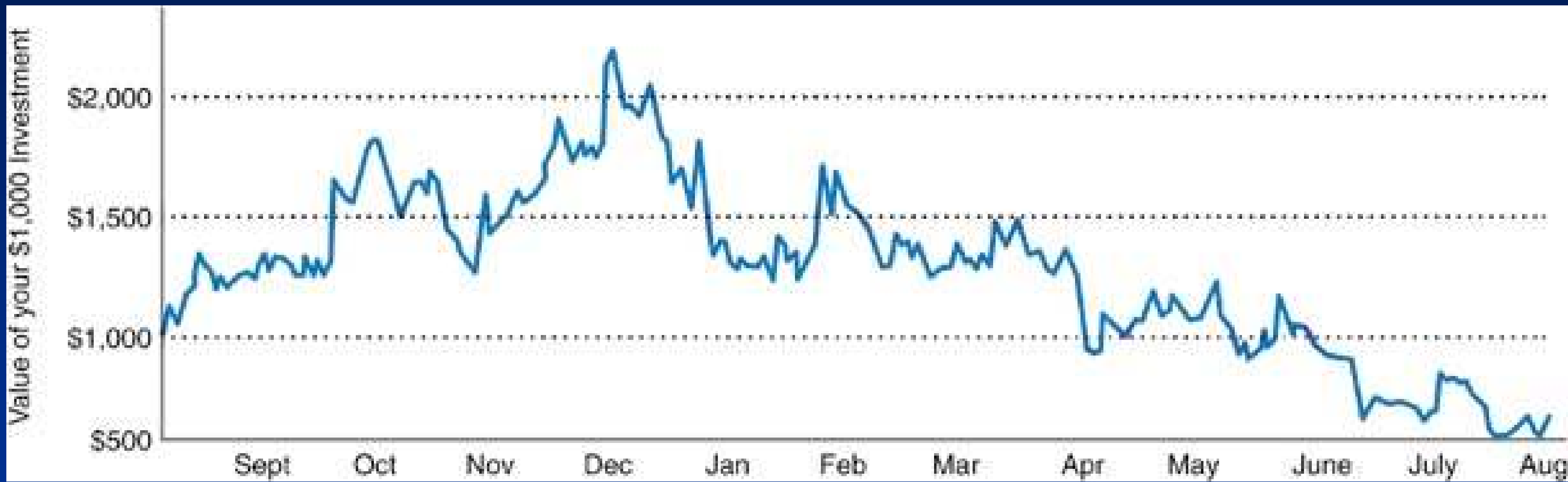
Uncertainty

BEE2017 Microeconomics



Uncertainty: The share prices of Amazon and the difficulty of investment decisions

Contingent consumption

1. What consumption or wealth will you get in each possible outcome of some random event?
2. Example: rain or shine, car is wrecked or not, etc.
3. Consumer cares about pattern of contingent consumption: $U(c_1, c_2)$.
4. Market allows you to trade patterns of contingent consumption — insurance market, stock market.
5. Insurance premium is like a relative price for the different kinds of consumption.
6. Can use standard apparatus to analyze choice of contingent consumption.

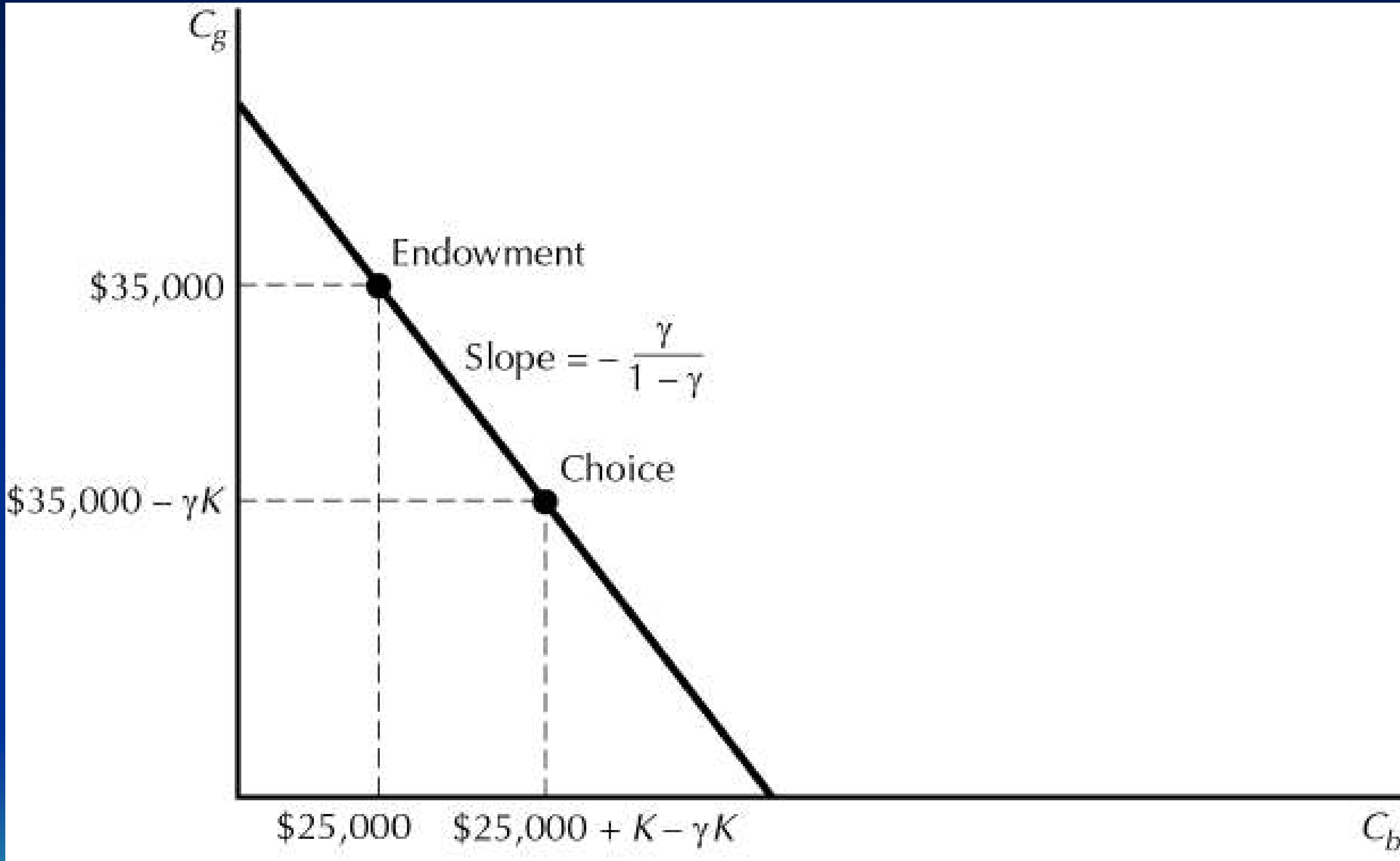


Figure 12.1 Insurance

Utility functions

1. preferences over the consumption in different events depend on the probabilities with which the events will occur.
2. So $u(c_1, c_2, p_1, p_2)$ will be the general form of the utility function.
3. Under certain plausible assumptions, utility can be written as being linear in the probabilities, $p_1u(c_1) + p_2u(c_2)$. That is, the utility of a pattern of consumption is just the expected utility over the possible outcomes.

Subjective probability

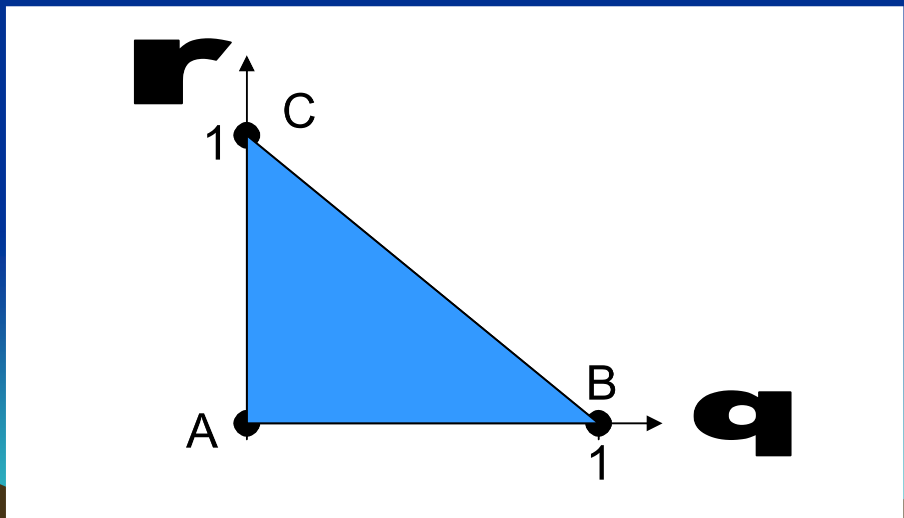
- Von Neumann / Morgenstern:
  - "Roulette lotteries": known probabilities (risk)
- Savage / Ramsey / de Finetti
  - "Horse Races": unknown probabilities (if any) (uncertainty)
- Savage: Rational individuals behave ***as if*** they would maximize expected utility given subjective utilities and beliefs
- Aumann / Anscombe: allow for both

Lotteries and Expected Utility

- Suppose there 3 prizes A, B and C to be won by the consumer with $A<B<C$
- Let $0=u(A)<u(B)<u(C)=1$ be the utilities of the consumer.
- A lottery is described by probabilities p, q, r with which the prices A, B, C can be won.
- $p=1-q-r$ since the probabilities must add up to 1.

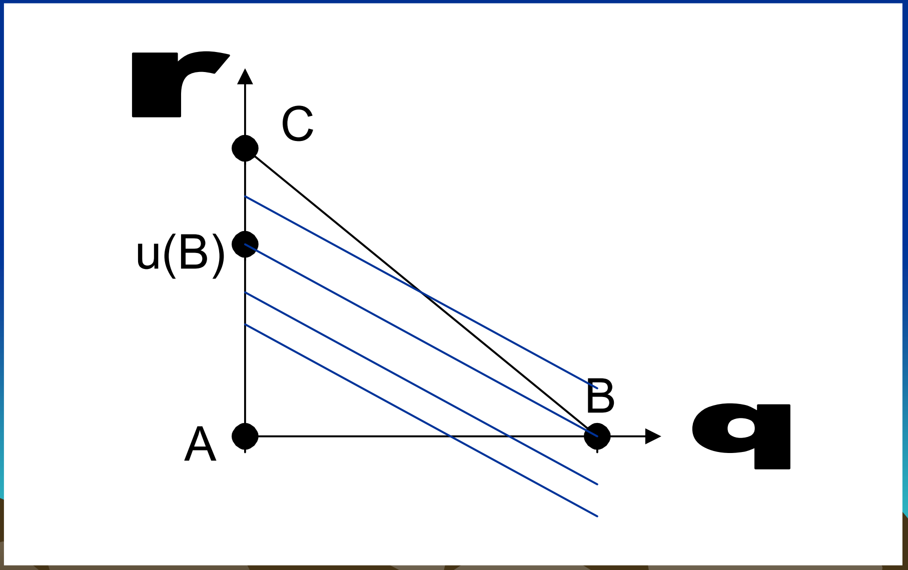
Lotteries 2

- Any possible lottery over the prizes can be described by the pairs (q, r) in the q-r-plane.
- Restriction to a triangle: q, r ≥ 0, q+r ≤1.



Lotteries 3

- Linear expected utility from a lottery:
- $U(q,r)=(1-q-r)*u(A) + q*u(B) + r*u(C) = q*u(B) + r$
- Indifference curve: $q*u(B) + r = c_{onstant}$
- $r=c - u(B)q$

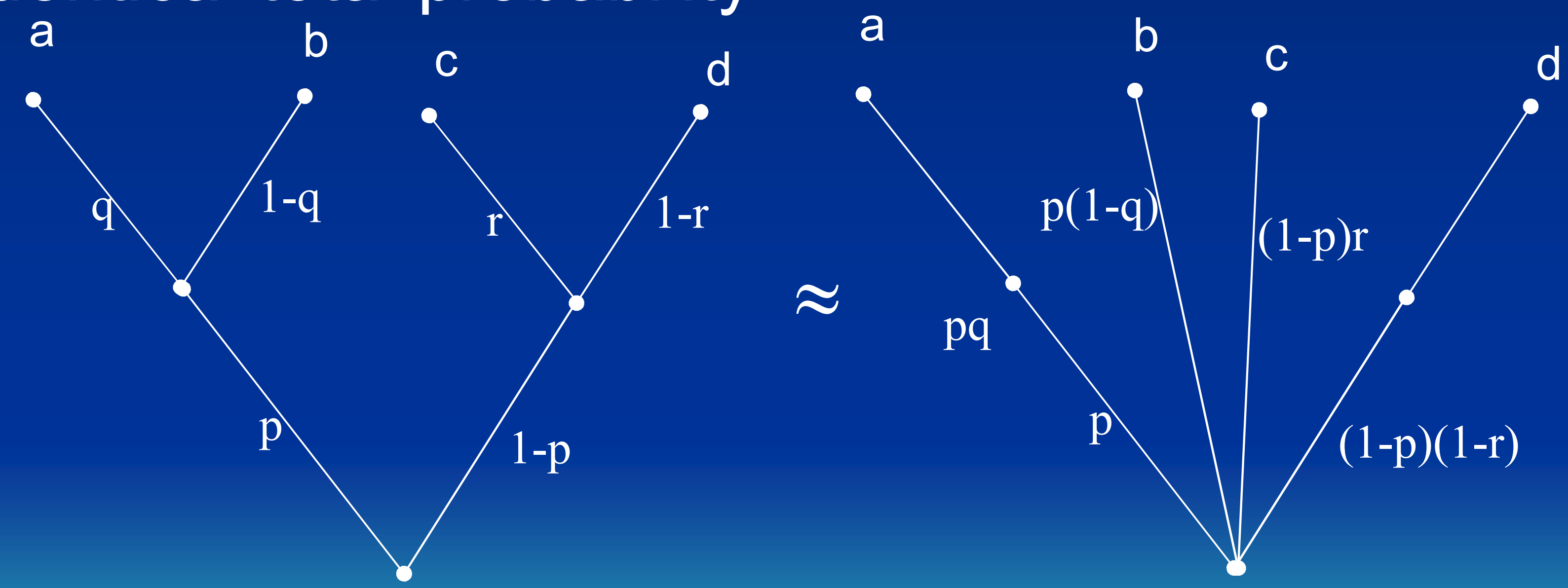


Lotteries 4

- Expected utility implies linear and parallel indifference curves over lotteries.
- Conversely “Consequentialism” + “Substitution Axiom” + “Continuity Axiom” imply that indifference curves are parallel lines.

Consequentialism

- The consumer identifies compound lotteries, where the prizes are themselves lotteries, with simple lotteries which give identical prizes with identical total probability.



On the left is a compound lottery which give with probability p a lottery which gives “a” with probability q and “b” with probability 1-q. With probability 1-p the consumer gains the lottery which gives c with probability r and d with probability 1-r.

Lotteries 5

- Continuity (simplified): If $a<b<c$ then there exists a probability p such that the consumer is indifferent between prize b and the lottery which gives him prize c with probability p and prize a with probability 1-p.

Lotteries 6

- Substitution Axiom (simplified): Suppose consumer is indifferent between prize B and a lottery which gives her C with prob. x and A with prob. 1 - x. She should hence be indifferent between a lottery (p,q,r) and

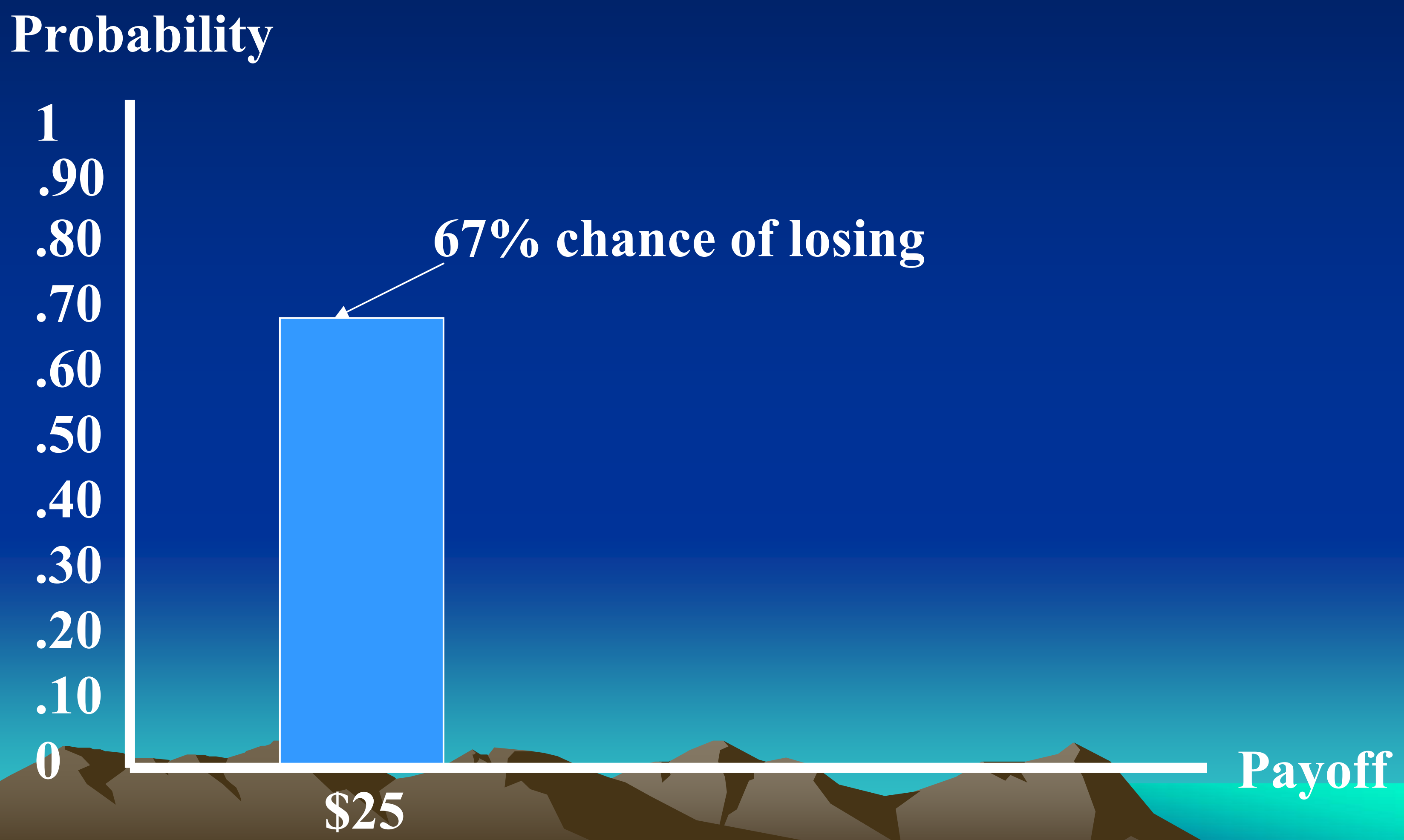
  (p+(1-x)q, 0, xq+r).).
- Plausible for lotteries, but not for consumption.

Money prizes

Now: infinitely many money-prizes

shape of utility function over money describes attitudes towards risk.

Example: A Probability



Example: A Probability

Definition: The **expected value** of a lottery is a measure of the average payoff that the lottery will generate.

EV = Pr(A)xA + Pr(B)xB + Pr(C)xC

Where: Pr(.) is the probability of (.)
A,B, and C are the payoffs if
outcome A, B or C occurs.

In our example lottery, which pays \$25 with probability .67 and \$100 with probability 0.33, the expected value is:

$$EV = .67 \times \$25 + .33 \times 100 = \$50.$$

Notice that the expected value need not be one of the outcomes of the lottery.

<u>Definition:</u> The **variance** of a lottery is the average deviation between the possible outcomes of the lottery and the expected value of the lottery. It is a measure of the lottery's riskiness.

$$\text{Var} = (A - EV)^2(\Pr(A)) + (B - EV)^2(\Pr(B)) + (C - EV)^2(\Pr(C))$$

<u>Definition:</u> The **standard deviation** of a lottery is the square root of the variance. It is an alternative measure of risk.

For our example lottery, the squared deviation of winning is:

$(\$100 - \$50)^2 = 50^2 = 2500.$

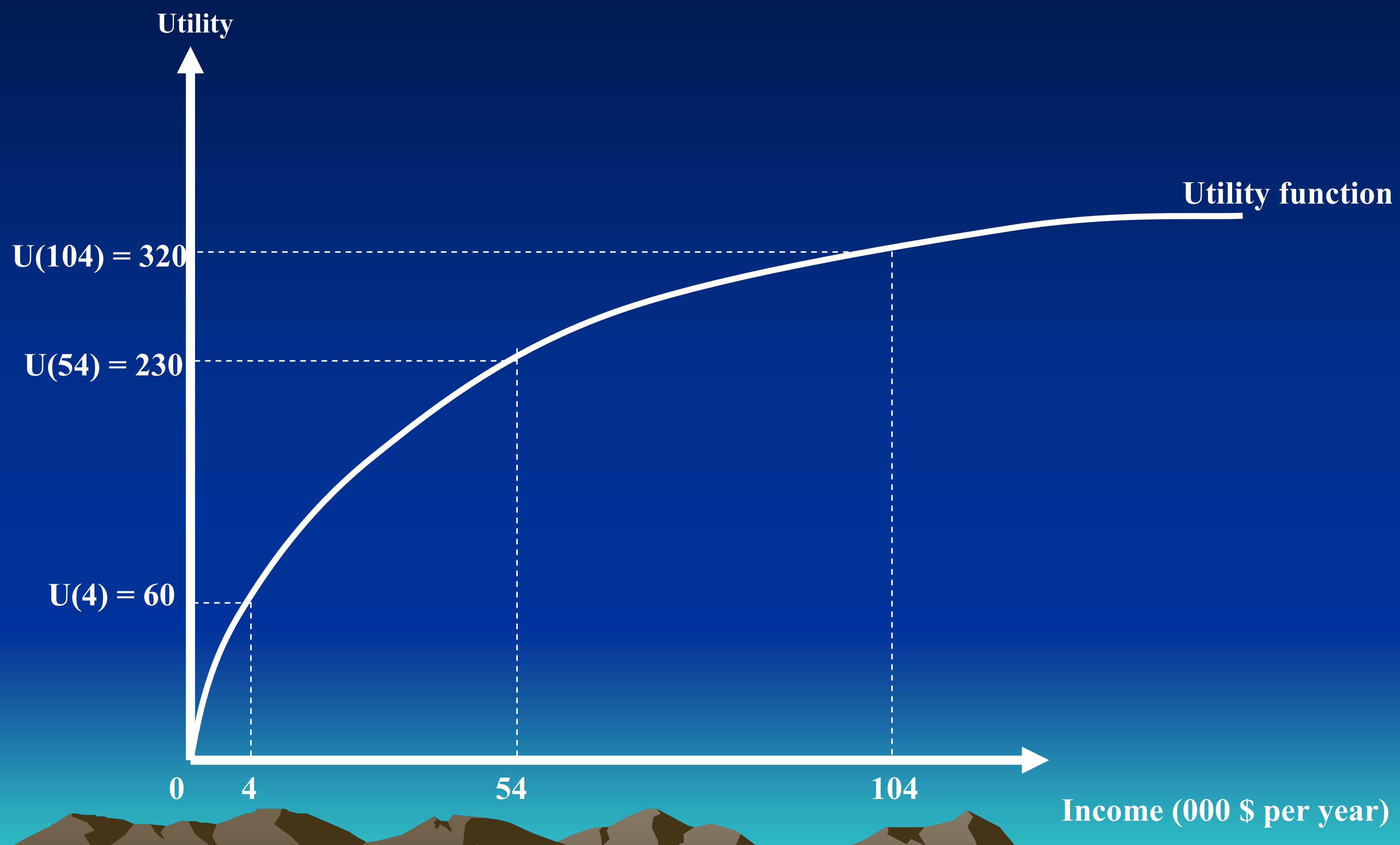
The squared deviation of losing is:

$(\$25 - \$50)^2 = 25^2 = 625.$

The variance is:

$(2500 \times .33) + (625 \times .67) = 1250$

Example: Evaluating Risky Outcomes



Evaluating Risky Outcomes

Example: Work for IBM or Amazon.Com?

Suppose that individuals facing risky alternatives attempt to maximize expected utility, i.e., the probability-weighted average of the utility from each possible outcome they face.

U(IBM) = U($54,000) = 230

U(Amazon) =

.5xU($4,000) + .5xU($104,000) =

.5(60) + .5(320) = 190

Note:

EV(Amazon) = .5($4000)+.5($104,000) = $54,000

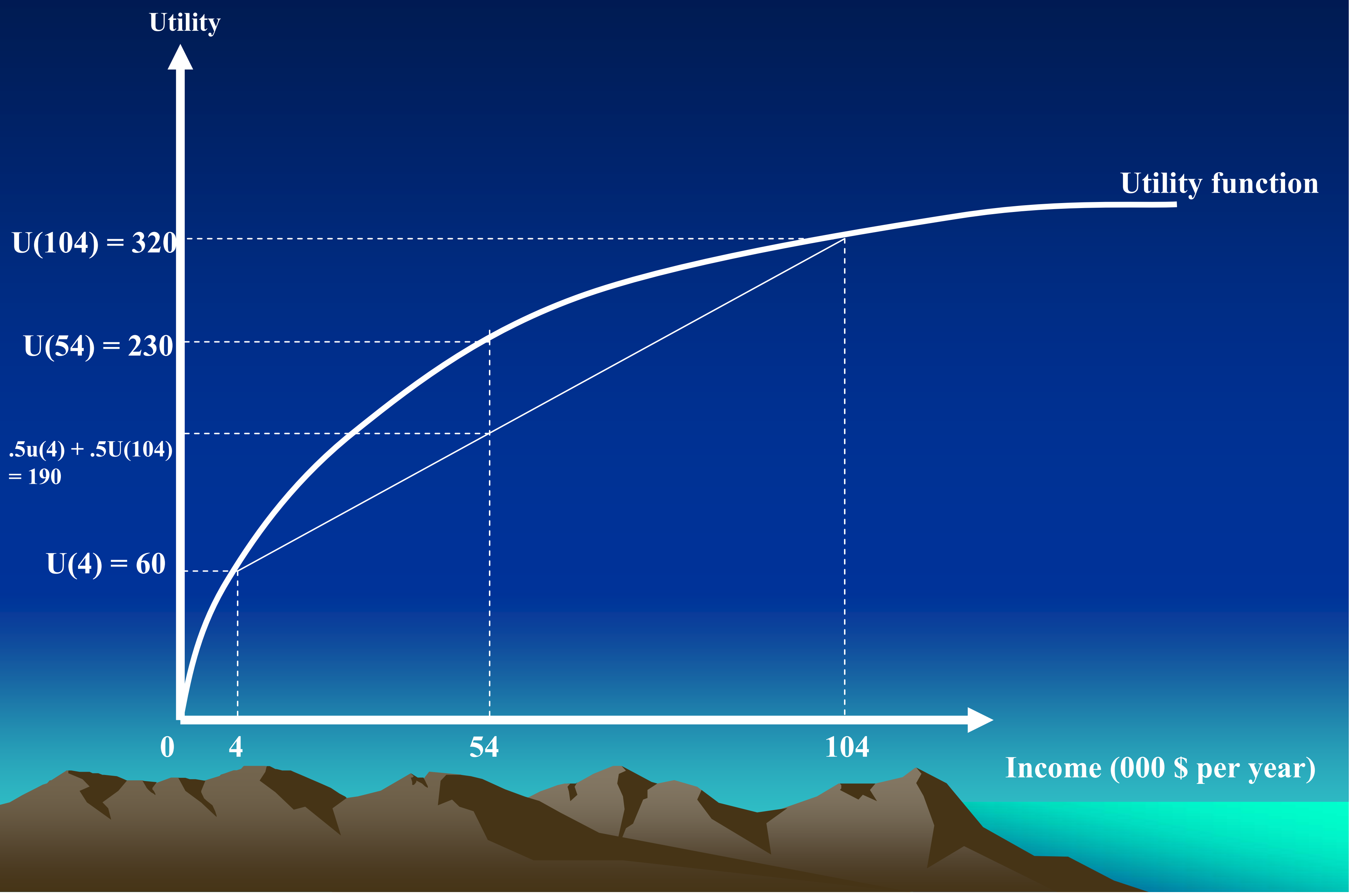
Notes:

- Utility as a function of yearly income only
- Diminishing marginal utility of income

Expected utility over money

- Assume two monetary prizes $x<y$, p=prob(x)
- Expected monetary value of lottery:
  - EL=(1-p)x+py=x+p(y-x)
  - p=(EL-x)/(y-x)
- Expected utility of lottery
  - Eu(L)=(1-p)u(x)+pu(y)
    =u(x)+p(u(y)-u(x))=u(x)+(u(y)-u(x))(EL-x)/(y-x)
- Thus Eu(L) is linear in EL

Example: Evaluating Risky Outcomes



Risk Preferences

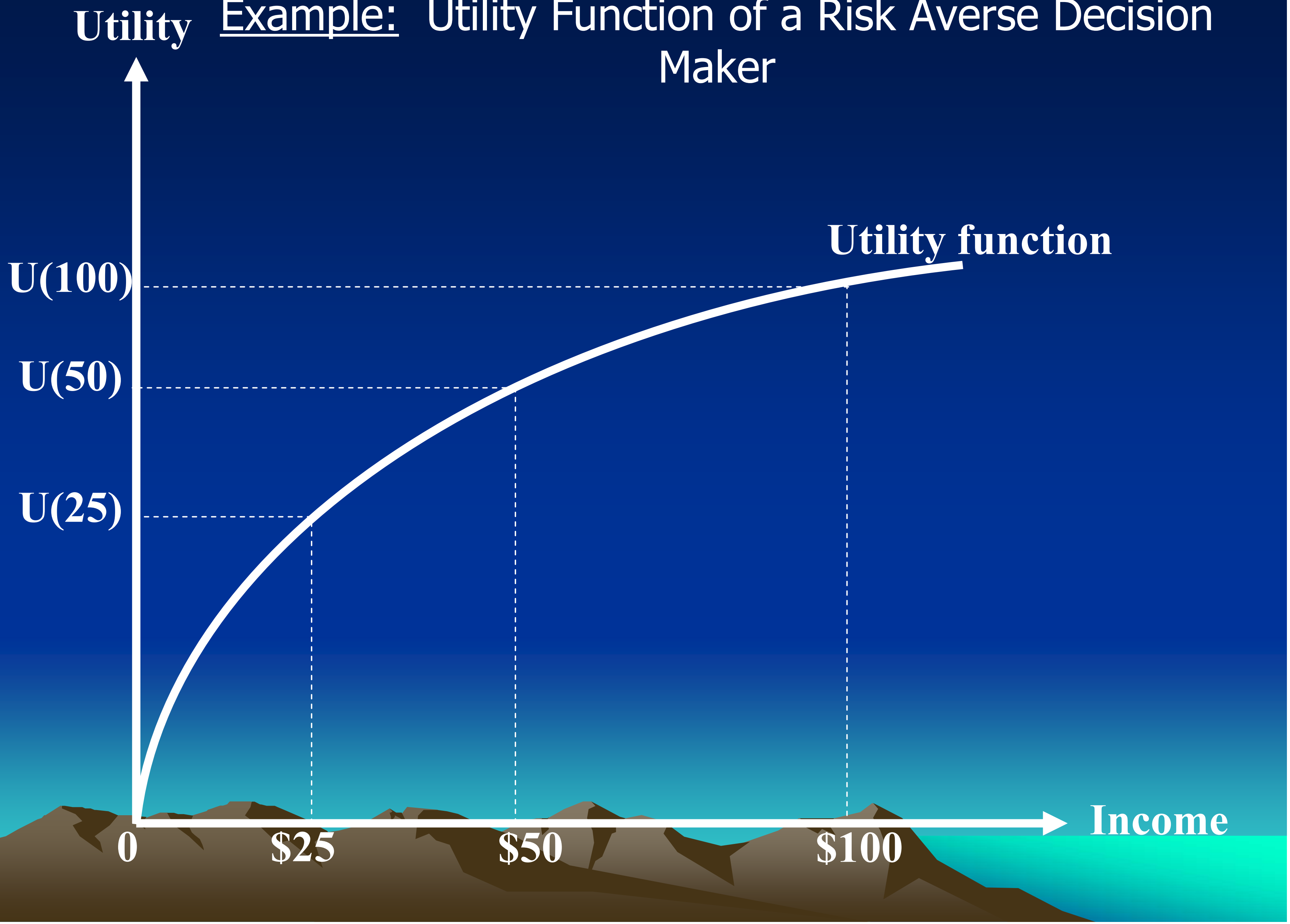
Definition: The risk preferences of individuals can be classified as follows:

An individual who prefers a sure thing to a lottery with the same expected value is **risk averse**

An individual who is indifferent about a sure thing or a lottery with the same expected value is **risk neutral**

An individual who prefers a lottery to a sure thing that equals the expected value of the lottery is **risk loving** (or **risk preferring**)

Example: Utility Function of a Risk Averse Decision Maker



Riskaverse ⇔ concave utility fn. ⇔ decreasing marginal utility

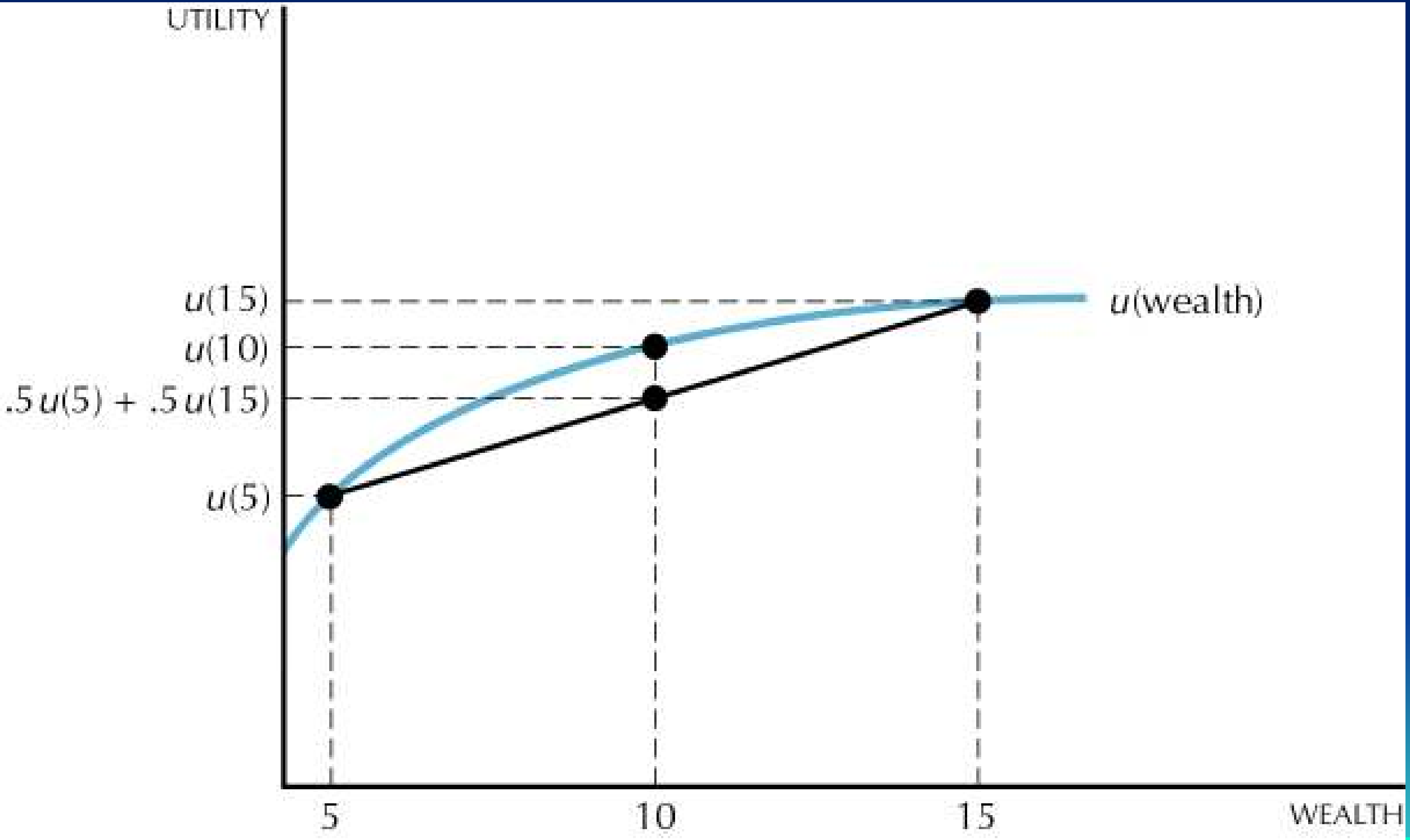
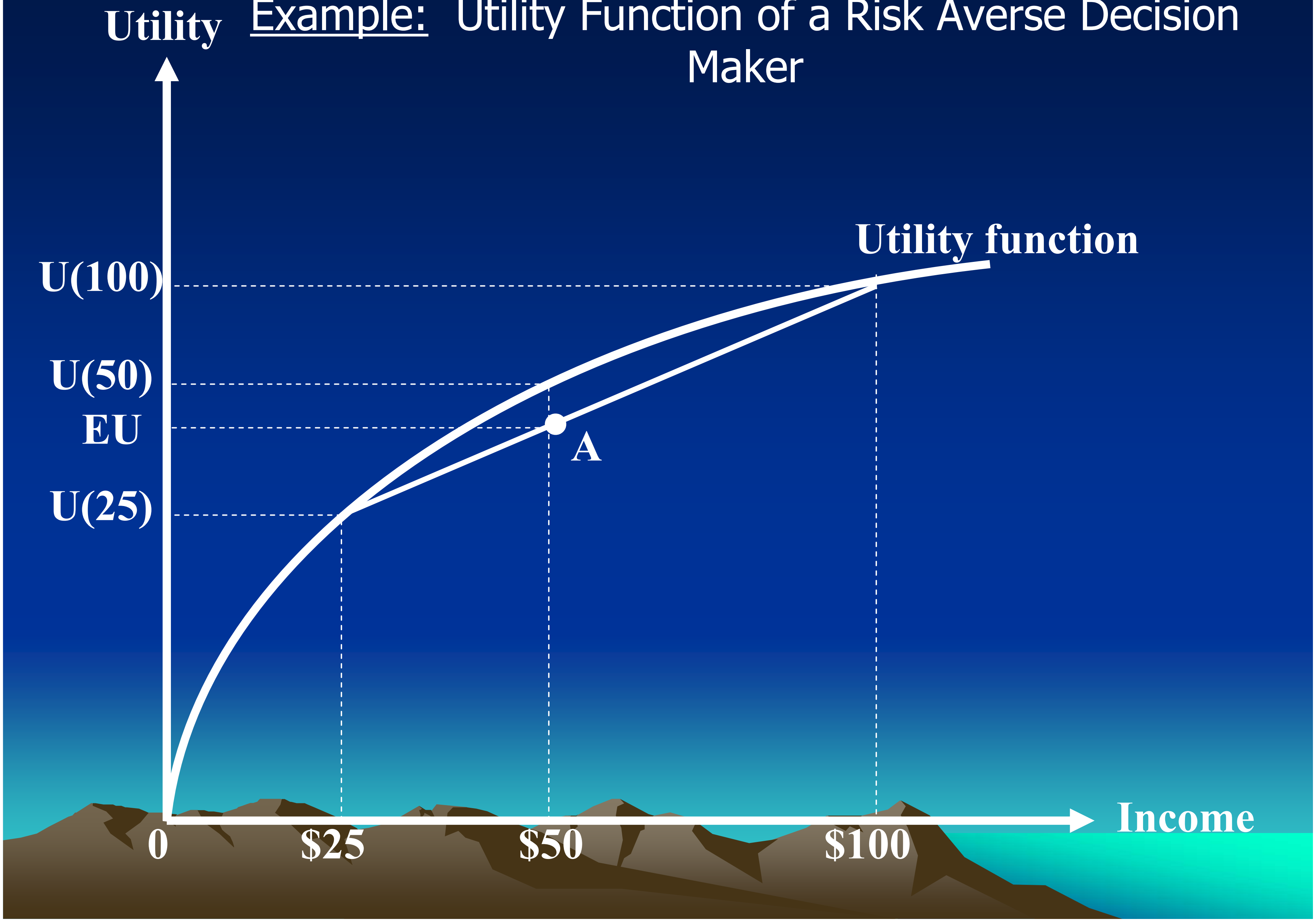
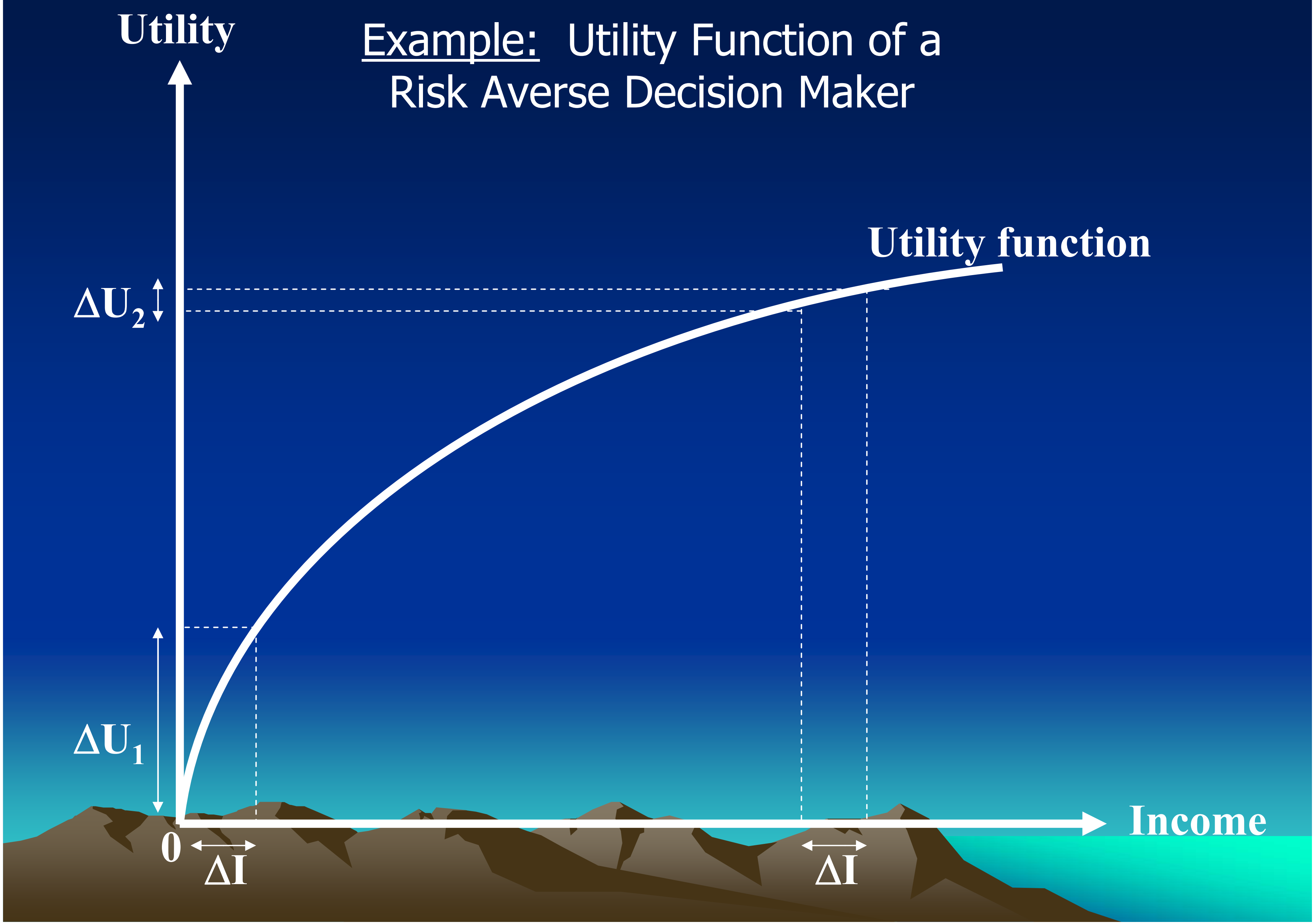


Figure 12.2 Risk aversion

Example: Utility Function of a Risk Averse Decision Maker

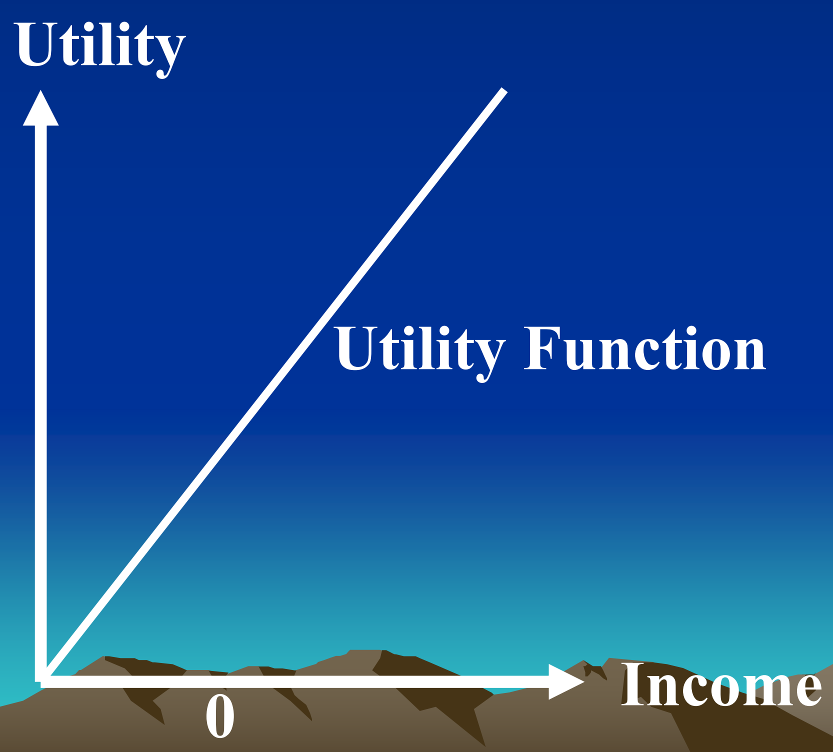


Example: Utility Function of a Risk Averse Decision Maker

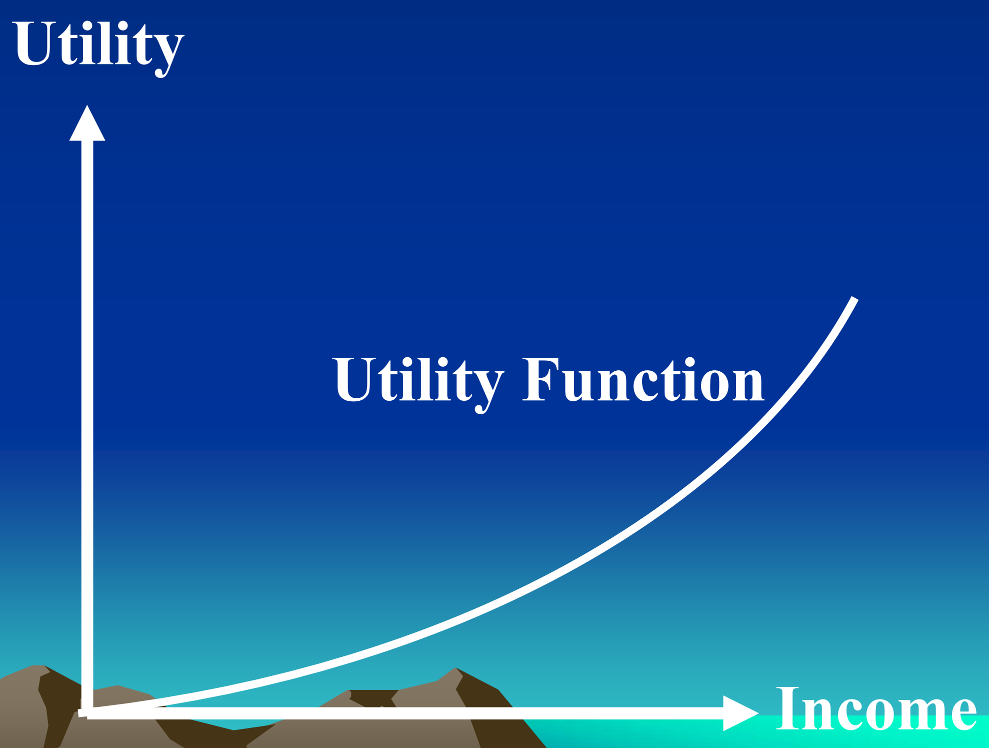


Example: Utility Functions of a Risk Neutral and a Risk Loving Decision Maker

Risk Neutral Preferences



Risk Loving Preferences



Example:

Suppose that an individual must decide between buying one of two stocks: the stock of an Internet firm and the stock of a Public Utility. The values that the shares of the stock may take (and, hence, the income from the stock, I) and the associated probability of the stock taking each value are:

Internet firm		Public Utility	
I	Probability	I	Probability
$80	.3	$80	.1
$100	.4	$100	.8
$120	.3	$120	.1

Which stock should the individual buy if she has utility function $U = (100I)^{1/2}$? Which stock should she buy if she has utility function U = I?

EU(Internet) = .3U(80) + .4U(100) + .3U(120)

EU(P.U.) = .1U(80) + .8U(100) + .1U(120)

a. $U = (100I)^{1/2}$:

$U(80) = (8000)^{1/2} = 89.40$
$U(100) = (10000)^{1/2} = 100$
$U(120) = (12000)^{1/2} = 109.5$

→ EU(Internet) = .3(89.40)+.4(100)+.3(109.50) = 99.70

→ EU(P.U.) =

.1(89.40) + .8(100) + .1(109.50) = 99.90

The individual should purchase the public utility stock.

b. U = I:

→ EU(Internet) = .3(80)+.4(100)+.3(120)=100

→ EU(P.U.)

.1(80) + .8(100) + .3(120) = 100

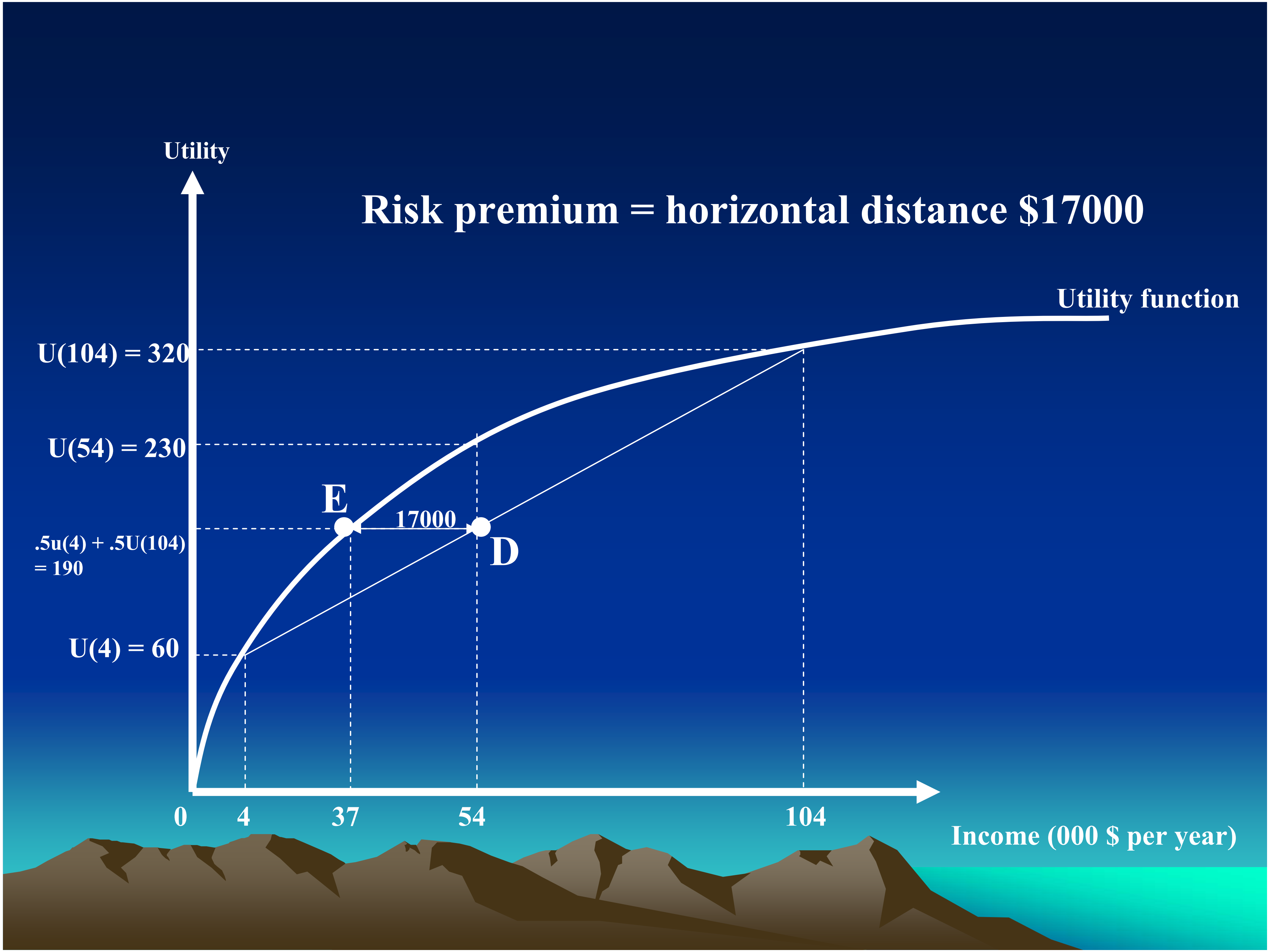
This individual is indifferent between the two stocks.

Avoiding Risk

1. Insurance

    <u>Example:</u> The Risk Premium

    Refer to Graph on Next Slide


Utility
Risk premium = horizontal distance $17000
Utility function
U(104) = 320
U(54) = 230
E
17000
.5u(4) + .5U(104)
= 190
D
U(4) = 60
0
4
37
54
104
Income (000 $ per year)

Definition: The **risk premium** of a lottery is the necessary difference between the expected value of a lottery and the sure thing (the "certainty equivalent") so that the decision maker is indifferent between the lottery and the sure thing.

$$pU(I_1) + (1\text{-}p)U(I_2) = U(pI_1 + (1\text{-}p)I_2 - RP)$$

$$EU(L)=U(EL\text{-}RP)$$

NB: We can see from the graph that the larger the variance of the lottery, the larger the risk premium

Example: Computing a Risk Premium

$U = I^{1/2}$; $p = .5$

$I_1 = \$104,000$
$I_2 = \$4,000$

a. *Verify that the risk premium for this lottery is approximately $17,000*

$$.5(104{,}000)^{1/2} + .5(4{,}000)^{1/2} = (.5(104{,}000) + .5(4{,}000) - RP)^{1/2}$$

$$\$192.87 = (\$54{,}000 - RP)^{1/2}$$

$$\$37{,}198 = \$54{,}000 - RP$$

$$RP = \$16{,}802$$

b. *Let I_1 = \$108,000 and I_2 = \$0. What is the risk premium now?*

$$.5(108{,}000)^{1/2} + 0 = (.5(108{,}000) + 0 - RP)^{1/2}$$

$$.5(108{,}000)^{1/2} = (54{,}000 - RP)^{1/2}$$

RP = \$27,000

(Risk premium rises when variance rises, EV the same...)

The Demand for Insurance

Lottery: \$50,000 if no accident (p = .95)
\$40,000 if accident (1-p = .05)

(i.e. "Endowment" is that income in the good state is 50,000 and income in the bad state is 40,000)

EV = .95(\$50000)+.05(\$40000) = \$49,500

Insurance: coverage = $10,000
Price = $500

$49,500 sure thing.

Why?

In a good state, receive 50000-500 = 49500

In a bad state, receive 40000+10000-500=49500

If you are risk averse, you prefer to insure this way over no insurance…why?

- *Full coverage (⇔ no risk so prefer all else equal)*

- *<u>Definition:</u> A **fairly priced** insurance policy is one in which the insurance premium (price) equals the expected value of the promised payout. i.e.:*

$$500 = .05(10{,}000) + .95(0)$$

Here, then the insurance company expects to break even *and* assumes all risk! *Why would an insurance company ever offer this policy?*

The Supply of Insurance

- *Does not require risk lovers in population*

Price above the "fair" price

The Supply of Insurance

Definition: **Adverse Selection** ("Hidden knowledge") is "*opportunism"* characterized by an informed person's benefiting from trading or otherwise contracting with a less informed person who does not know about an *unobserved characteristic* of the informed person.

Example: good and bad driver

Definition: **Moral Hazard** is "*opportunism*" characterized by an informed person's taking advantage of a less informed person through an *unobserved action*.

Example: Insurance and risky behavior

Adverse Selection and Market Failure

Lottery: \$50,000 if no blindness (p = .95)
\$40,000 if blindness (1-p = .05)

EV = \$49,500

(fair) insurance: Coverage = \$10,000
Price = \$500

\$500 = .05(10,000) + .95(0)

Suppose that each individual's probability of blindness *differs* and is known to the individual, but not to the insurance company. The insurance company only knows the average probability.

EXTREME CASE: Each individual knows before accepting the insurance whether he will have the blindness or not.

→ Only the “bad risks” accept the insurance. With the above contract the insurance no longer breaks even. The market breaks down.

The Value of Information

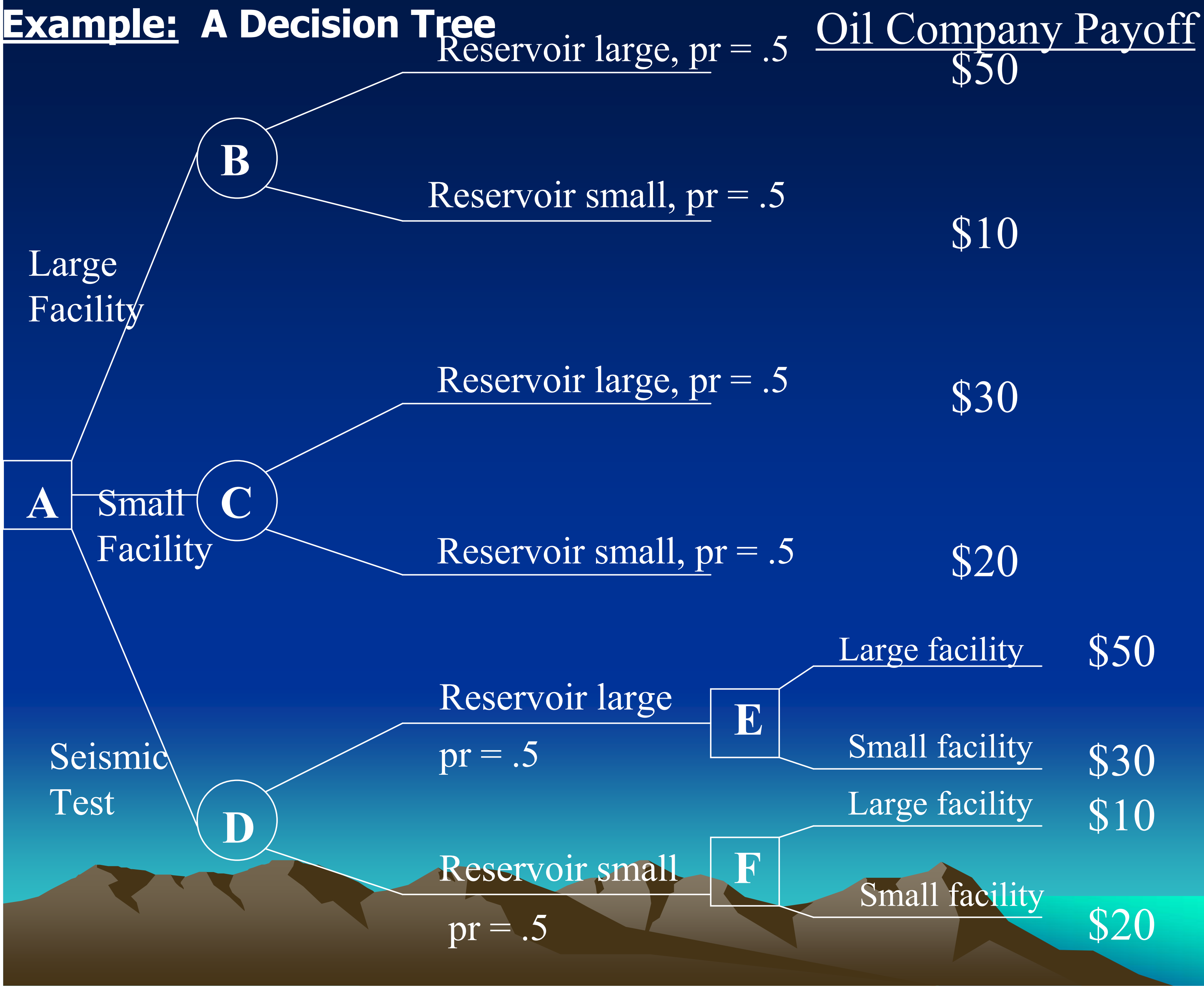
Consider the following Decision Problem:

Definition: A **decision tree** is a diagram that describes the options available to a decision maker, as well as the risky events that can occur at each point in time.

Elements of the decision tree:

1. Decision Nodes
2. Chance Nodes
3. Probabilities
4. Payoffs

Example: A Decision Tree



We analyze decision problems by working backward along the decision tree to decide what the optimal decision would be…

Steps in constructing and analyzing the tree:

1. map out the decision and event sequence
2. identify the alternatives available for each decision
3. identify the possible outcomes for each risky event
4. assign probabilities to the events
5. identify payoffs to all the decision/event combinations
6. find the optimal sequence of decisions

Definition: The **value of perfect information** is the increase in the decision maker's expected payoff when the decision maker can -- at no cost -- obtain information that reveals the outcome of the risky event.

Example:

Expected payoff to conducting test: $35M
Expected payoff to not conducting test: $30M

The value of information: $5M

The value of information reflects the value of being able to tailor your decisions to the conditions that will actually prevail in the future. It should represent the agent's willingness to pay for a "crystal ball".

Summary

1. We can think of risky decisions as lotteries.

2. Expected utility theory is an adjustment of standard utility theory, with additional assumptions appropriate for contingent consumption

2. Individuals are assumed to maximize expected utility when faced with risk.

3. Individuals differ in their attitudes towards risk: those who prefer a sure thing are risk averse. Those who are indifferent about risk are risk neutral. Those who prefer risk are risk loving.

Summary Ctd...

4. Insurance can help to avoid risk. The optimal amount to insure depends on risk attitudes.

5. The provision of insurance by individuals does not require risk lovers.

6. Adverse Selection and Moral Hazard can cause inefficiency in insurance markets.

7. We can calculate the value of obtaining information in order to reduce risk by analyzing the expected payoff to eliminating risk from a decision tree and comparing this to the expected payoff of maintaining risk.