BEE1024 – Mathematics for Economists	Juliette Stephenson Amr Algarhi
Homework -Solution **Week 8**	Department of Economics University of Exeter

Problem 1 *Solve by partial integration. Check your answers by differentiation.*

$$a)\ \int xe^{\frac{x}{2}}dx$$
$$b)\ \int (3-2x)\,e^{-x}dx$$

Solution 1 a) We set

$$\begin{array}{ll} u = x & v = 2e^{\frac{x}{2}} \\ u' = 1 & v' = e^{\frac{x}{2}} \end{array}$$

(Check:

$$\frac{d}{dx}\left(2e^{\frac{x}{2}}\right) = 2\frac{de^y}{dy}\frac{d\left(\frac{x}{2}\right)}{dx} = e^y = e^{\frac{x}{2}}$$

where $y = \frac{x}{2}$) Then

$$\int xe^{\frac{x}{2}}dx = 2xe^{\frac{x}{2}} - \int 2e^{\frac{x}{2}}dx = 2xe^{\frac{x}{2}} - 4e^{\frac{x}{2}} + C.$$

Check:

$$\frac{d}{dx}\left(2xe^{\frac{x}{2}} - 4e^{\frac{x}{2}} + C\right) = xe^{\frac{x}{2}} + 2e^{\frac{x}{2}} - 2e^{\frac{x}{2}} = xe^{\frac{x}{2}}$$

b) We set

$$\begin{array}{ll} u = 3-2x & v = -e^{-x} \\ u' = -2 & v' = e^{-x} \end{array}$$

Then

$$\begin{aligned} \int (3-2x)\,e^{-x}dx &= -(3-2x)\,e^{-x} - \int (-2)\left(-e^{-x}\right)dx \\ &= -(3-2x)\,e^{-x} - 2\int e^{-x}dx = -(3-2x)\,e^{-x} + 2e^{-x} + C \\ &= 2xe^{-x} - e^{-x} + C \end{aligned}$$

Check:

$$\frac{d}{dx}\left(2xe^{-x} - e^{-x} + C\right) = -2xe^{-x} + 2e^{-x} + e^{-x} = (3-2x)\,e^{-x}$$

Problem 2 *Solve by substitution using $u = x^2 - 1$ and $u = 3x+5$ respectively. Check your answers by differentiation.*

$$a)\ \int 2xe^{x^2-1}dx$$
$$b)\ \int \frac{1}{3x+5}dx$$

Solution 2 a) $u = x^2 - 1$ implies $\frac{du}{dx} = 2x$ or $dx = \frac{1}{2x}du$. Hence

$$\int 2xe^{x^2-1}dx = \int 2xe^u \left(\frac{1}{2x}du\right) = \int e^u du = e^u + C = e^{x^2-1} + C.$$

Test:

$$\frac{d}{dx}\left(e^{x^2-1} + C\right) = \frac{d}{du}\left(e^u\right)\frac{d}{dx}\left(x^2 - 1\right) = e^u 2x = 2xe^{x^2-1}$$

b) $u = 3x + 5$ implies $du = 3dx$ or $dx = \frac{1}{3}du$. Hence

$$\int \frac{1}{3x+5}dx = \int \frac{1}{u}\left(\frac{1}{3}du\right) = \frac{1}{3}\int u^{-1}dx = \frac{1}{3}\ln|u| + C = \frac{1}{3}\ln|3x+5| + C$$

Test:

$$\begin{aligned}\frac{d}{dx}\left(\frac{1}{3}\ln|3x+5| + C\right) &= \frac{1}{3}\frac{d}{dx}\left(\ln|3x+5|\right) = \frac{1}{3}\frac{d\ln|u|}{du}\frac{d\left(3x+5\right)}{dx} \\ &= \frac{1}{3}\frac{1}{u} \times 3 = \frac{1}{3x+5}\end{aligned}$$

Problem 3 *"Multiply"*

$$\begin{bmatrix} 1 & 2 & 3 & 4 & 5 \end{bmatrix}\begin{bmatrix} 5 \\ 4 \\ 3 \\ 2 \\ 1 \end{bmatrix} =?$$

Solution 3

$$\begin{bmatrix} 1 & 2 & 3 & 4 & 5 \end{bmatrix}\begin{bmatrix} 5 \\ 4 \\ 3 \\ 2 \\ 1 \end{bmatrix} = 1 \times 5 + 2 \times 4 + 3 \times 3 + 4 \times 2 + 5 \times 1 = 35$$

Problem 4 *Consider the matrices*

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix}, \ B = \begin{bmatrix} 5 & 4 & 3 \\ 2 & 1 & 0 \end{bmatrix}, \ C = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}$$

a) Which of the following products is defined: AC', CA, CB, CC, CA', CB'?
b) Calculate BC' and CC'.

Solution 4 a)

AC'	CA	CB	CC	CA'	CB'
2×3, 3×3	3×3, 2×3	2×3, 2×3	3×3, 3×3	3×3, 3×2	3×3, 3×2
yes(2×3)	no	no	yes (3×3)	yes (3×2)	yes (3×2)

b)

$$
\begin{aligned}
BC' &= \begin{bmatrix} 5 & 4 & 3 \\ 2 & 1 & 0 \end{bmatrix} \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 8 & 4 & 8 \\ 2 & 1 & 2 \end{bmatrix} \\
CC' &= \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 2 & 0 & 2 \\ 0 & 1 & 0 \\ 2 & 0 & 2 \end{bmatrix}
\end{aligned}
$$