BEE2017 Intermediate Microeconomics
week 1
A simple example of price discrimination.

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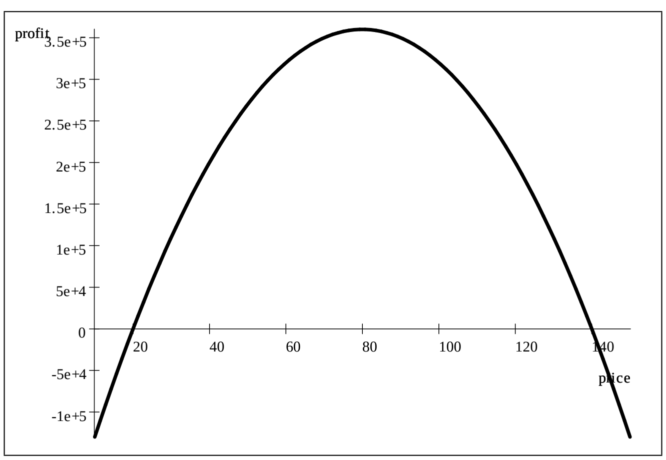
We consider the case of a monopolist who produces a local newspaper. His demand function is given by

$$Q = 14000 - 100P$$

where Q is the number of newspapers and P the price in pence. We can safely assume that nobody wants to buy a second newspaper even at a discount. So Q is the number of actual buyers. For instance, if the monopolist charges 90p for the paper, five thousand people in the city will buy the newspaper (since $14000 - 9000 = 5000$). Stated differently, there are 5000 consumers who are willing to pay 90p or more for the newspaper. We can also write

$$P = 140 - Q/100$$

which is called the *inverse* demand function. This information tells us, for instance, that in order to sell 7000 copies (or more) the monopolist must take a price no higher than 70p.



The profit function

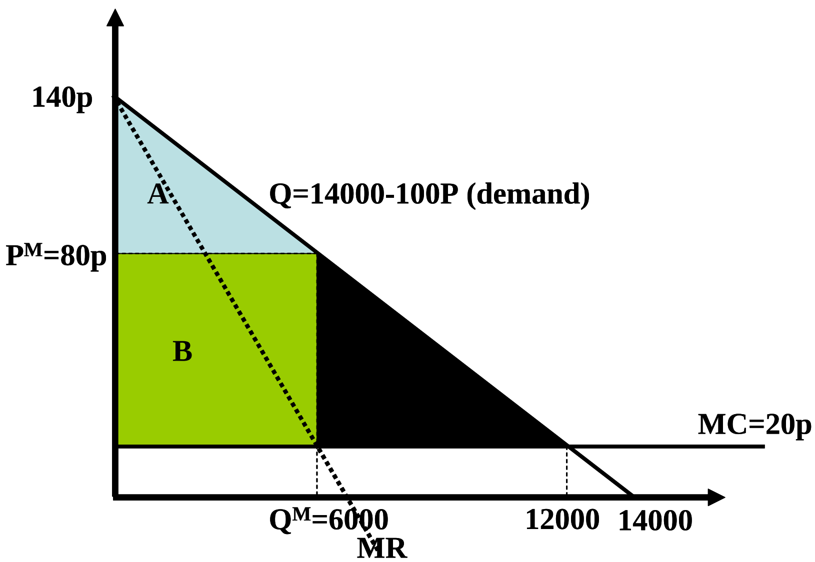
We assume that there are no fixed costs and that there are constant marginal costs of 20p per newspaper, i.e., $c = 20$.

If he sets the price P the monopolist makes the total revenue (price times units sold) $PQ = P(14000 - 100P)$ and has the total costs $cQ = 20Q = 20(14000 - 100P)$. His profit is hence.

$$\Pi = PQ - cQ = (P - c)Q = (P - 20)(14000 - 100P)$$

We can draw the graph of this profit function:The graph shows that the maximum occurs at $P^M = 80$p.[1] At this price the quantity demanded is $Q^M = 14000 - 80 = 60$ and the profit is $\Pi^M = (80 - 20) \times (14000 - 8000) = 60 \times 6000 = 360000$. Since 6000 consumers yield a net pay of 60p each, the profit is 360000p or £3.600.

$\Pi^M = 36000$ is the area of the square B in the following graph and gives the producer surplus. (Recall that producer surplus and profits coincide if there are no fixed costs.) Consumer surplus is the area of the triangle A which has base 6000 and height $(140 - 80)$. Consumer surplus is hence $\frac{6000 \times (140-80)}{2} = \frac{360000}{2} = 180000$p, which corresponds to £1.800.



[1] If you unsure, calculate the profit for $P = 20, 40, 60, ..., 140$ yourself and draw the graph. Here are some alternative (mathematically safer) ways to find the maximum: a) Since $\Pi = 100(P - 20)(140 - P)$ profit is zero for $P = 20$ and for $P = 140$. (Why is this intuitive?) The maximum (or minimum) of this quadratic function must hence be *exactly* in the middle between these two roots, i.e. at $\frac{20+140}{2} = 80$. (Intuitively this holds because the graph of a quadratic function is symmetric around its maximum (or minimum) and so the two roots must be at equal distance from the maximum or minimum.) For a quadratic function of type $aP^2 + bP + c$ the sign of the constant a decides whether the graph of the function is upward or downward bowed. If a is positive the graph is upward bowed like a smile "$\smile$" and has a minimum. If a is negative it is downward bowed like a frown "$\frown$" and has a maximum. In our case this is the case here since In our case coefficient $(P - 20)(140 - P) = -P^2 + 160P - 2800$, so $a = -1$ and we have a maximum.

b) You can differentiate the profit function to show that $P = 8$ 0is a profit maximum.

c) The inverse demand function is $P = 140 - Q/100$. Whenever the inverse demand function is of the form $P = A - BQ$ the marginal revenue curve is given by $MR = A - 2BQ$, in our case by $MR = 140 - 2Q/100$. Equating MR and MC give $140 - 2Q/100 = 20$ or $Q = \frac{140-20}{2} = 60$. thus profit is maximized at the *quantity* $Q^M = 60$ where the price is $P^M = 140 - 60 = 80$. See the learning-by-doing exercise 12.1 in the textbook for this approach.

In our example we can illustrate why consumer surplus measures something reasonable. Each person has a maximum willingness to pay. His maximum willingness to pay minus the price he actually pays is his gain. We claim that if we sum these gains over all consumers who buy the product, we get the consumer surplus. Calculating this sum is not easy. We can, however, use a trick which the famous mathematician Gauss used at school to add up the numbers one to hundred when he was six years old. This works as follows.

6000 consumers buy the good at 80p. The first one would be willing to pay up to 139.99p=$\left(140-\frac{1}{100}\right)$p for it, the last one 80p=$\left(140-\frac{6000}{100}\right)$p. The first one wins 59.99p, the last one 0p. Together they gain 59.99p. The second consumer would be willing to pay up to 139.98p=$\left(140-\frac{2}{100}\right)$p, the second last 80.01=$\left(140-\frac{5999}{100}\right)$p. The second one gains 59.98p, the second last one 1p. Together they gain also 59.99p. The third consumer would be willing to pay up to 139.97p=$\left(140-\frac{3}{100}\right)$p, the third last 80.02=$\left(140-\frac{5998}{1000}\right)$p. The third one gains 59.997p, the third last one 2p. Together they gain also 59.999p. We can continue this calculation until we reach consumer 3000 who would be willing to pay 110p=$\left(140-\frac{3000}{100}\right)$p while the 3000th-last consumer (i.e., number 3001) would be willing to pay 109.99p=$\left(140-\frac{3001}{100}\right)$p. Number 3000 gains 30p, number 3001 gains 29.99p, overall the gain is 59.99p. Thus we have 3000 pairs who gain 59.99p each. Consumer surplus is

$$3000 \times 59.99\text{p} \approx 3000 \times 60\text{p} = £1800$$

The welfare loss in this example is the area of the black triangle $\frac{(14000-6000)\times(80-20)}{2} = \frac{360000}{2} = £1800$.

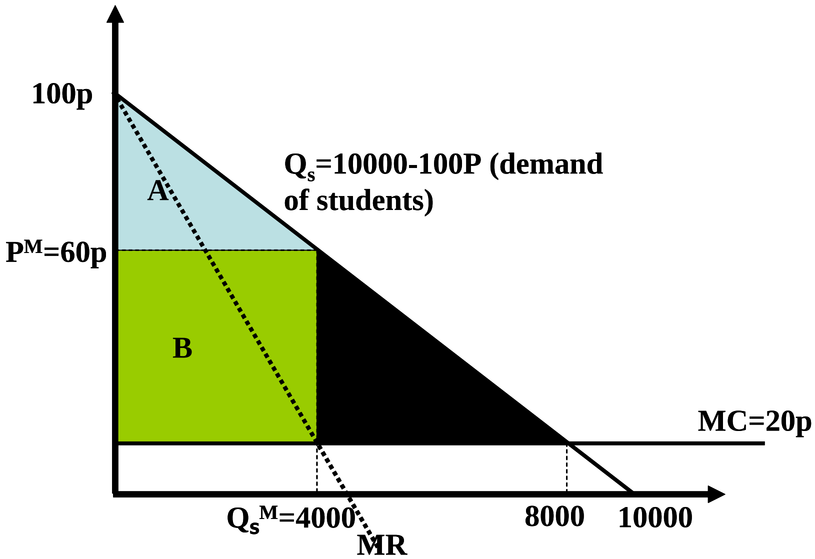
Now for *third-degree price discrimination*: Suppose our town of 14.000 inhabitants consists of a large student population, say 10.000 students. Let us suppose that the students (who have a lower income) have a lower willingness to pay for the newspaper then any non-student. This means that there are 4000 non-students in the town who are each willing to pay at least 100p while no student is willing to pay a pound or more.

Because students have students identities, the monopolist can charge students a different price. Which price should he charge? He clearly must take a price below £1. If he sets, for instance, the price 80p, then from the whole population 14000-8000=6000 people would buy. This counts in all the non-students and so 2000 students would buy the newspaper. Thus we obtain the demand function for students

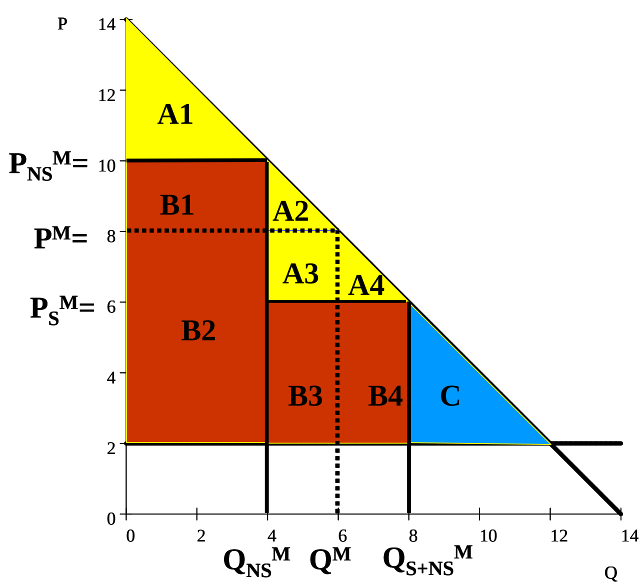
$$Q_S = 10000 - 100P$$

and the profit is $\Pi_S = 100\,(P-20)\,(100-P)$. Thus profit is maximized at $P_S^M = \frac{20+100}{2} = 6o p$ where the students' quantity demanded is $Q_S^M = 4000$. The monopoly profit is $\Pi_S^M = (60-20) \times 4000p=$ £1600. The diagram below show that consumer surplus is

$\frac{(100-60)\times 4000}{2}$p= £800 and the welfare loss is $\frac{(60-20)\times(10000-8000)}{2}$p= £800.[2]



What price should the monopolist charge to the non-students? All non-students are willing to pay at least £1 and so he should certainly not take less. At any price $P > 10$ only non-students buy and his profit is hence as given before by $(P - 20)(14000 - 100P)$. The graph of the profit function decreases in the range $100 \leq P \leq 140$ and therefore $P_{NS}^M = 100$ is the optimal price for the non-students. $Q_{NS}^M = 4000$, i.e. 4000 newspapers are sold and there is no welfare loss. The profit from non-students is $(100 - 20) \times 40$p= £3200. The last figure shows that consumer surplus for the non-students is $\frac{(140-100)\times 4000}{2}$p= £800.



The welfare effects of price discrimination.

[2]In this grqaph the price is given in multiples of 10 pennies and the quantity in multiples of 1000 papers.

Summary:

- By price discriminating the monopolist wins. His profit increases from £3,600 to £3200+£1600=£4800. (Profit before: B2+B3+A3, Profit after: B1+B2+B3+B4).
- Total output increases from 6000 to 8000 newspapers.
- Non-students lose due to the price discrimination. They pay £1 instead of 80p. Their consumer surplus decreases from £1600 to £800. (CS before: B1+A1, CS after: A1)
- The monopolist loses money on the students with a willingness to pay between 80p and £1. He loses £400 in profits on these students due to price discrimination while their consumer surplus goes up from £200 to £600. (CS before: A2, CS after: A3+A2)
- The monopolist gains a profit of £400 on the students with willingness to pay between 60p and 80p since these did not buy before (area B4). These students gain. Their consumer surplus increases from 0 to £200 (area A4).
- Price discrimination generates an *efficiency gain*. The welfare loss decreases from £1800 (area A4+B4+C) to £800 (area C only). Correspondingly, *social welfare increases* from £7200-£1800=£5400 to £7200-£800=£6400.
- *Consumer surplus decreases* from £1800 (area A1+B1+A2) to £1600 (area A1+A2+A3). Consumer surplus is reduced by area B1 and increased by areas A3+A4.