Game Theory

- Game theory models strategic behavior by agents who understand that their actions affect the actions of other agents.
- Aumann: Interactive Decision Theory
- Strategic interaction
- Been used to study
  - Company behavior
  - Military strategies.
  - Bargaining/Negotiations
  - Biology
  - Auction design
- A game consists of players, strategies, and payoffs.

Battle of Bismarck Sea

		Imamura: sail North	Imamura: sail South
Kenney	search North	2, -2	2, -2
	search South	1, -1	3, -3

Imamura wants to run convoy from Rabaul to Lae

Battle of Bismarck Sea

		Imamura: sail North	Imamura: sail South
Kenney	search North	2, -2	2, -2
	search South	1, -1	3, -3

The Capacity Game

		GM: DNE	GM: Expand
Ford	DNE	18, 18	15, 20
	Expand	20, 15	16, 16

Where would the companies like to be?

The Capacity Game

		GM: DNE	GM: Expand
Ford	DNE	18, 18	15, 20
	Expand	20, 15	* 16, 16

What is the equilibrium here?
Where would the companies like to be?

The Prisoners' Dilemma

Clyde

Bonnie	**S**	**C**
S	-5 / -5	-1 / -15
C	-15 / -1	-10 / -10

Bonnie and Clyde are caught.
They can confess or be silent.

The Prisoners' Dilemma

Clyde

Bonnie	**S** →	**C**
S ↓	-5 / -5	-1 / -15 ↓
C	-15 / -1 →	* -10 / -10

Bonnie and Clyde are caught.
They can confess or be silent.

War

Mars

Venus	**Not Shoot** →	**Shoot**
Not Shoot ↓	-5 / -5	-1 / -15 ↓
Shoot	-15 / -1 →	* -10 / -10

- **Are we doomed to the bad outcome?**
- **Not in trench warfare of WWI.**
- **This happens since the game is repeated.**
- **Tit-for-tat can work (see later lectures).**
- **Not so easy if uncertainty of action.**

Nash equilibrium *

- Not a self-destroying prophecy (Selten)
- A combination of strategies for each player such that it is optimal for each player to stick to his strategy if he expects everybody else to do it.

Nash equilibrium *

- A Nash equilibrium is a pair of strategies
  - Where no player has an incentive to deviate.
  - (or) Given other equilibrium strategies a player would (be willing to) choose his equilibrium strategy.
- A pure strategy equilibrium is where each player only chooses a particular strategy with certainty. Is there always a pure strategy Nash equilibrium? Is it unique?

Nash equilibrium

- Is there always a pure strategy Nash equilibrium?

Penalty Kick

		Goalie	
		Dive L	Dive R
Kicker	Kick L	-1, 1	1, -1
	Kick R	1, -1	-1, 1

- **A Kicker can kick a ball left or right.**
- **A Goalie can dive left or right.**
- **Zero-sum game**

Penalty Kick

		Goalie	
		Dive L	Dive R
Kicker	Kick L	-1, 1	1, -1
	Kick R	1, -1	-1, 1

- **A Kicker can kick a ball left or right.**
- **A Goalie can dive left or right.**

Mixed Strategy equilibrium

- Happens in the Penalty kick game.
- Notice that if the Kicker kicks 50-50 (.5L+.5R), the Goalie is indifferent to diving left or right.
- If the Goalie dives 50-50 (.5L+.5R), the Kicker is indifferent to kicking left or right.
- Thus, (.5L+.5R,.5L+.5R) is a mixed-strategy N.E.
- Nash showed that there always exists a Nash equilibrium.

Do you believe it?

- Do they really choose only L or R? Yes. Kickers 93.8% and Goalie 98.9%.
- Kickers are either left or right footed. Assume R means kick in "easier" direction. Below is percentage of scoring.

	Dive L	Dive R
Kick L	58.3	94.97
Kick R	92.91	69.92

- Nash prediction for (Kicker, Goalie)=(41.99L+58.01R, 38.54L+61.46R)
- Actual Data =(42.31L+57.69R, 39.98L+60.02R)

Parking Enforcement Game

		Student Driver	
		Park OK	Park in Staff
University	Check	-5, -5	5, -95
	Don't	5, -5	0, 5

- **Student can decide to park in staff parking.**
- **University can check cars in staff parking lot.**

What happens?

- If the University checks, what do the students do?
- If the students park ok, what does the Uni do?
- If the uni doesn't check, what do the students do?
- If the students park in the staff parking, what does the uni do?
- What is the equilibrium of the game?
- What happens if the university makes it less harsh a punishment to only –10. Who benefits from this? Who is hurt by this?

Best replies of students

- Suppose university controls with probability 1-p.
- Payoff for student
  - If he parks ok: -5
  - If he parks in staff area: -95(1-p)+5p
- It is better to park ok if

  -5> -95(1-p)+5p

  90>100p, p<0.9, 1-p>0.

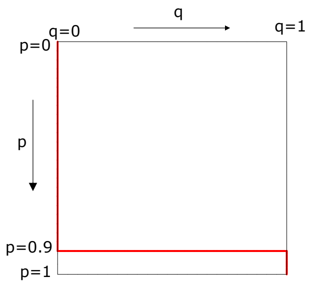
  i.e. if student gets controlled with more than 10% probability.

Best replies of students

It is optimal for the student

- to park ok if and only if p<0.9;
- to park in staff area if and only if p>0.9
- To randomize between the two options in any way the student likes if and only if p=0.9

Best replies of students



Terminology

- IO: Response function
- (not really a function): best reply *correspondence*

Best replies of university

- Suppose student parks in staff area with probability q.
- Payoff for university
  - If university checks: -5(1-q)+5q
  - If not: 5(1-q)
- It is better to check if

  -5(1-q)+5q>5(1-q); 5q>10(1-q);

  15q>10; q>2/3

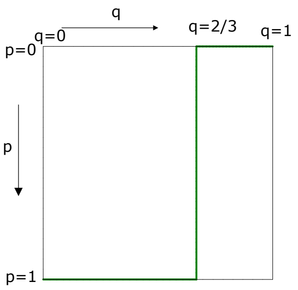
  i.e. if student parks in staff area with a probability greater than 66.6%

Best replies of university

It is optimal for the university

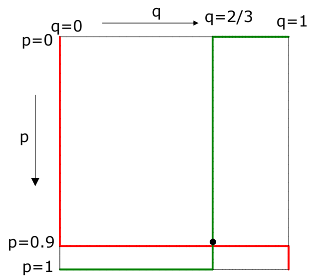
- to check if and only if q>2/3;
- not to check if and only if q<2/3
- to randomize between the two options in any way if and only if q=2/3

Best replies of university



q: probability to park in staff area

Nash equilibrium



Nash equilibrium in mixed strategies: (0.1C+0.9NC,(1/3)OK+(2/3)S)

Nash Equilibrium 1

- Student parks legally 1/3 of the time and the Uni checks 1/10 of the time.
- Penalty:95

Change in penalty?

- Suppose the because of students parking the penalty has been recently increased from 10 to 95. To find out how this has improved the situation we have to calculate the previous Nash equilibrium at the penalty 10….

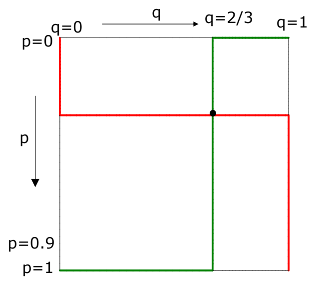
Parking Enforcement Game

Student Driver

University	Park OK	Park in Staff
Check	-5, -5	5, -10
Don't	5, -5	0, 5

- **Student can decide to park in staff parking.**
- **University can check cars in staff parking lot.**

Nash equilibrium



Nash equilibrium in mixed strategies: ((1/3)C+(2/3)NC,(1/3)OK+(2/3)S)

Answer

- With lower penalty, student parked legally 1/3 of the time and the uni checked 2/3 of the time.
- Who's expected payoff changes? No one.
- Parking "inspectors" were more "lazy", probability of "illegal" parking has not changed…

The watchman and the thief

- Experimental observations confirm with comparative statics, but there are strong own-payoff effects.

Stag hunt

Hunter 2

Hunter 1	Stag	Rabbit
Stag	9, 9 *	0, 7
Rabbit	7, 0	7, 7 *

Rousseau

Pure coordination

Hunter 2

Hunter 1	Stag	Rabbit
Stag	9, 9 *	0, 0
Rabbit	0, 0	7, 7 *

Minimal effort games

- Payoff: minimal effort of all minus 0.5x(my effort)

Experimental evidence:

- Large groups: race to the bottom
- Small groups, fixed matching: coordination on high outcomes

Coordination Problem

Jim

Sean	VHS	Beta
VHS	1, 1 *	0, 0.5
Beta	0.5, 0	2, 2 *

- **Jim and Sean want to have the same VCR.**
- **Beta is a better technology than VHS.**

Network experiment

- Multiple equilibria
- 0%, 40% or 60% participation
- Classroom experiments seems to conform with 60% participation

Battle of the sexes

Bob \ Alice	Boxing	Ballet
Boxing	3, 1 *	0, 0
Ballet	0, 0	1, 3 *

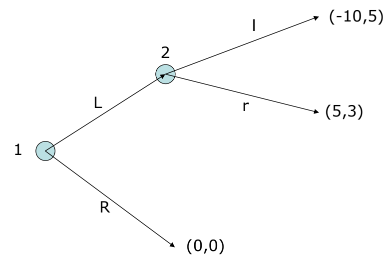
All these games have also mixed strategy equilibria. Here: Both play their preferred strategy with 2/3 Probability. Then each party gets the same expected payoff with each strategy and so all strategies are optimal

Chicken

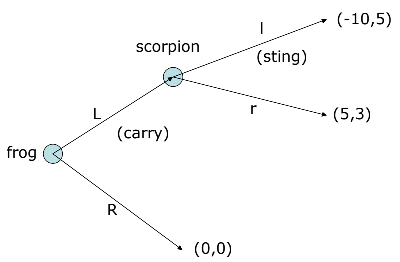
Teen 1 \ Teen 2	Chicken	Dare
Chicken	5, 5	4, 7 *
Dare	7, 4 *	0, 0

Bertrand Russel

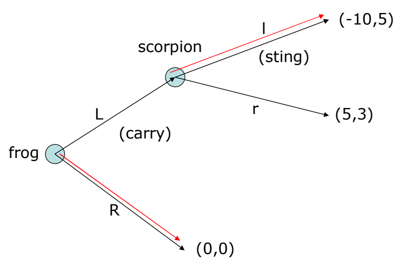
Extensive games



The frog and the scorpion

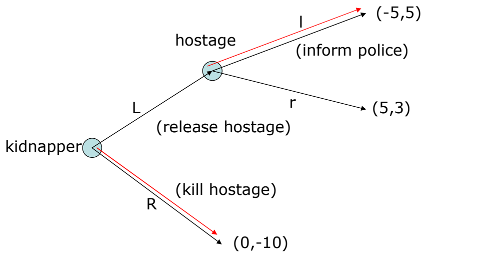


The subgame-perfect equilibrium point (Selten 1965)



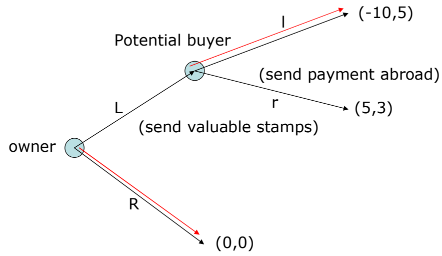
(do not carry, sting)

Kidnapping



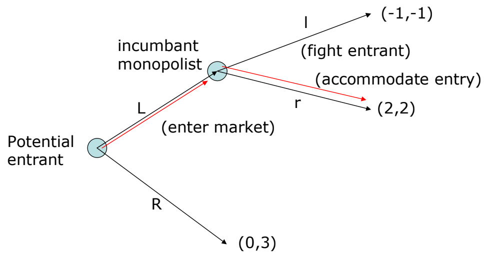
(do not carry, sting)

e-bay



Problem: incredible promise

Potential entry



Subgame perfect Nash equilibrium: (L,r)

Problem: incredible threat

Other Nash equilibria?

1 \ 2	l	r
L	-1, -1	2, 2 **
R	0, * 3	0, 3

(L,r) subgame-perfect Nash equilibrium, (R,r) also Nash equilibrium, but not subgame perfect.
Schelling, Selten: (L,l) is not a credible Nash equilibrium because if 2 would have to move he would play R. (non-credible threat)

Repeated Games

Venus \ Mars	Not Shoot	Shoot
Not Shoot	-5, -5	-15, -1
Shoot	-1, -15	-10, * -10

Experiments on PD

- Pure one shot game versus random matching: cooperation dies out quickly
- Mild gender effects
- Does Studying Economics Inhibit Cooperation? Frank, Gilovich, Regan claim that economic students are less cooperating than other students
- Now: THEORY of repeated PD's with fixed matching

Repeated games

- 1. if game is repeated with same players, then there may be ways to enforce
  a better solution to prisoners' dilemma
- 2. suppose PD is repeated 10 times and people know it
  - then backward induction says it is a dominant strategy to cheat every
    round
- 3. suppose that PD is repeated an indefinite number of times
  - then it may pay to cooperate
- 4. Axelrod's experiment: tit-for-tat

Continuation payoff

- Your payoff is the sum of your payoff today plus the discounted "continuation payoff"
- Both depend on your choice today
- If you get punished tomorrow for bad behaviour today and you value the future sufficiently highly, it is in your self-interest to behave well today
- Your trade-off short run against long run gains.

Infinitely repeated PD

- Discounted payoff, $0<d<1$ discount factor ($d^0=1$)
- Normalized payoff: $(d^0u_0+ d^1u_1+ d^2u_2+\ldots +d^tu_t+\ldots)(1-d)$
- Geometric series:

$(d^0+ d^1+ d^2+\ldots +d^t+\ldots)(1-d)$

$=(d^0+ d^1+ d^2+\ldots +d^t+\ldots)$

$-(d^1+ d^2+ d^3+\ldots +d^{t+1}+\ldots)= d^0=1$

Infinitely repeated PD

- Constant "income stream" $u_0= u_1=u_2=\ldots =u$ each period yields total normalized income u.
- Grim Strategy: Choose "Not shoot" until someone chooses "shoot", always choose "Shoot" thereafter
- Is non-forgiving, problem: Not "renegotiation proof"

- Payoff if nobody shoots:

$(-5d^0- 5d^1-5d^2-\ldots -5d^t+\ldots)(1-d)=-5$

$=-5(1-d)-5d$

- Maximal payoff from shooting in first period ($-15<-10$!):

$(-d^0-10d^1-10d^2-\ldots -10d^t-\ldots)(1-d)$

$=-1(1-d)-10d$

- $-1(1-d)-10d< -5(1-d)-5d$ iff $4(1-d)<5d$ or $4<9d$ $d>4/9 \approx 0.44$
- Cooperation can be sustained if $d> 0.45$, i.e. if players weight future sufficiently highly.

Selten / Stöcker,1986

- Students play 25 times a 10-round fixed pair repeated PD
- New, random assignment for each play of the repeated game.
- Results initially: chaos, players learn to cooperate and use punishments
- With experience: cooperation emerges
- With more plays: players learn to defect in the last periods (end-effect)
- Final periods of defection get longer