Auctions

- An auction is a mechanism for trading items by means of bidding.
- Dates back to 500 BC where Babylonians auctioned off women as wives.
- Position of Emperor of Rome was auctioned off in 193 ad
- Can have the bidders trying to buy an item: Christie's, ebay, snapnames.
- Can have the bidders trying to sell an item : Procurement, priceline.com
- Spectrum auctions

Rules to Auctions

- **First-Price Sealed-Bid Auction**: Everyone writes down a bid in secret. The person with the highest bid wins the object and pays what he bids.
- **Second-Price Sealed-Bid (Vickrey) Auction:** Everyone writes down a bid in secret. The person with the highest bid wins the object and pays the **second highest** bid. (used for stamps and by Goethe)
- **English Auction:** The auctioneer starts at a reserve price and increases the price until only one bidder is left.
- **Dutch Auction (**Not Demonstrated**):** The auctioneer starts at a high price and decreases the price until a bidder accepts the price.

Auction types

- **All Pay Auction**: Everyone writes down a bid in secret. The person with the highest bid wins. Everyone pays.

Two types of Settings: Common and Private

- Examples of Common Value Auctions:
  - Spectrum.
  - Oil Drilling
  - Book Example.
- Examples of Private Value Auctions:
  - Consumption items.
  - Memorabilia

Private Value Auctions: Example:

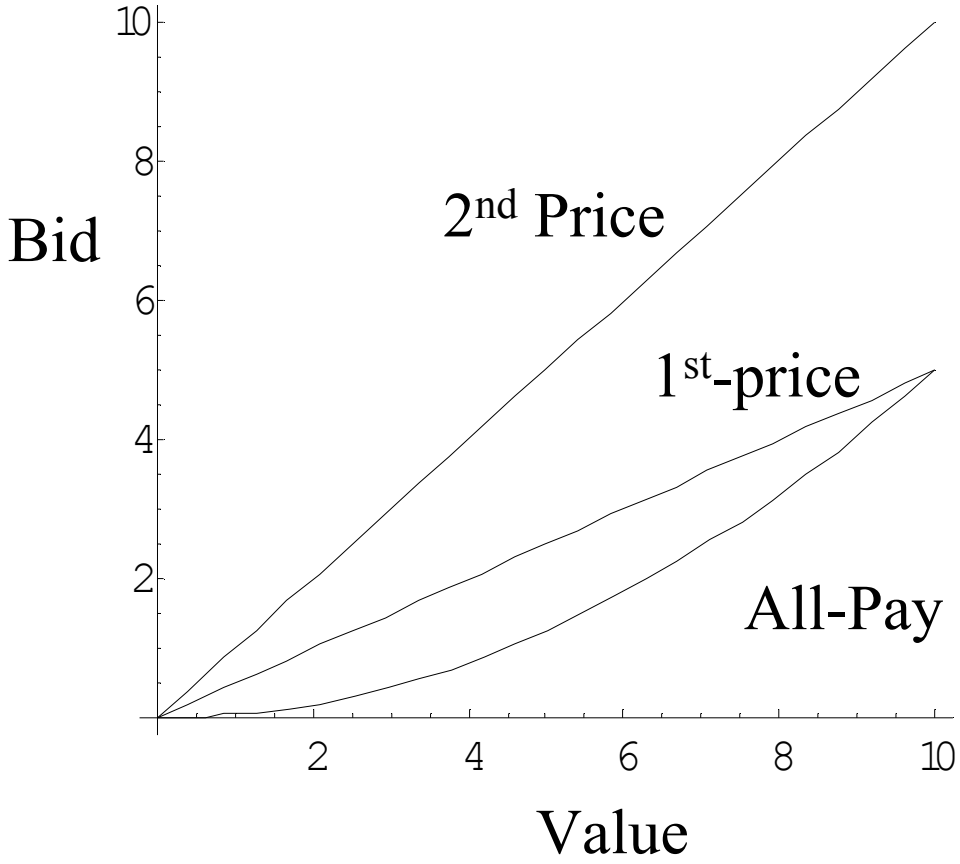
- I am auctioning off the right shoe of the star of the Exeter City Football team.
- Al has value £30. Bob has value £40. Chris has value £55.

- What is revenue in the following situations?
  - 2nd-price sealed bid.
  - 1st price sealed bid. Each bids 2/3 of his value.
  - English, Each bids up to his value
  - Dutch, each bids 2/3 of his value.

Uniformly distributed values.

- Values are drawn from 0 to £10 with an equal chance of each amount (like in the lab). N is the number of bidders.

Private values: Equilibrium Bid Functions



Strategies with Private values: English Auction

- The English: stay in the auction until either
  - you win
  - or the bid goes higher than your value.
  - If not one either makes one lose when it is worthwhile to win or win when it is worthwhile to lose.
- The key to understanding this is to understand that staying in does not affect the price one pays if they win only whether one wins (it does affect others' prices).

Strategies with Private Values: 2nd Price Auctions

- 2nd price similar logic to English auction.
- It is optimal to bid one's value.
  - One's bid does not affect the price one pays only whether or not one pays.
  - Raising one's bid will cause one to win when it is not worthwhile.
  - Lowering one's bid will cause one to lose when it was worthwhile to win.

Strategies with Private Values: First Price

- Strategies in the first-price should shade bid below your value
  - This is because one's bid affects one's price.
  - Bidding your value will earn zero surplus.
  - Shading one's bid lowers the probability of winning, but increases the surplus gained when winning.
- There is a natural trade-off between probability of winning and profit if one wins.
  - If bid is b, value is v, expected profit is

    Probwin(b)*(v-b)
  - Derivative of this w.r.t. b yields

    Probwin'(b)*(v-b)-Probwin(b)=0
  - First term is marginal benefit of prob of winning.
  - Second term is marginal cost of the profit if one wins.

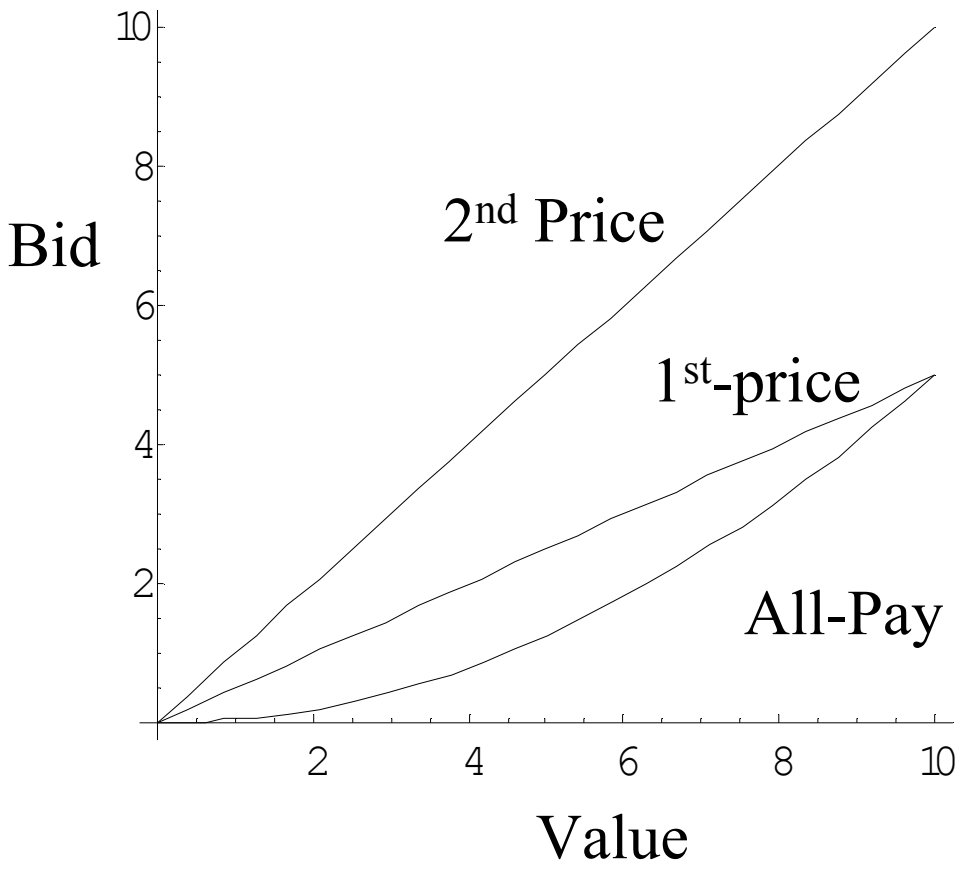
All-pay auction

- May seem like a strange auction to run/study but…
- It is used in charity auctions and from the lab one can see why. (Losers don't complain so much.)
- Extremely useful modelling tool.
  - Patent Races.
  - Political Campaigns.
  - Technology contests.
  - Procurement contests – Architecture, Next Generation Fighter Jet.
  - Sports contests. (Think of Chelsea, Man U, Arsenal all buying the best players.)

Strategies with Private Values: All Pay

- In the all-pay auction, you should again shade bid below your value.
- The natural trade-off is now between probability of winning and cost of bidding.
- This cost is incurred whether you win or not.
- It only makes sense to incur a high cost if the probability of winning is fairly high.
- For low values, bids are shaded much more than with first-price auctions.

Equilibrium Bid Functions



Strategies with uniform values.

- Values are drawn from 0 to £10 with an equal chance of each amount (like in the lab). N is the number of bidders.
- 1st-price the equilibrium bid (N-1)*v/N (that is if v=£5.50 and N=2, bid £2.75.
- Dutch auction is the same as the 1st-price.
- 2nd-price, optimal to bid value. English optimal to bid up to value.
- All-pay auction, should bid $(N-1) * (v/10)^N * 10/N$ (looks complicated but only we can see for low values shade bid more than for high values).

Revenue Equivalence

- For private values, there is *revenue equivalence* among all designs.
- Not only that but all auctions are fully efficient – the buyer who values the object the most winds up buying it.
- If a seller wants to maximize revenue, he can simply use an appropriate minimum bid in any of the designs.
- *Problems happen if:* asymmetry, risk aversion, common values, seller info.

Revenue Equivalence

One of the major findings of Auction Theory is the celebrated **Revenue Equivalence Theorem**, which states, for private values, that any allocation mechanism/auction in which (i) the bidder with the highest type/signal/value always wins, (ii) the bidder with the lowest possible type/value/signal expects zero surplus, (iii) where all bidders are risk neutral and (iv) drawn from a strictly increasing and atomless distribution will lead to the same revenue for the seller (and player i of type v can expect the same surplus across auction types).

Common Value Auctions

- I am auctioning off the right to sell refreshments during lecture. The value is £200.
- Al thinks it is worth £180
- Bob thinks it is worth £190
- Chris thinks it is worth £200
- Doug thinks it is worth £210
- Eric thinks it is worth £220
- What is the revenue in a 2nd price auction where everyone bids their estimated value? What is the average estimate? What should they do to their bids?

Book pages auction

- **In our auction:** The object is fictitious and worth the number of pages in the book in pence. I will pay the difference between the value and the price: **v-p** (if price is above the number of pages, I will receive the difference between the price and value: **p-v**).

Book Pages Auction

- **First-Price Auction**: The person with the highest bid wins the object and pays what he bids.
- **Second-Price Auction:** The person with the highest bid wins and pays the **second highest** bid.
- **The Prize:** The number of pages in the book in pence. I will pay the difference between this value and the price offered: **v-p** (if price is above the value, I will receive the difference between the price and value: **p-v**).

Name: _____

Estimate the value : _____ Pence

Bid in First-Price auction :_____ Pence

Bid in Second-Price auction : _____ Pence

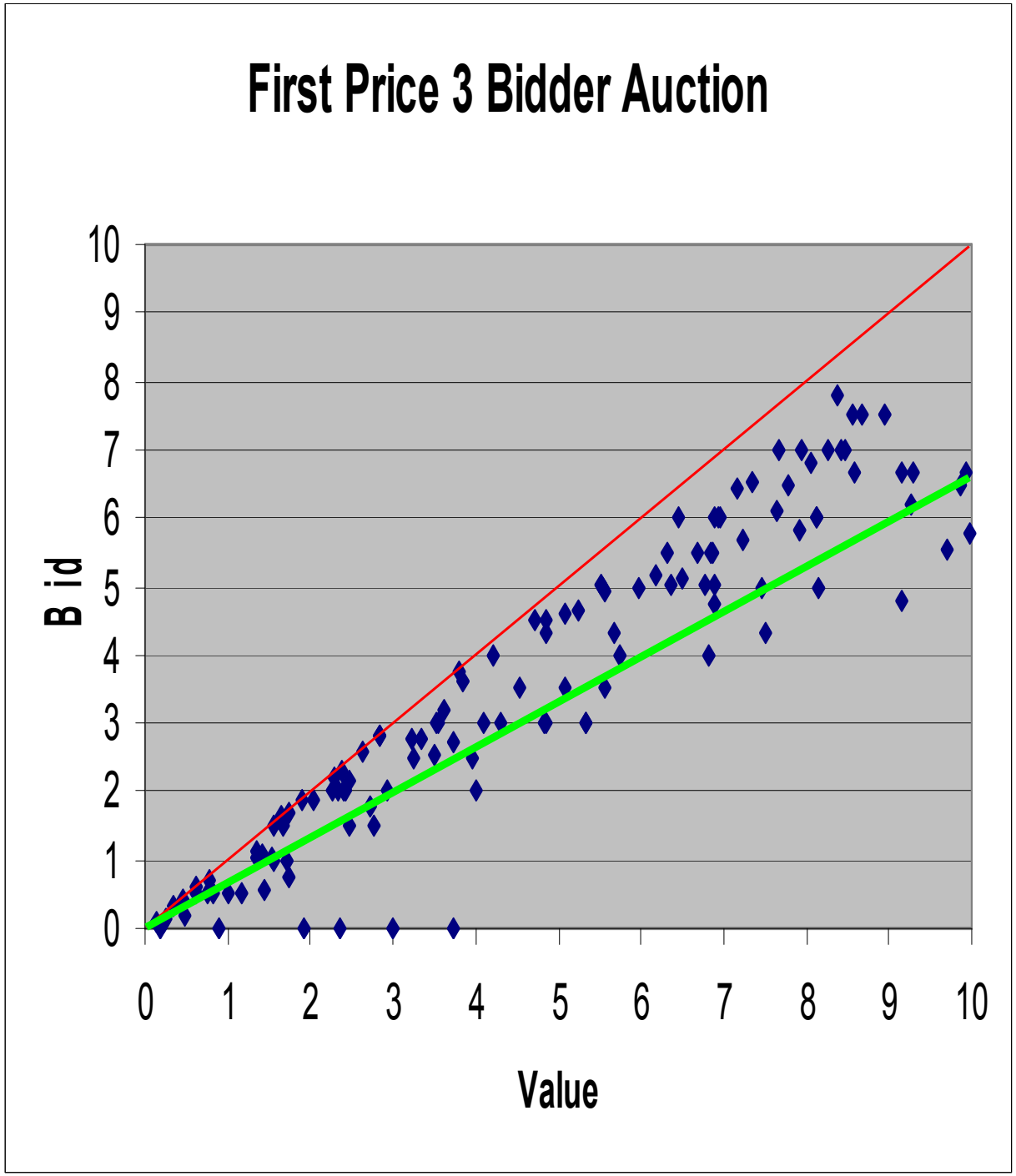
Results

- Book had approx 450 pages
- Averages were
  - 609 estimate
  - 459 first price
  - 590 second price
- Maxima were
  - 1250 estimate
  - 845 first price
  - 2000 /1250 /900 second price

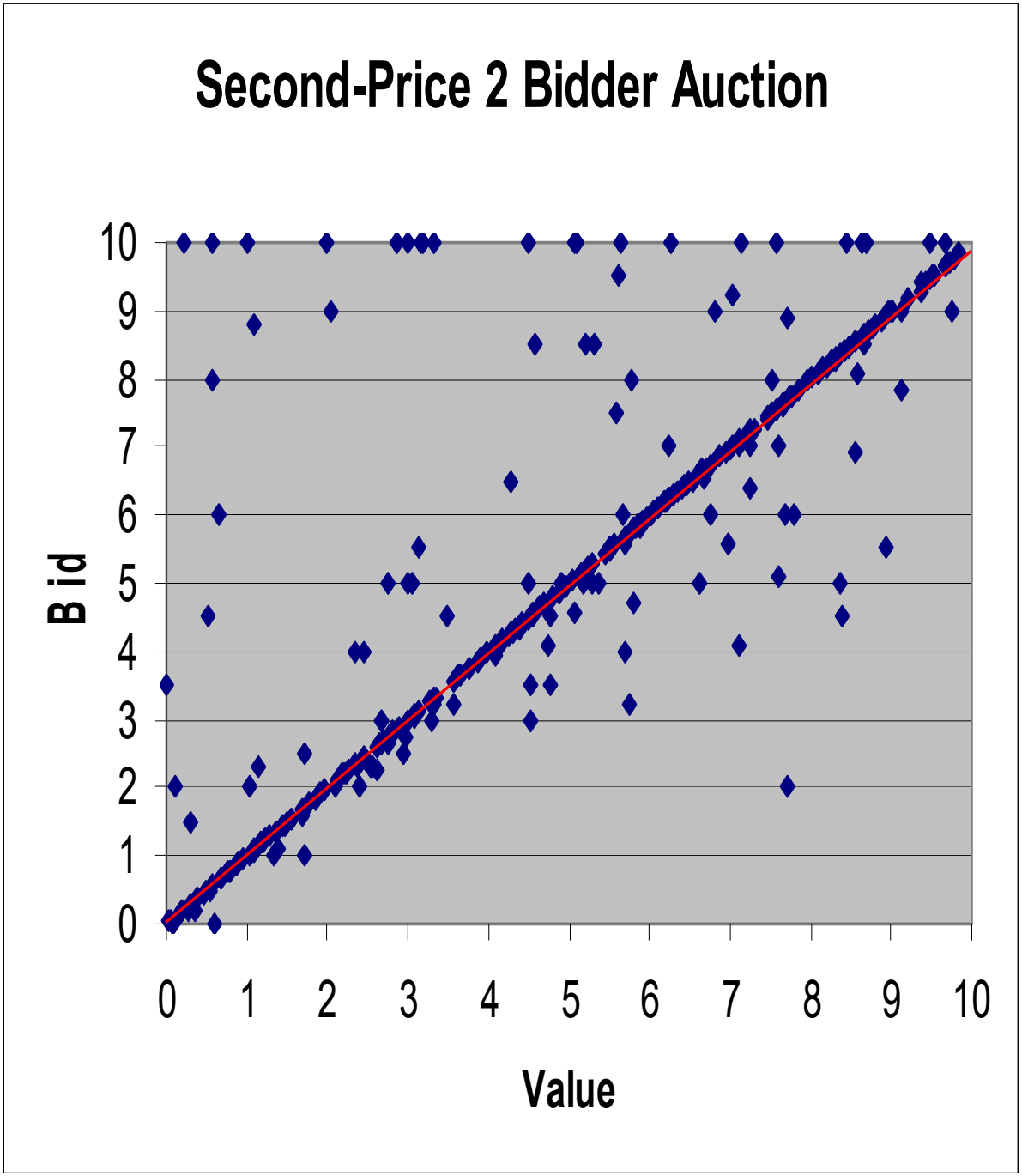
Classroom Experiments

- We ran a number of designs in the lab.
  - First-Price with two buyers (random partners)
  - Second-Price with two buyers (random partners)
- The buyers' surplus is their combined profit.
- The seller's surplus is the sale revenue.
- Total surplus is buyers' + seller's.
- Efficiency is 100*total surplus/potential surplus.

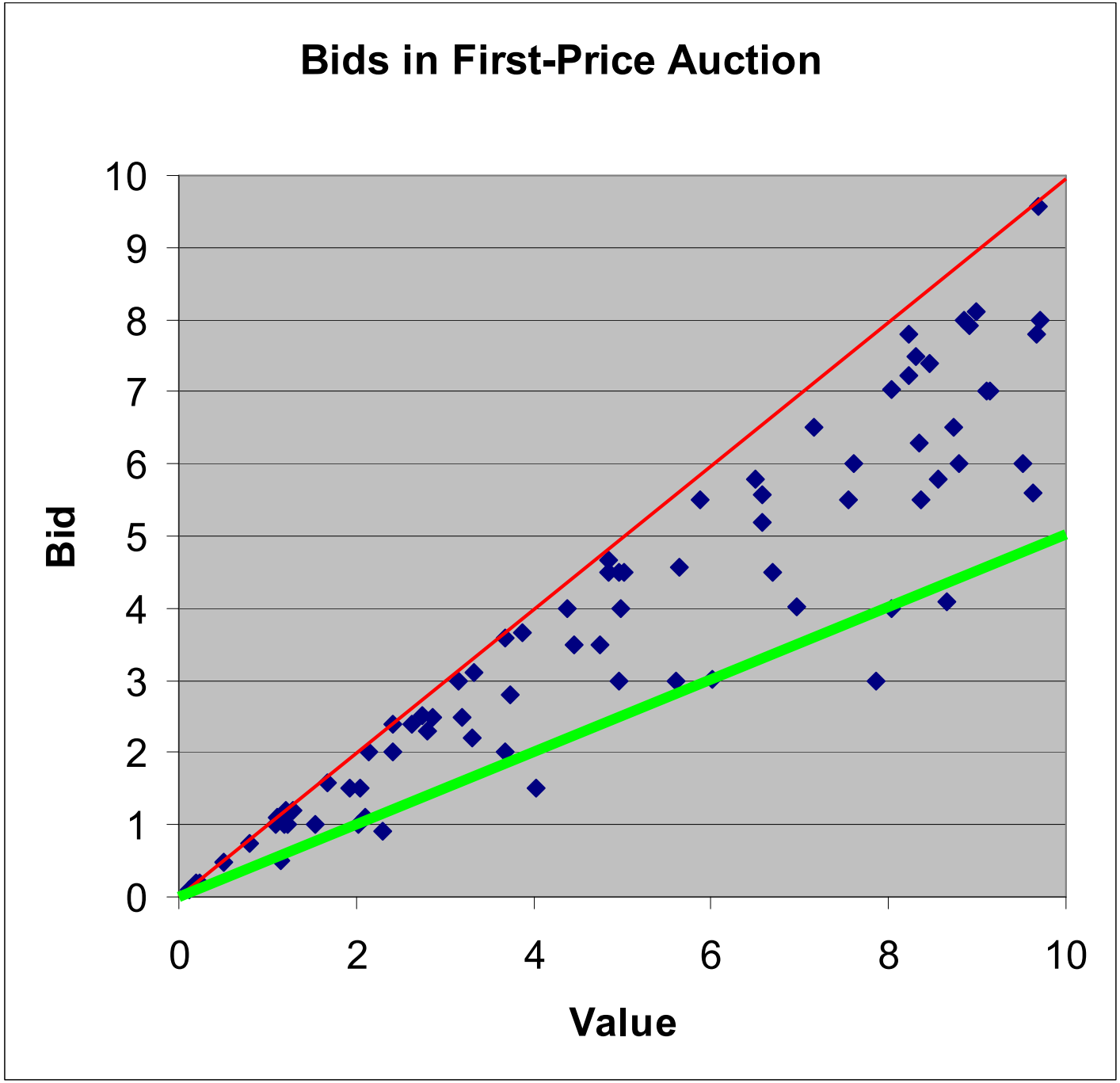
First-Price 2 bidders
Bid
Value
0
1
2
3
4
5
6
7
8
9
10

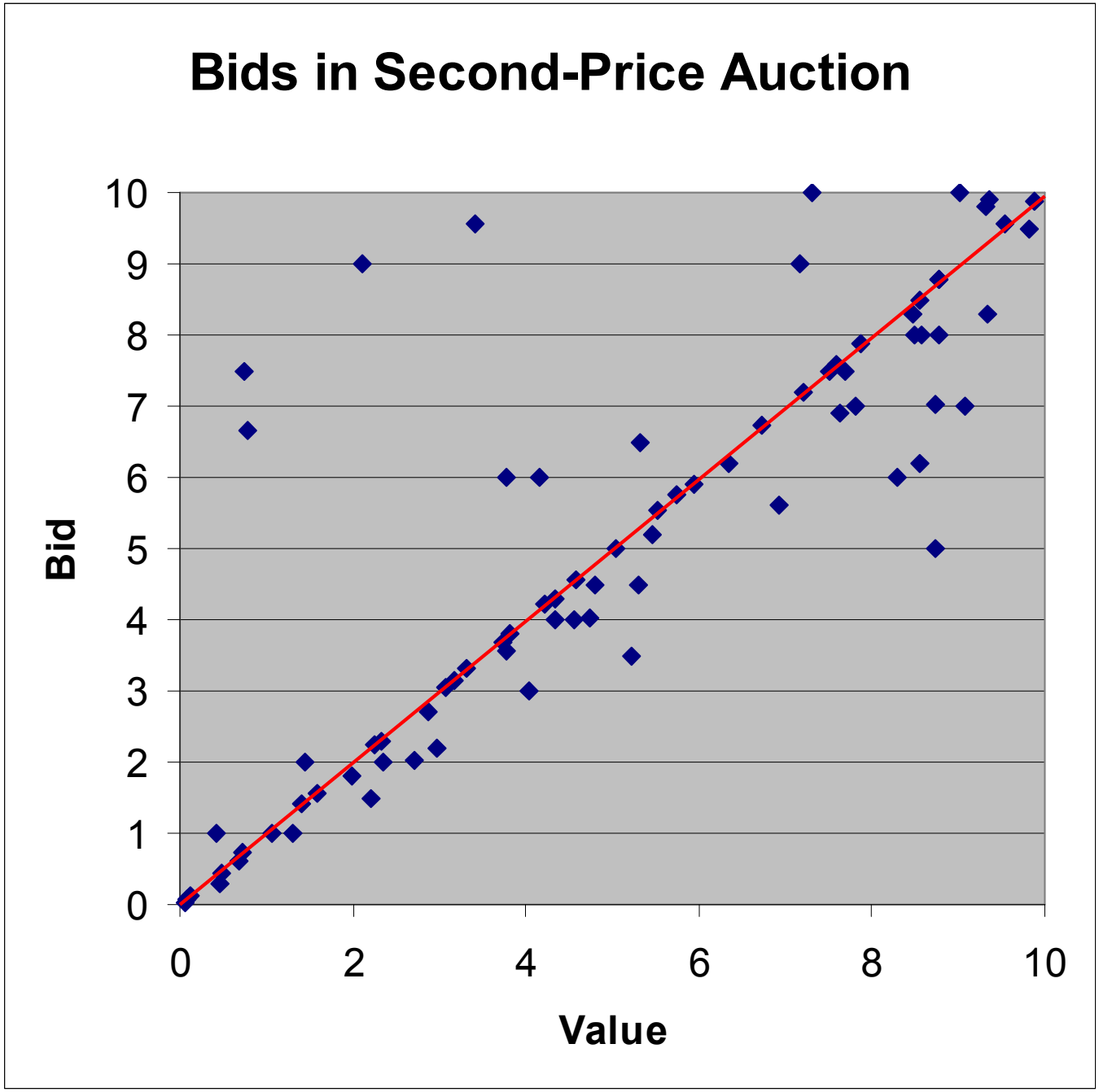

First Price 3 Bidder Auction
B id
Value
0
1
2
3
4
5
6
7
8
9
10

First-Price 4 bidder Auction
Bid
Value
0
1
2
3
4
5
6
7
8
9
10


Second-Price 2 Bidder Auction
Bid
Value
0
1
2
3
4
5
6
7
8
9
10

First-Price 2 bidder auction w/ rebate
Bid
Value
0
1
2
3
4
5
6
7
8
9
10


Bids in First-Price Auction
Bid
Value
0
1
2
3
4
5
6
7
8
9
10
0
2
4
6
8
10


Bids in Second-Price Auction
Bid
Value
0
1
2
3
4
5
6
7
8
9
10
0
2
4
6
8
10

Classroom Experiment Results

	Bidders' % pie	Seller's % of pie	Efficiency
FP 2 bidders	23%	74%	97%
SP 2 bidders	40%	54%	94%
AP 2 bidders	10%	84%	95%
Theory 2 bidders	50%	50%	100%
FP 3 bidders	22%		
Theory 3 bidders	25%		
FP 4 bidders	13%		
Theory 4 bidders	20%		
FP rebate	96%	0%	96%
Theory rebate	100%		100%

Classroom Experiment Results

- First Price is best for the seller.
- Second-Price is best for the buyers.
- First-Price is best for efficiency.

Design may matter.

- The airwave 3G auctions $34 billion in the UK using a design based on the English auction.
- A few months afterwards, mirroring their design Holland raised only $2.5 billion. On a per capita basis less than 30% of the per capital of the British Auction.
- Why? Klemperer suggests that bids were costly and limited number of buyers.

Careful design of rules

- In the FCC auction, bidders communicated which area they wanted by ending their bid with the area code. For example, $1,000,818.
- In Australia auctioned off segments of airways. No punishment for defaulting after auction. Companies placed several bids and defaulted with the high ones. Went for 20th highest bid to the same two companies.

Design Goals

- Efficiency - Want the bidder with the highest value to win.
- Revenue - May want to collect the highest revenue.
- Collusion, communication (FCC). Want to avoid.
- Entry - want to encourage.

Some tools

- Entry Fee- hurts entry, efficiency, helps revenue.
- Minimum Bid - same.
- Bid Cap – hurts efficiency, helps entry.
- Release information about object. Helps with winner's curse which may help revenue.
- Release information about buyers. Tends to help collusion.
- Binmore: changing institutions changes behavior, non-economists often do not forsee this