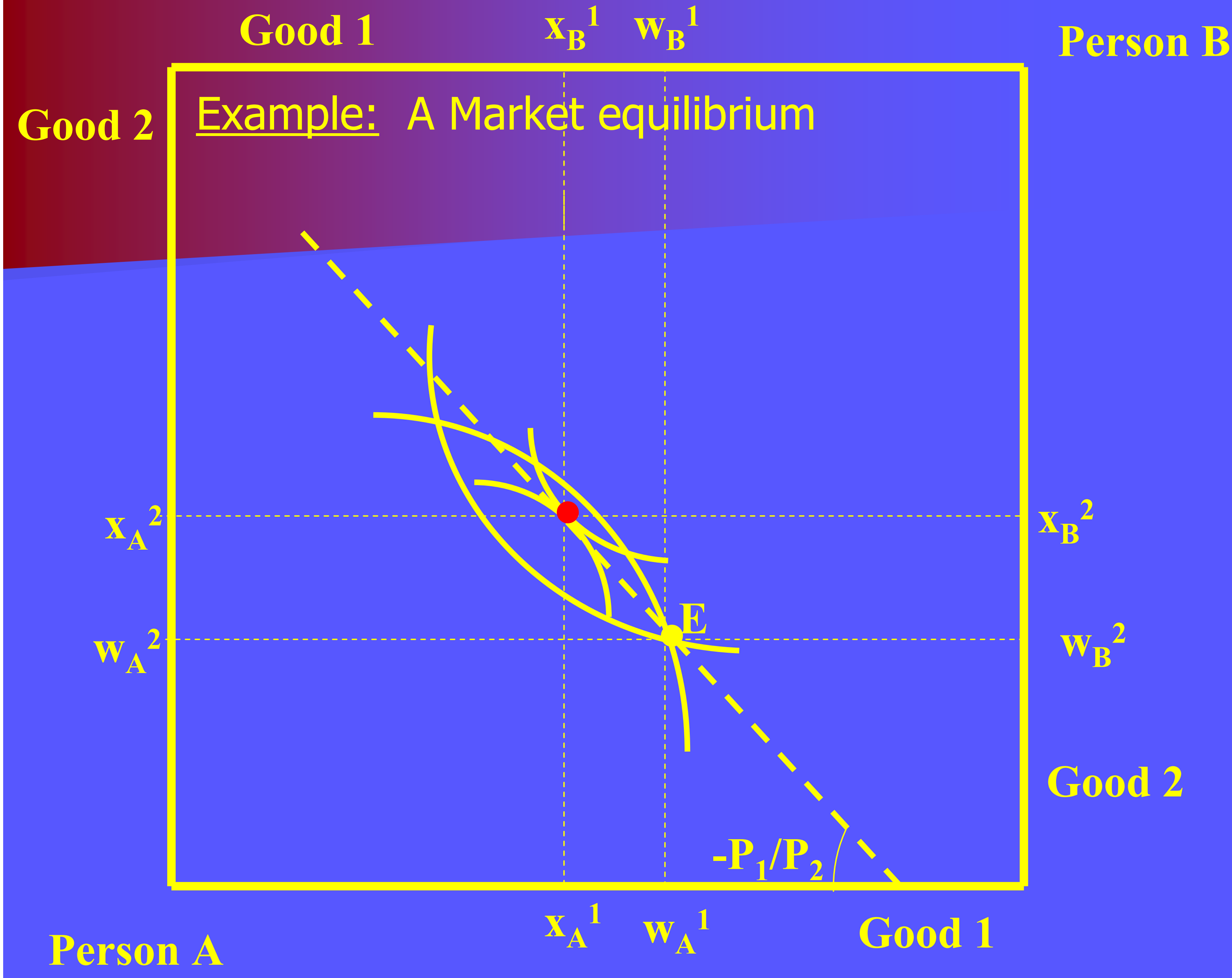

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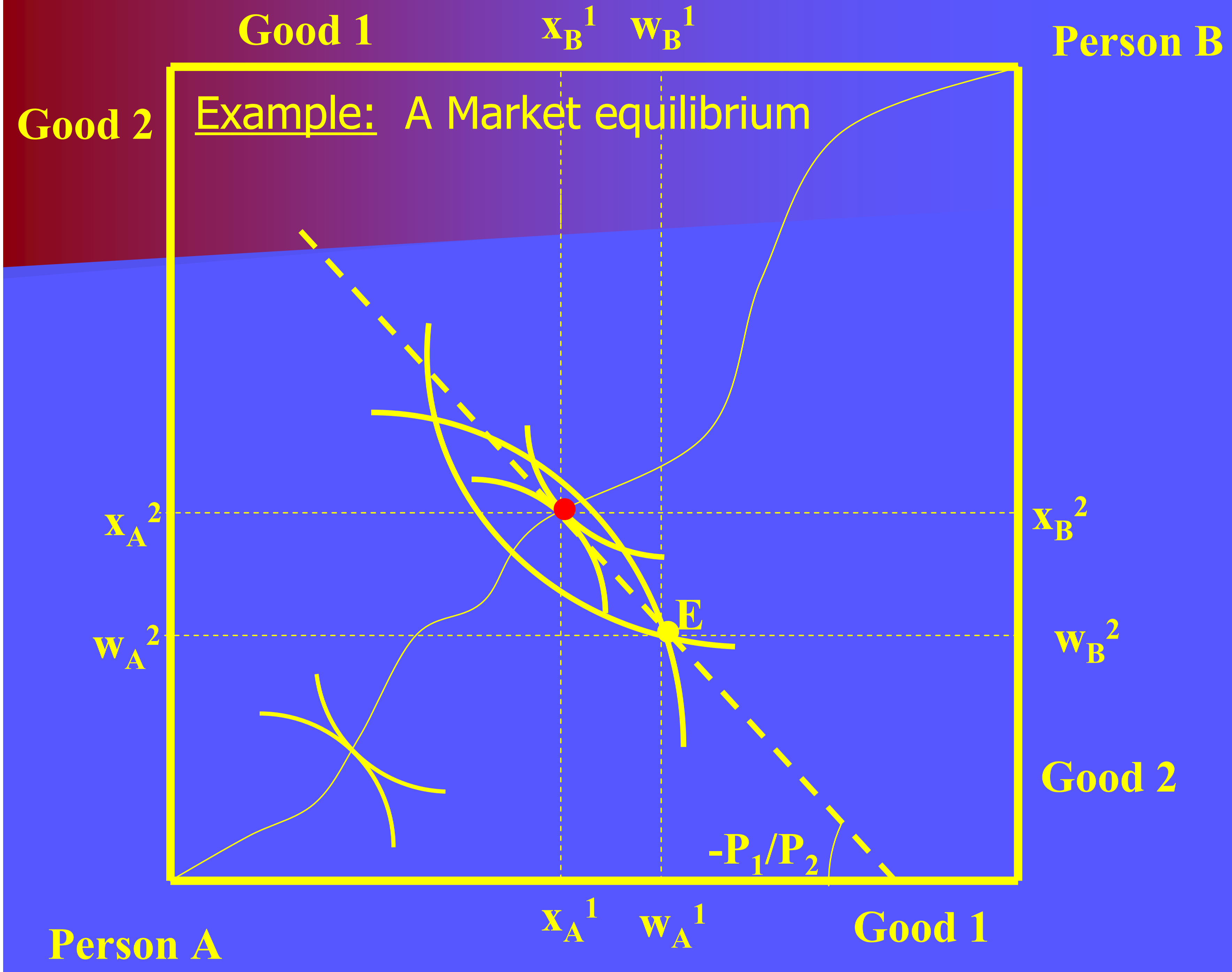


Market equilibrium

- We see: A market equilibrium must be a point in the core and hence Pareto-efficient. This means that the indifference curves of the two agents have the same tangent (i.e., both have the same MRS). Moreover, this tangent must go through the initial endowment and hence define the budget line for the agents. The latter implies that the price ratio equals the marginal rate of substitution in equilibrium.

- We have a system of two simultanoeus equations with two unknowns. The existence of a solution is not obvious.
- General conditions have been worked out by Arrow, Debreu and McKensey. Moreover, it follows:

The first welfare theorem

- Every market equilibrium is Pareto-efficient.

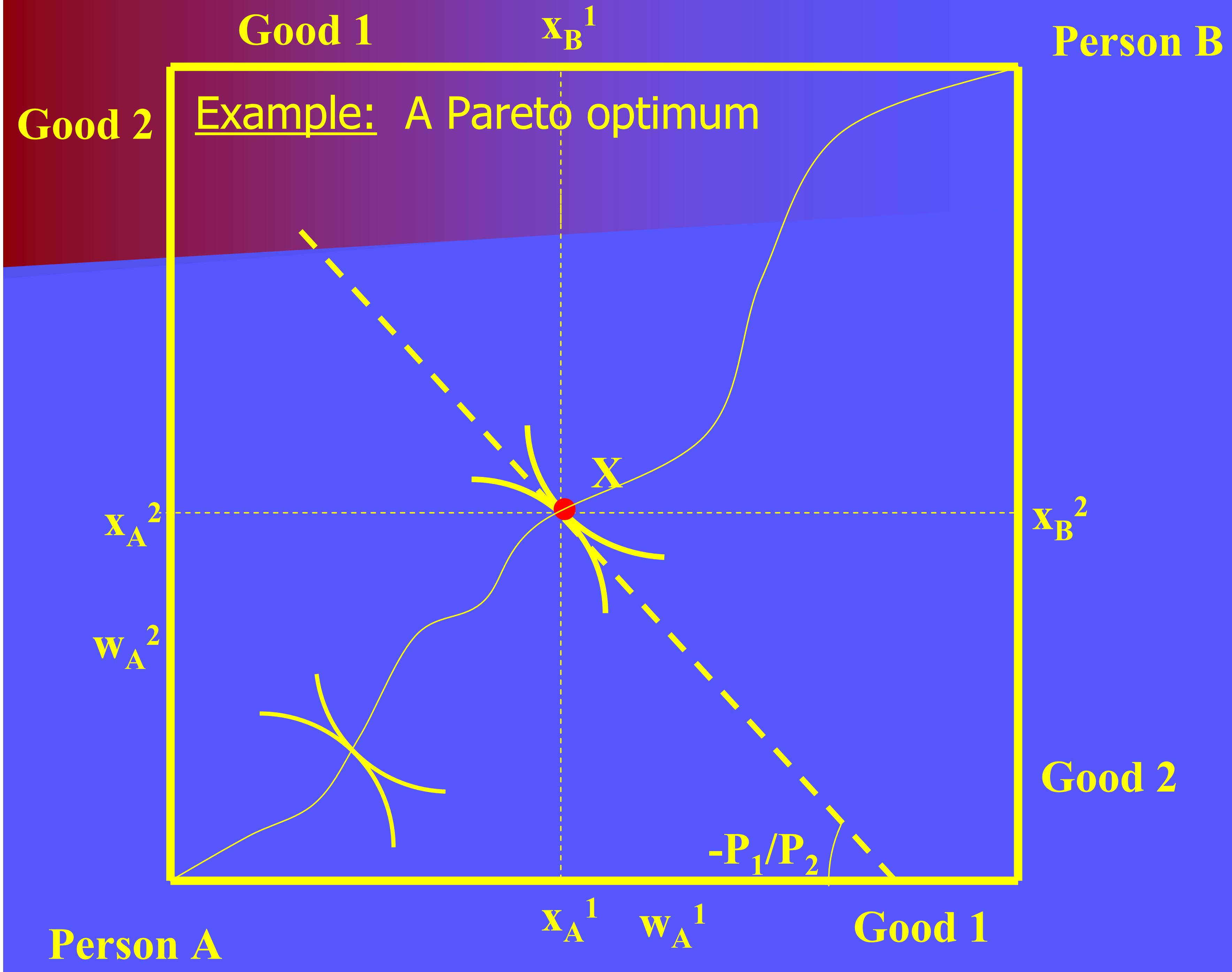
The second welfare theorem

- Every Pareto efficient allocation is a market equilibrium for a suitable initial allocation of commodities.

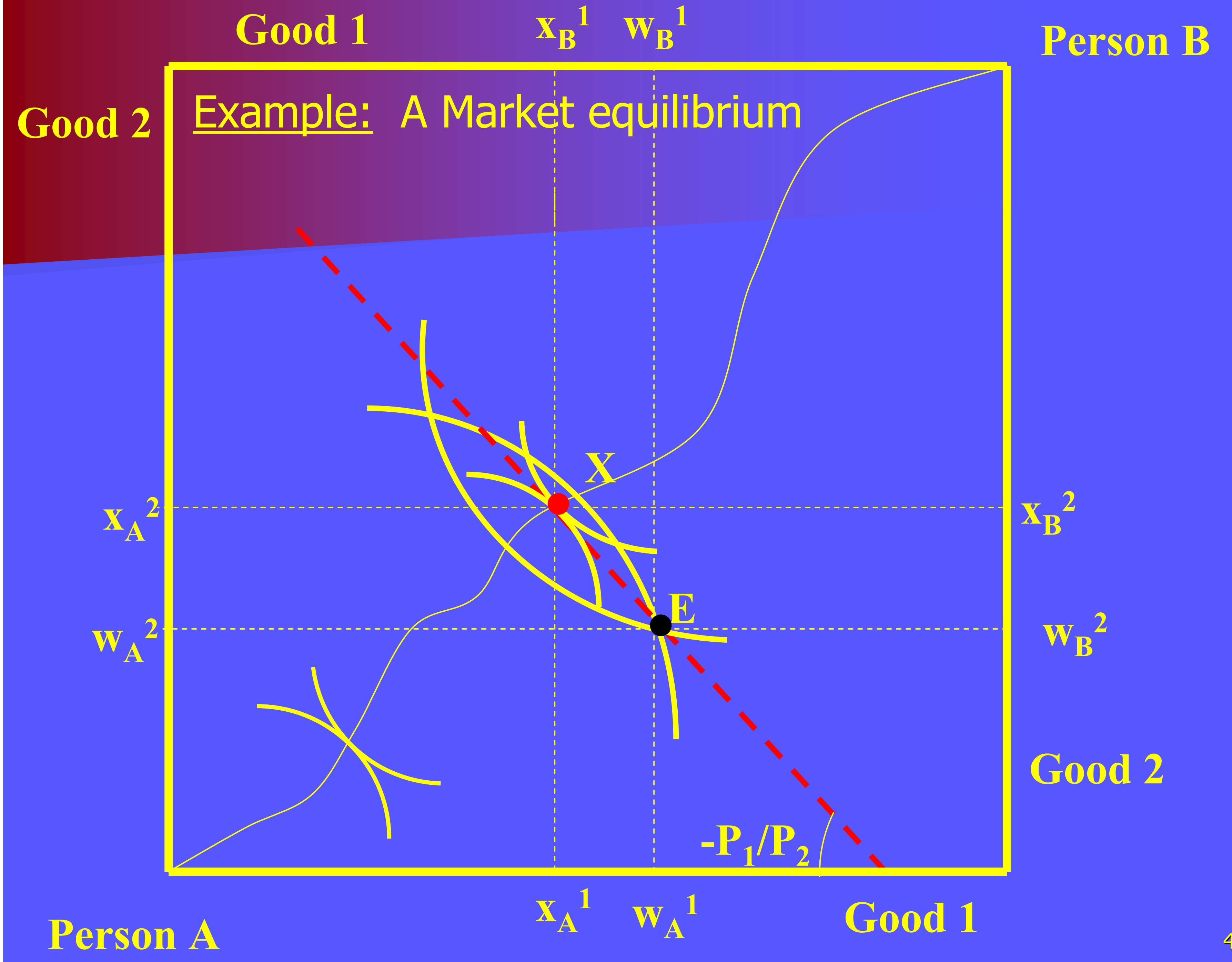
Take a point X on the contract curve. Set the relative price such that it is equal to the MRS of the two consumers. Choose as the initial endowment E a point on the tangent to the indifference curves. Relative to this initial endowment the prices and the point X form a market equilibrium.

Take X=E. Then no consumer wants to trade at these prices and a Market equilibrium is achieved. We obtain a no-trade equilibrium.

Example: A Pareto optimum



Example: A Market equilibrium



This means that society can achieve efficiency by allowing competition…

This equilibrium requires very little information (prices only) or co-ordination.

In fact, any Pareto-efficient equilibrium can be obtained by competition, given an appropriate endowment.

For example, any Pareto efficient allocation, x, can be obtained as a competitive equilibrium if the initial endowment is x.

This means that society can obtain a *particular* efficient allocation by appropriately redistributing endowments (income).

This can be achieved through taxes/subsidies to endowments (lump sum taxes) that do not affect choice (prices)

In fact, this redistribution could be viewed as the main role of government in the perfectly competitive model

Production

Suppose that all individuals in the economy have a dual role: they are consumers, but they also are the producers. In other words, the individual's role as a producer will determine their income…

Definition: The **production possibility frontier** (PPF) of an individual is the maximum combinations of goods A and B that can be produced with the individual’s input (e.g., labor) per unit of time.

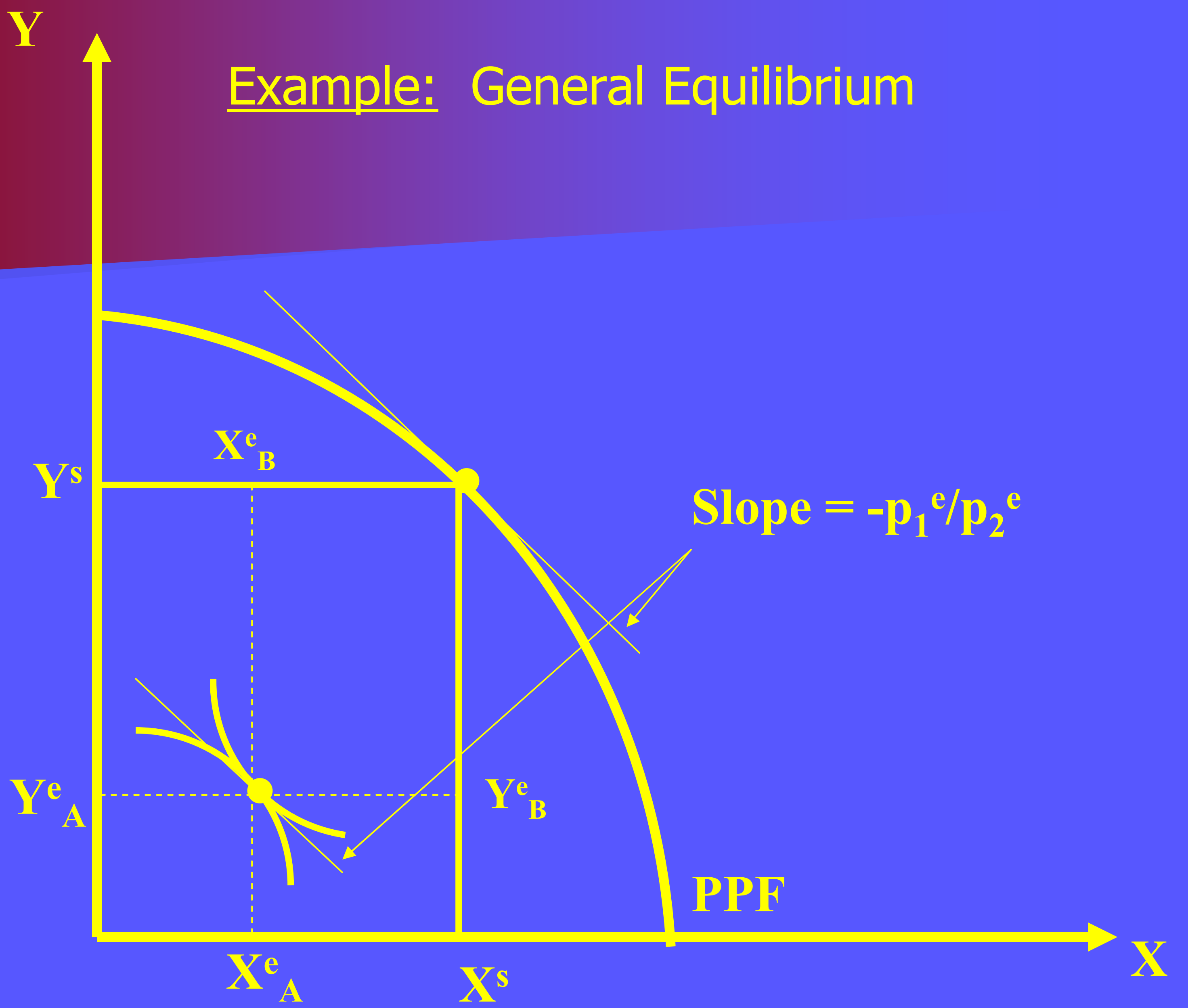
Definition: An individual achieves **efficiency in production** if s/he produces combinations of goods on the PPF (so that there is no "slacking off").

Definition: The slope of the production possibility frontier is the **marginal rate of transformation** (MRT).

The MRT tells us how much more of good Y can be produced if the production of good X is reduced by a small amount.

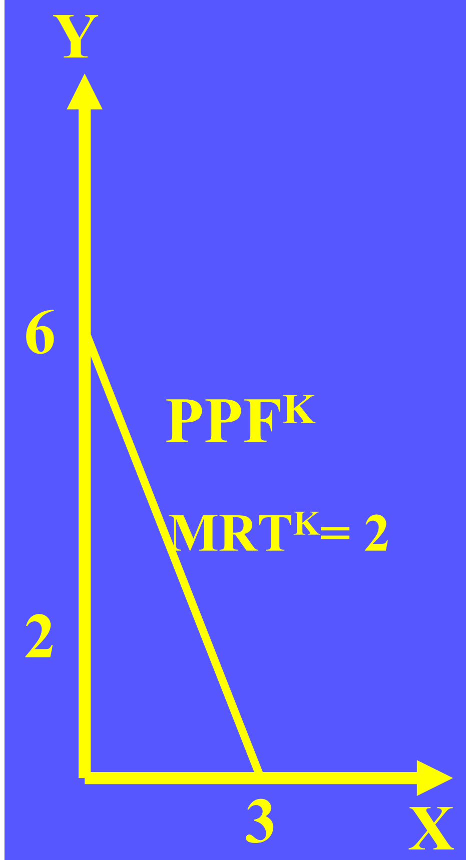
Or…the MRT tells us how much it costs to produce one good in terms of foregone production of the other good (opportunity cost).

Example: General Equilibrium



Example: The Production Possibility Frontier

Kate's production



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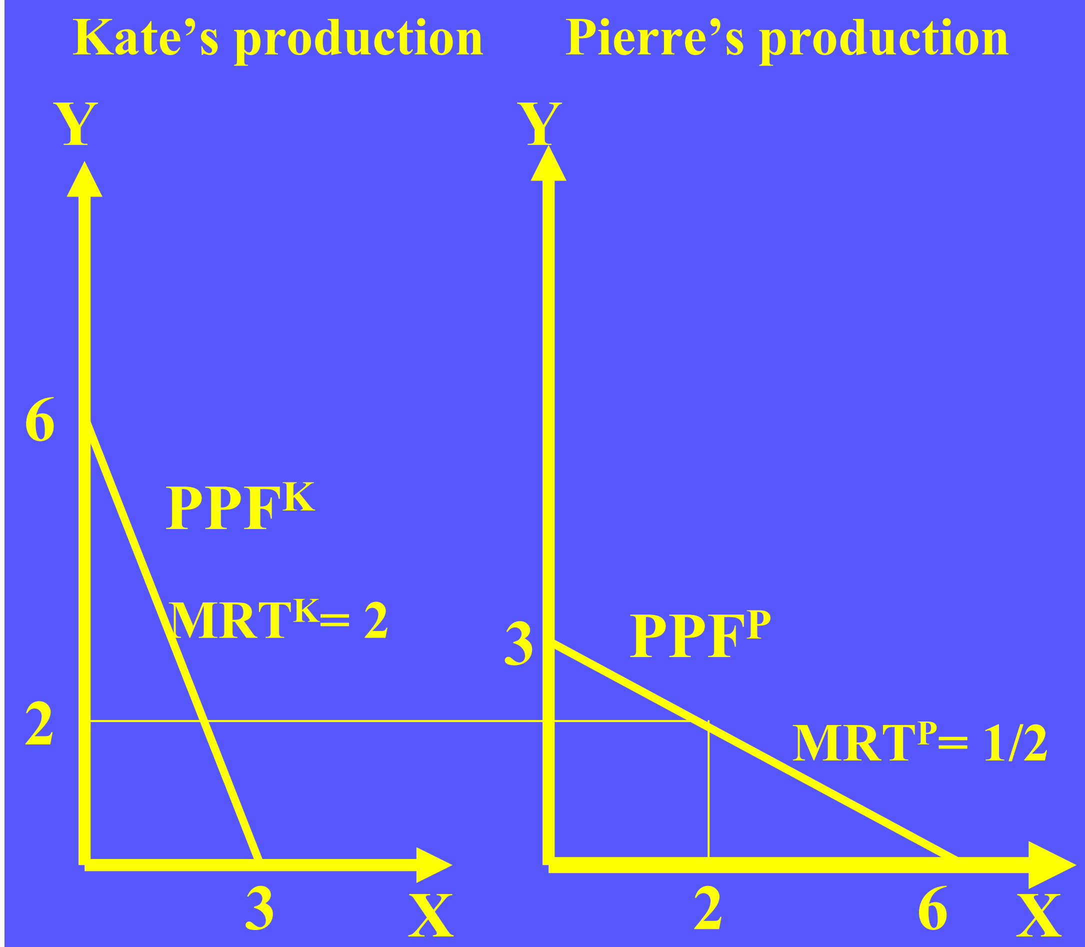
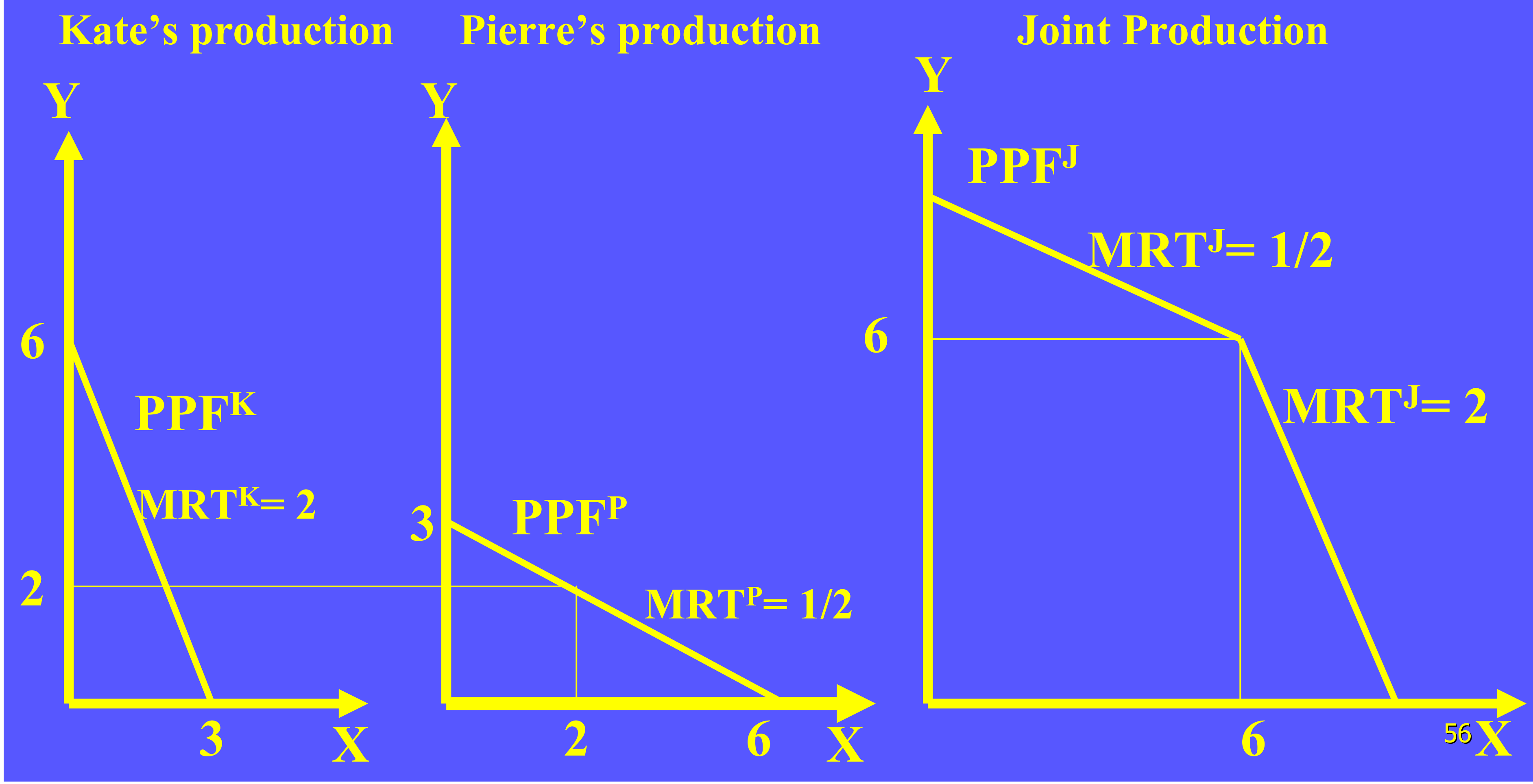


Example: The Production Possibility Frontier



Definition: The **joint PPF** for all possible technologies and all producers in the economy depicts the maximum amount of each good that could be produced in total by all producers.

Definition: A producer who, when producing one good, reduces production of a second good less compared to another producer is said to have a **comparative advantage** in producing the first good.

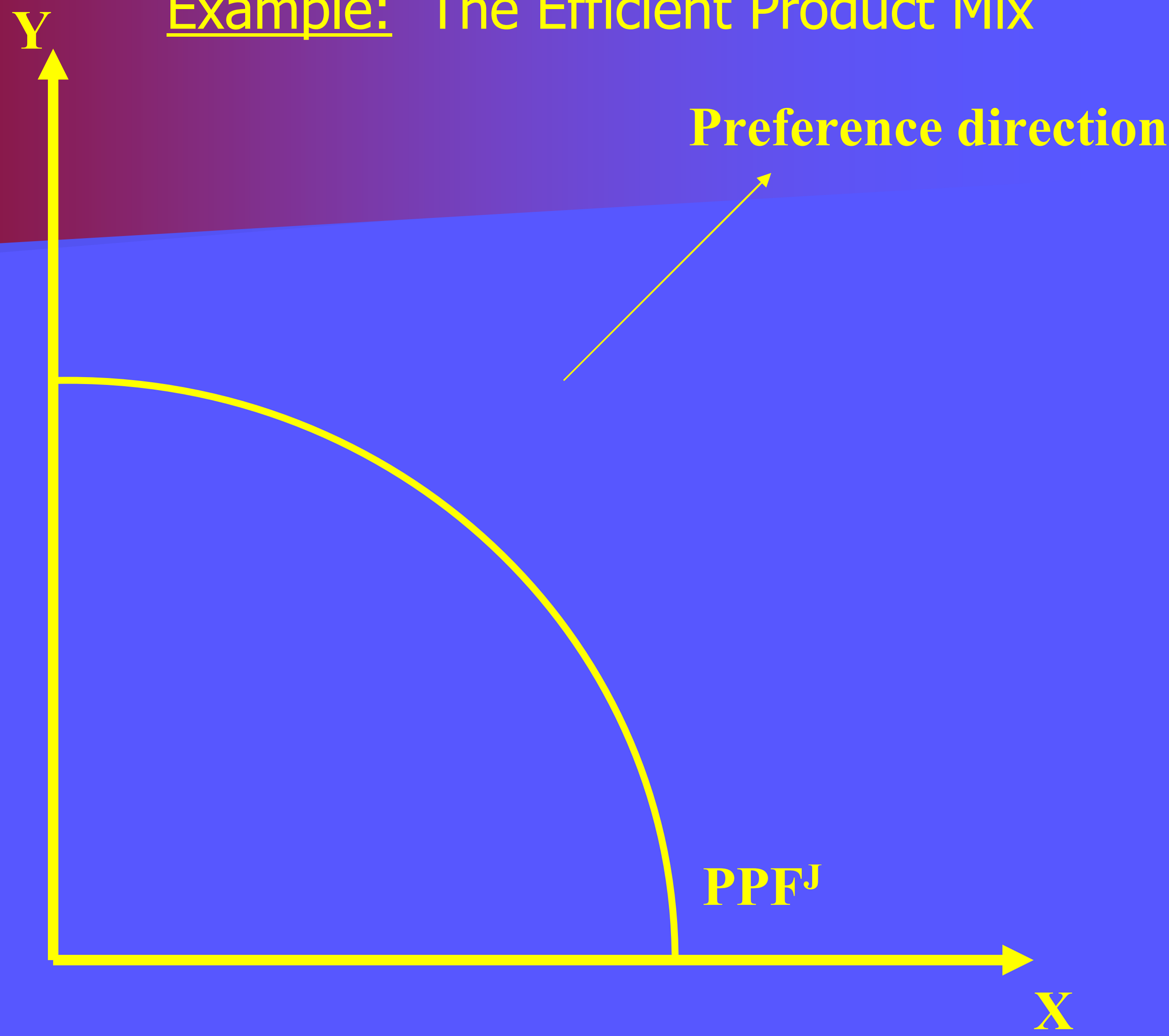
If the MRT of two different producers (and consumers) differs, then the individuals can potentially gain from trade

If many production methods are available, the joint PPF takes a typically "rounded" shape, representing the various MRT's available to the economy.

Now, let's look at the efficient product mix…At which point along the joint PPF would society operate?

Any *individual* consumer would prefer production to occur at a point where the consumer's indifference curve is just tangent to the PPF.

Example: The Efficient Product Mix



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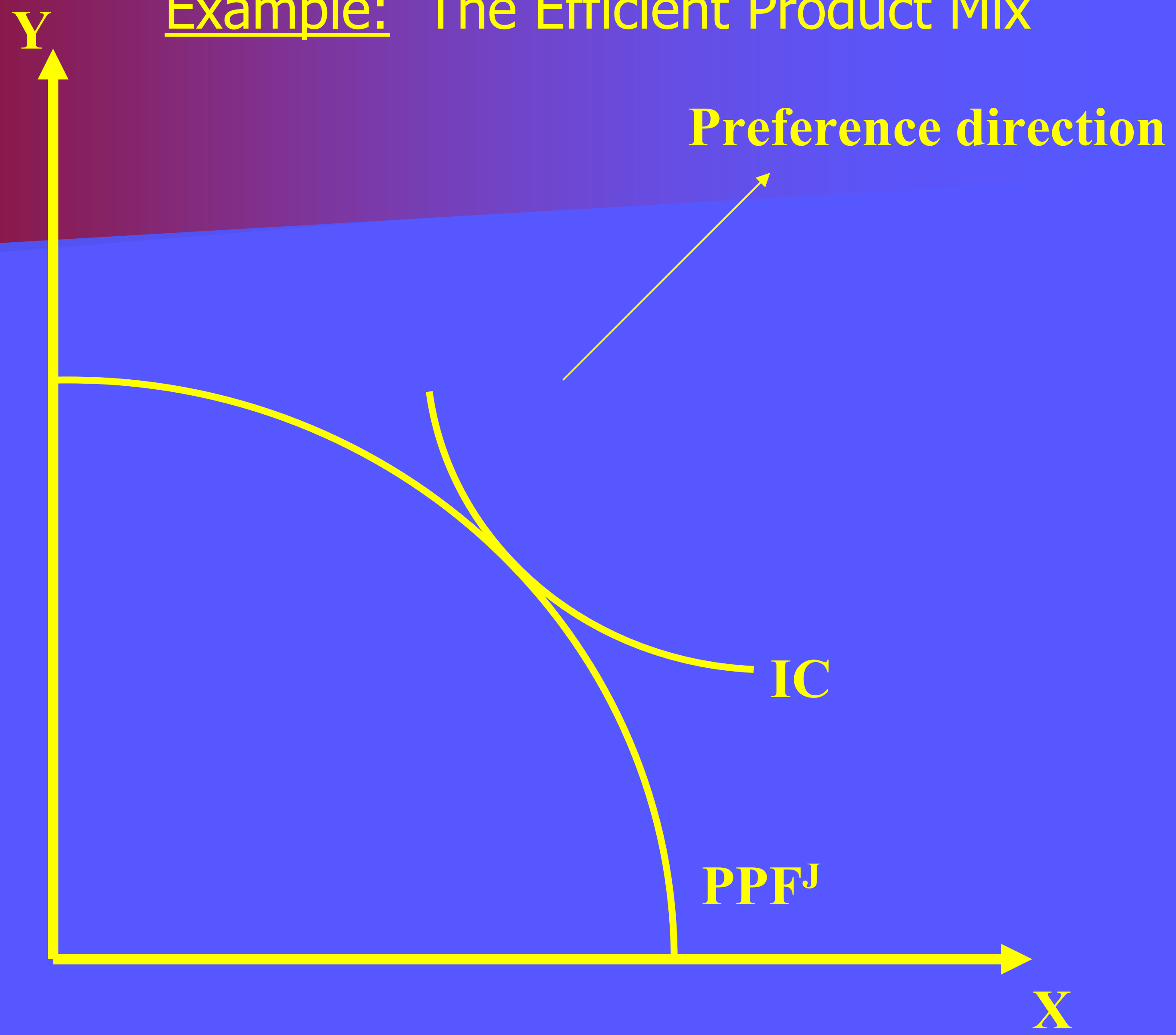

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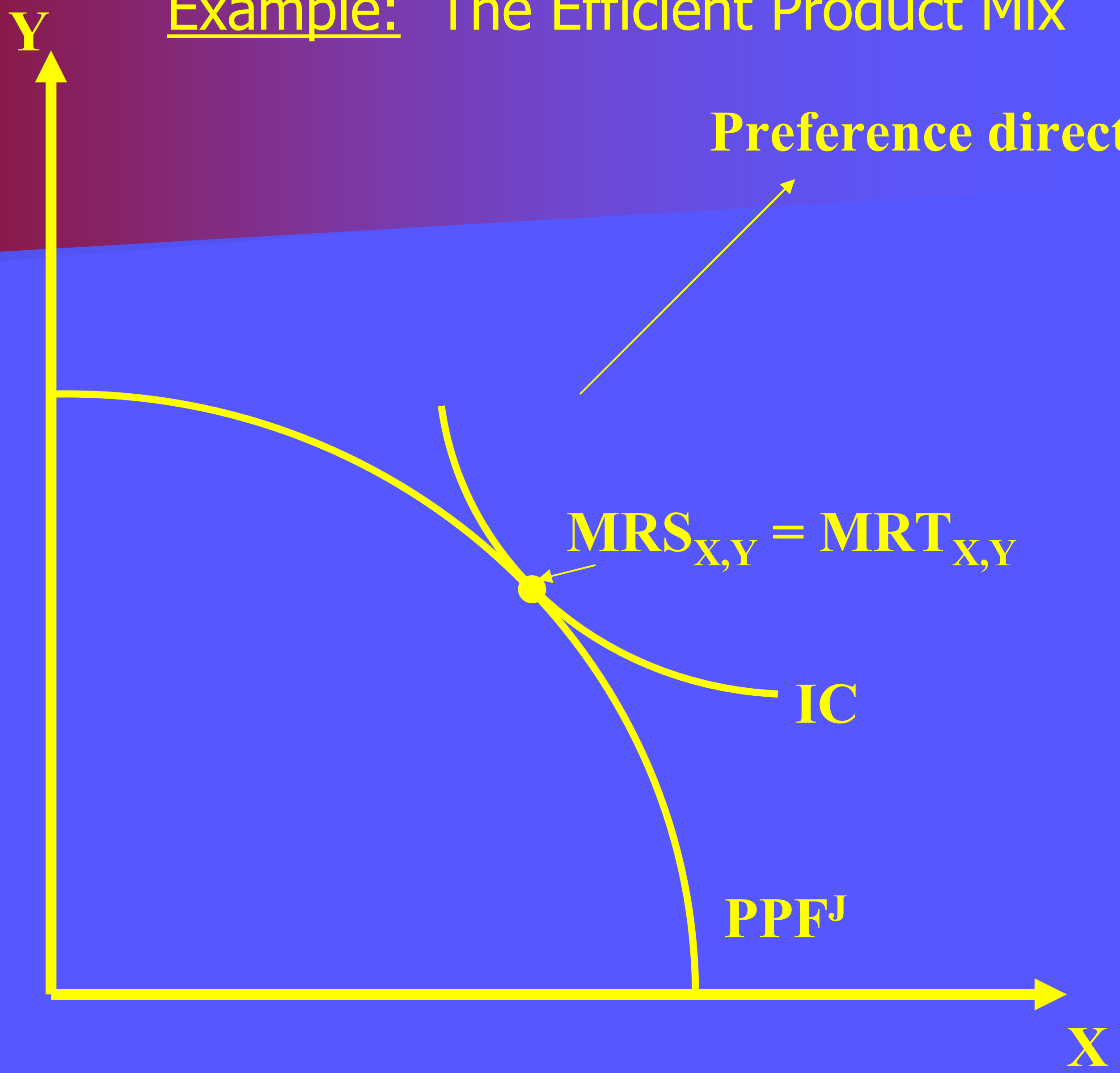


At this point, the consumer's willingness to give up good X in order to get good Y just equals the rate at which a producer has to give up good X in order to produce more of good Y.

$$MRT_{X,Y} = MRS_{X,Y}$$

But…This must be true for *all* consumers if the economy is to produce optimally for each consumer.

Can the competitive market help us to achieve this optimality?

At the Pareto efficient allocations, it is true for all consumers that:

$MRS_{X,Y} = p_X/p_Y$

Now, consider the producers' problem.

Suppose that the producers produce goods X and Y and choose the product mix so as to maximize profits given the prices p_X and p_Y:

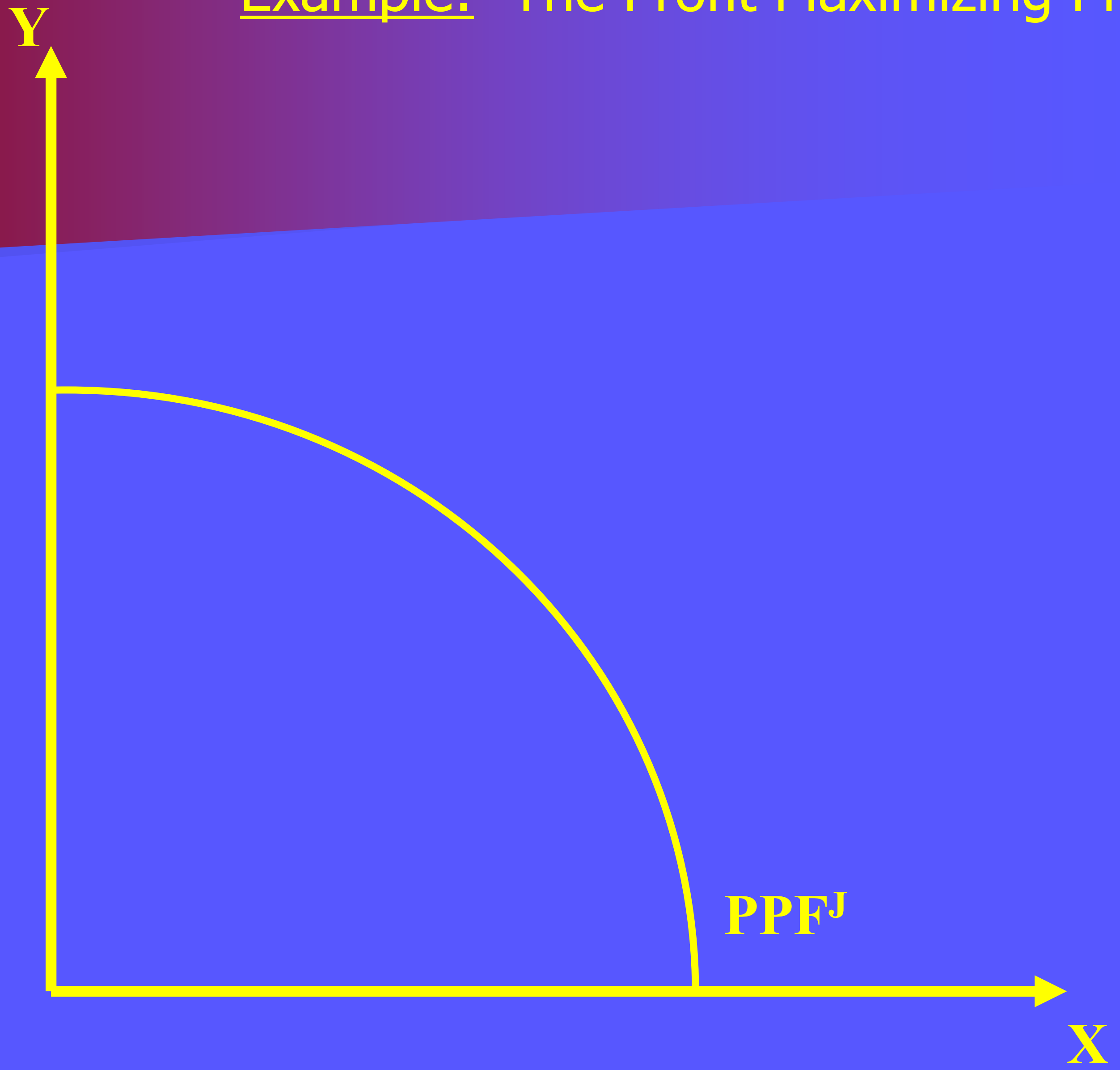
$$\max_{Q_X, Q_Y} \ \Pi = p_X Q_X + p_Y Q_Y - C^*$$

Where: we will suppose that the cost of production is fixed whatever the optimal output mix (e.g., we just want to know how to <u>employ</u> the labor we have contracted)

Definition: an **isoprofit** line shows the output combinations that result in a given level of profit, Π^0…or…

$$Q_Y = (\Pi^0 + C^*)/p_Y - p_X Q_X/p_Y$$

Example: The Profit Maximizing Product Mix



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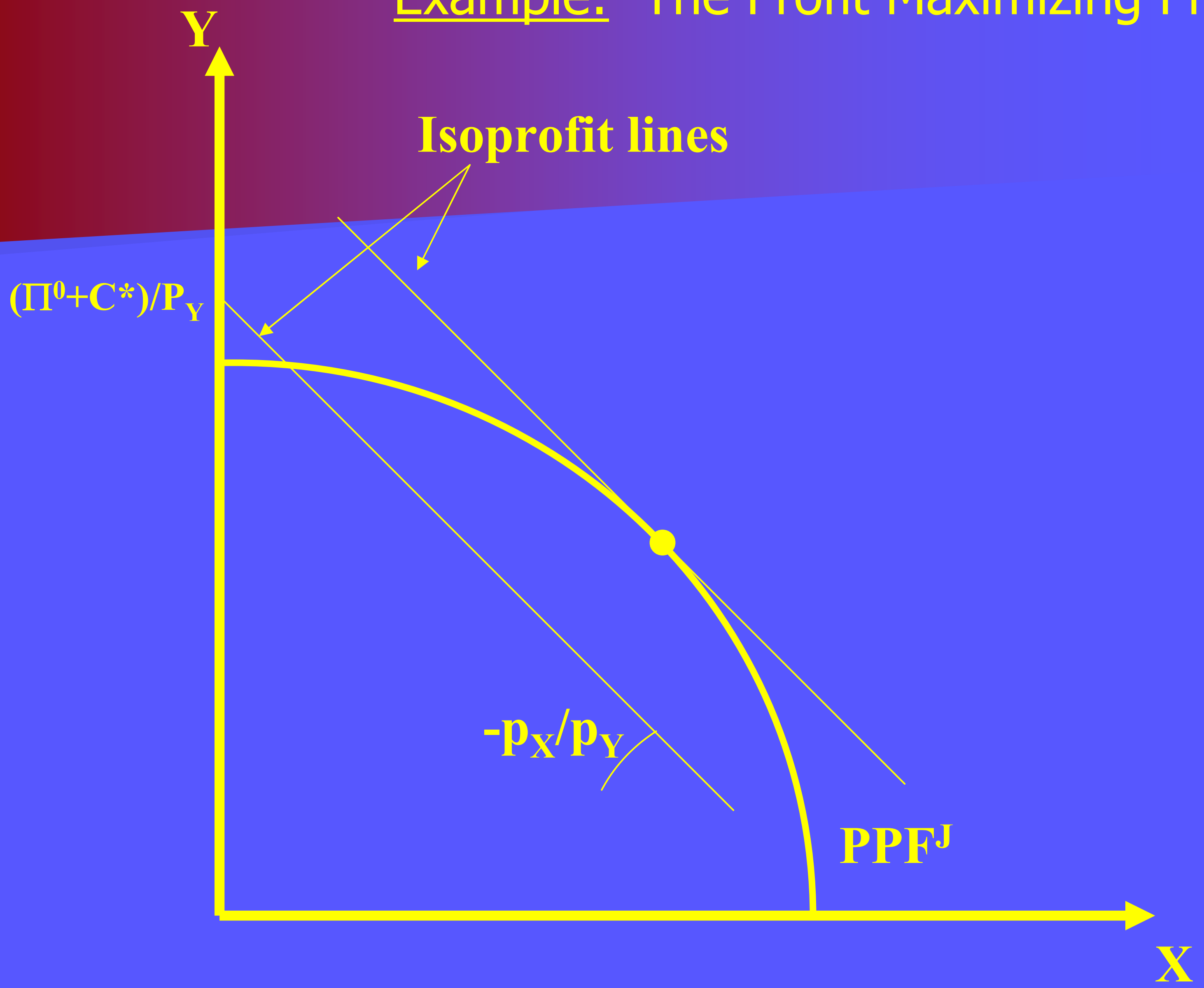
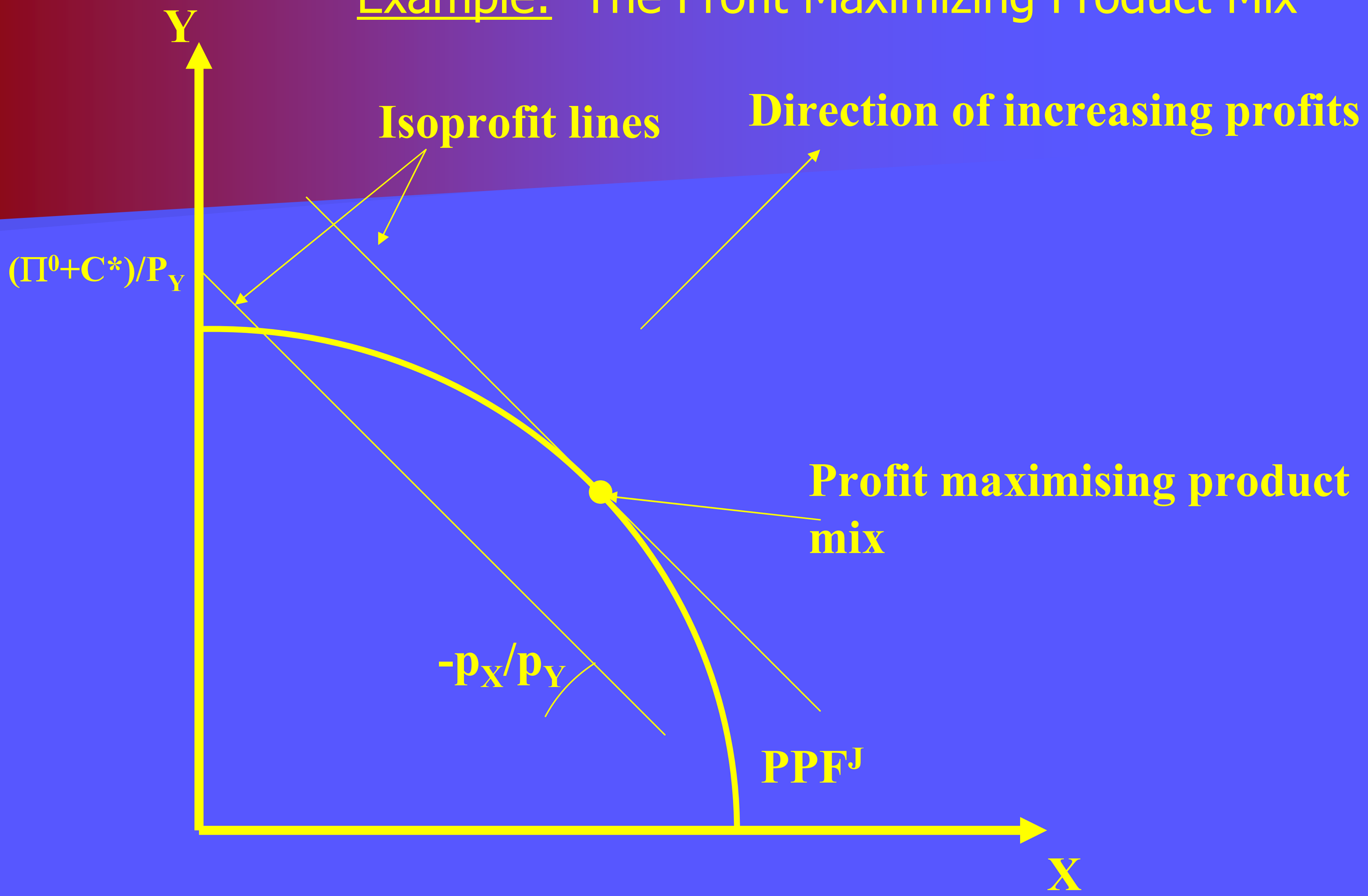


Example: The Profit Maximizing Product Mix



Hence, If the firm maximizes profits, then, it chooses the product mix that shifts out the isoprofit line as much as possible while remaining feasible. This is a tangency point such that for all producers:

$$MRT_{X,Y} = p_X/p_Y$$

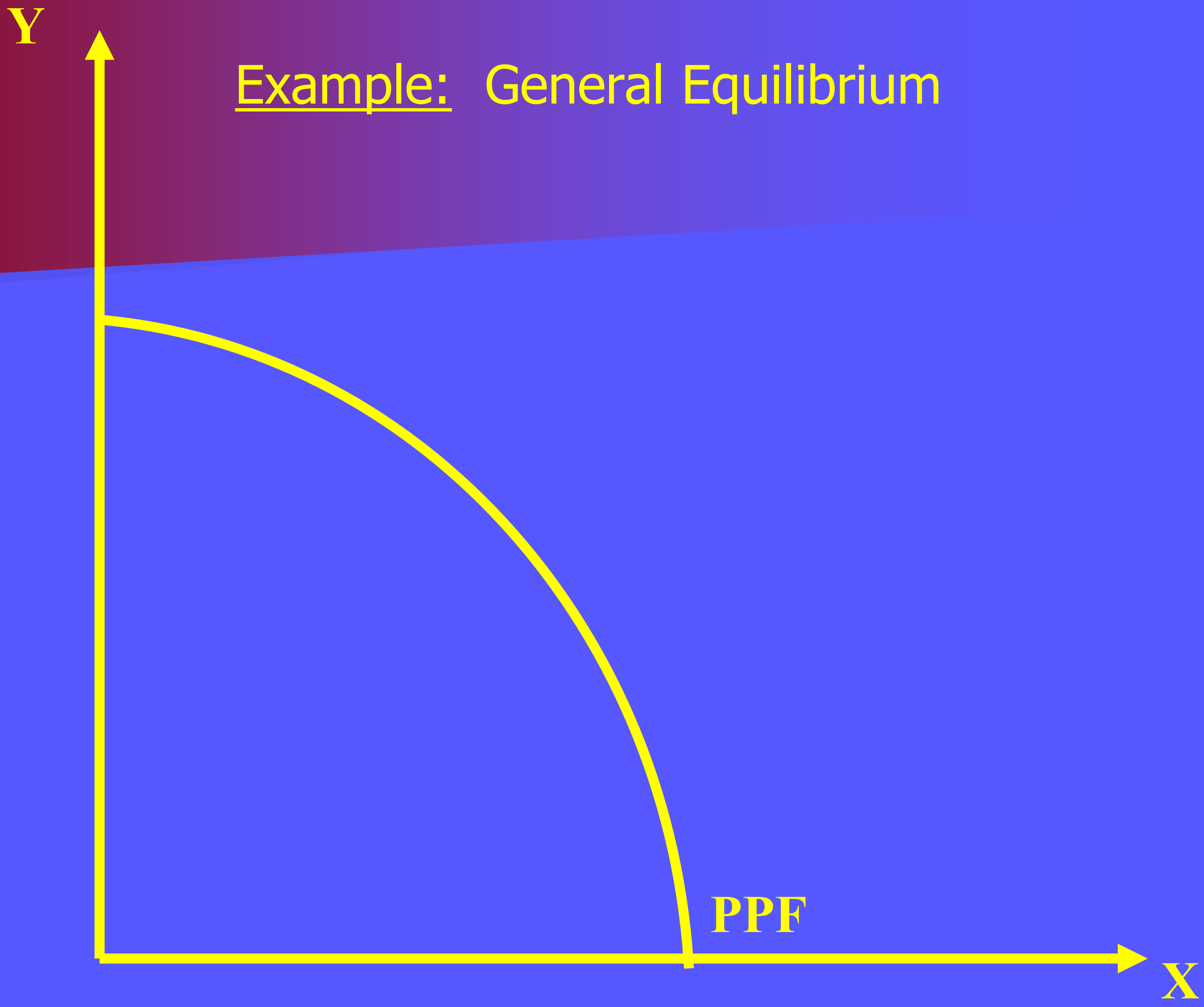
In other words, in equilibrium, the price ratio will measure the opportunity cost of production of one good in terms of production of the other good.

Therefore…

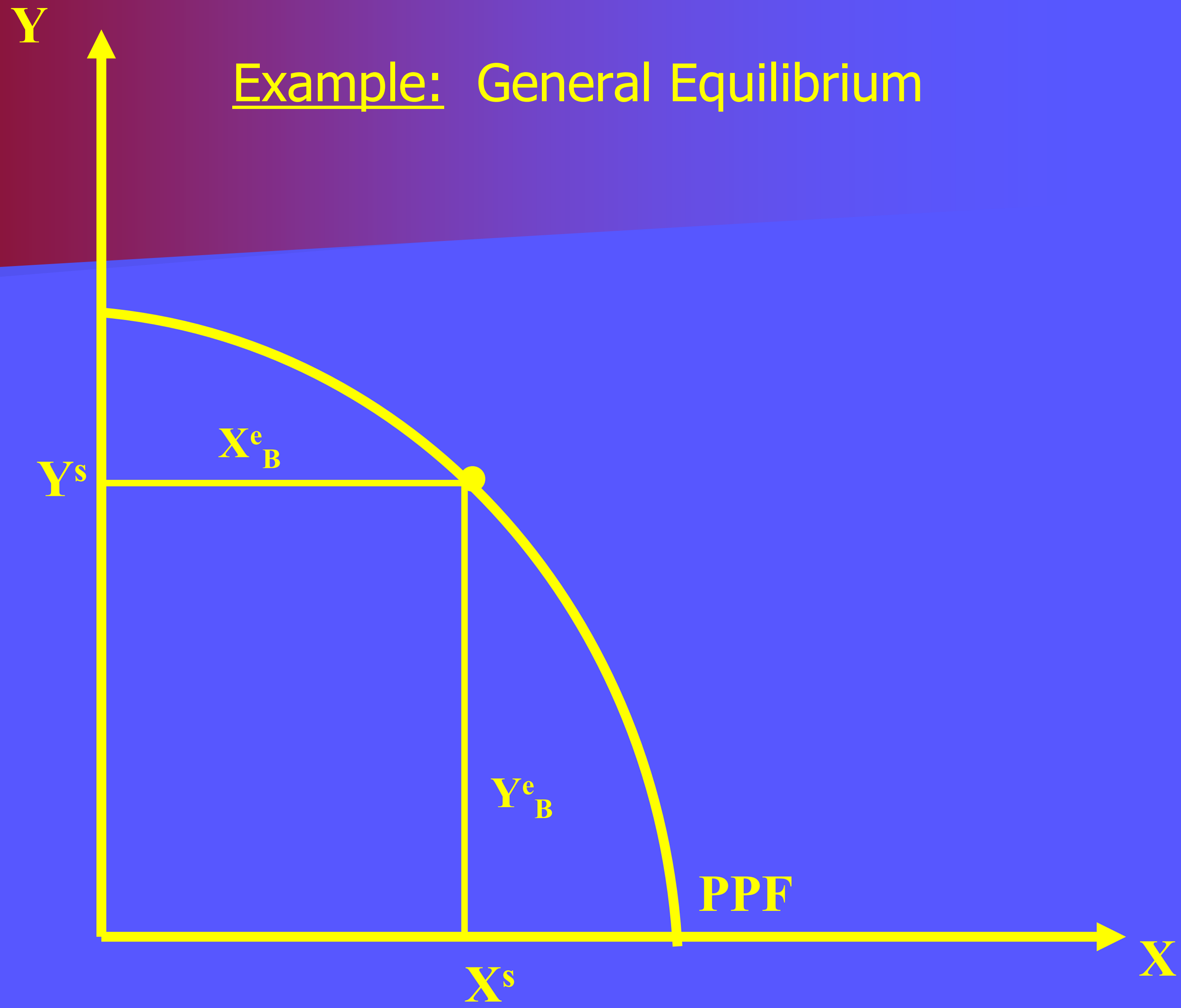
Because competition ensures that both the MRS and the MRT equal the (same) price ratio for all producers and all consumers, a competitive equilibrium achieves an efficient product mix for all producers and all consumers

Our earlier allocative efficiency results still hold with production…

Example: General Equilibrium



Example: General Equilibrium



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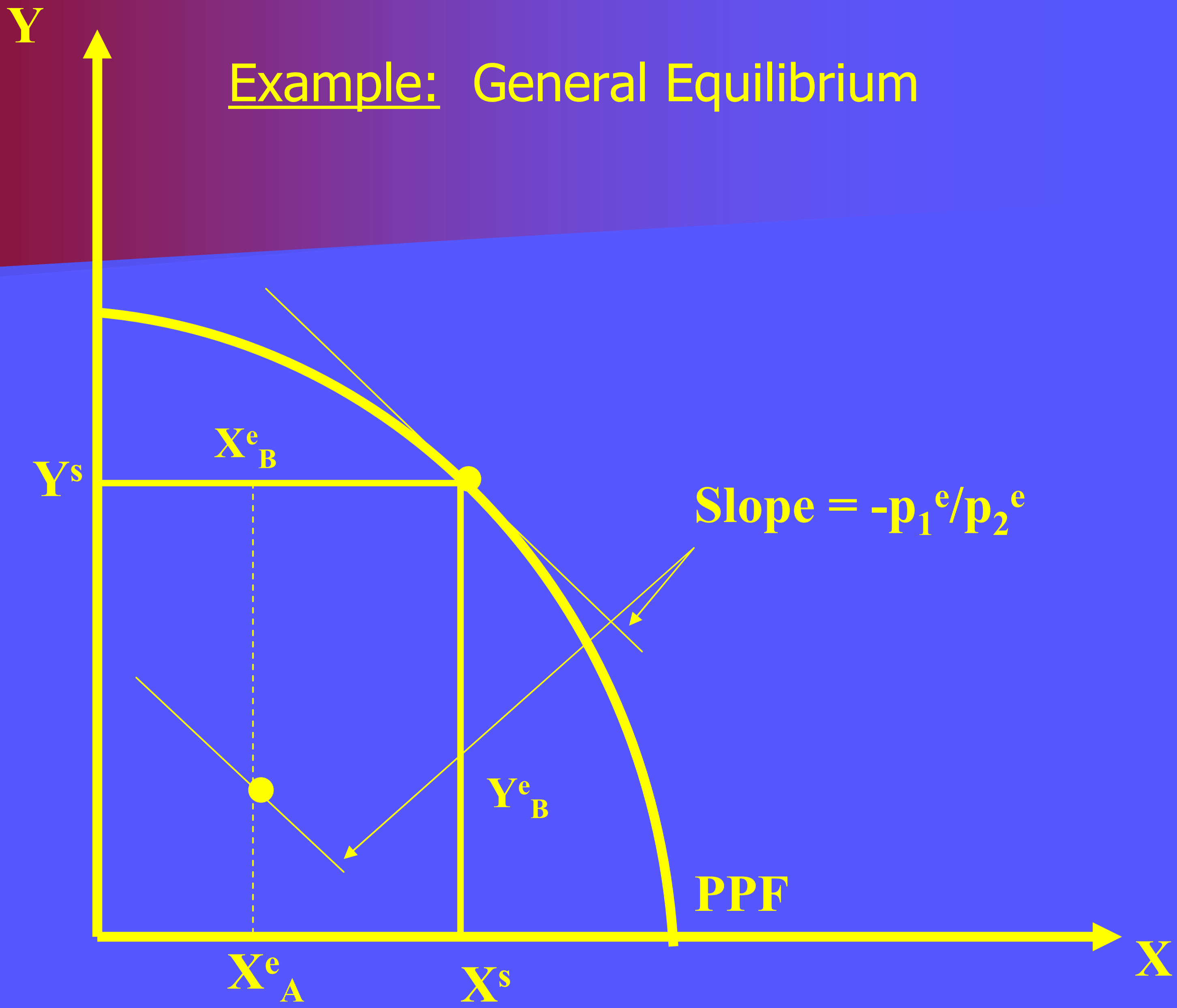
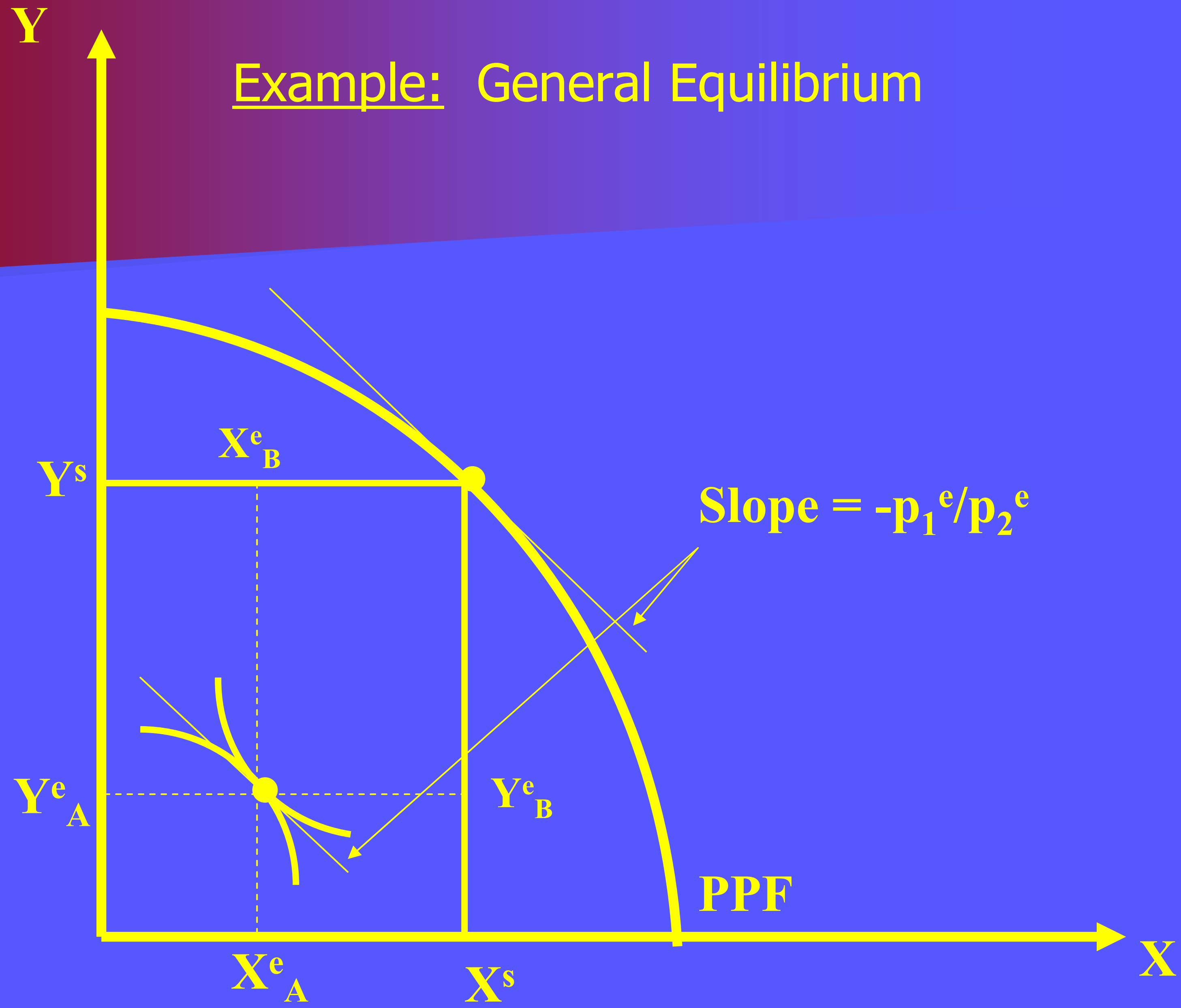
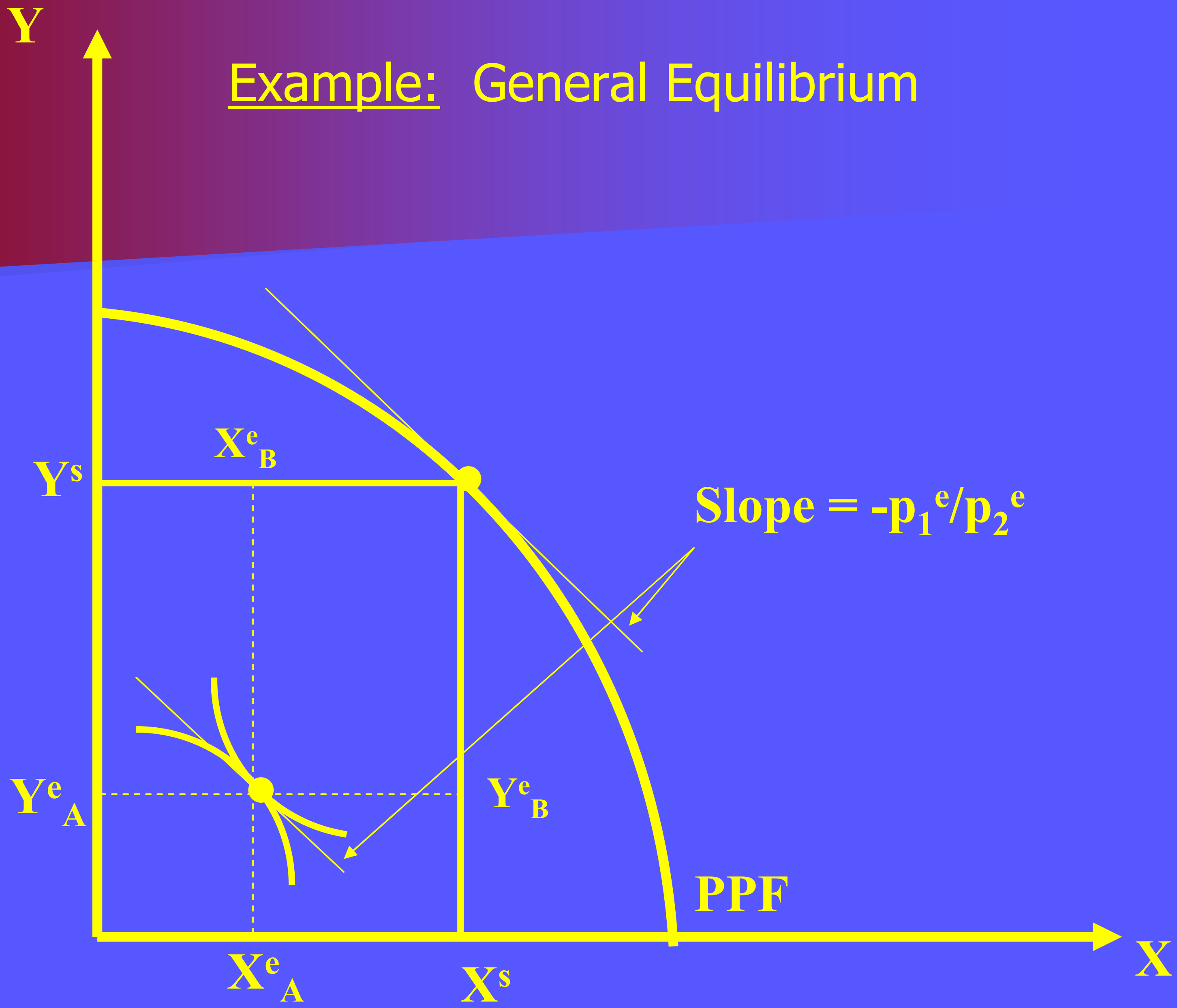


Example: General Equilibrium



Example: General Equilibrium



where: Y^s and X^s are the amounts of X produced in the economy; (X^e_A, Y^e_A) is the amount of X and Y consumed by person A and (X^e_B, Y^e_B) is the amount of X and Y consumed by person B.

- *Efficiency in exchange (on contract curve)*
- *Efficiency in use of inputs (on PPF)*
- *Efficiency in product mix(tangency with PPF)*