BEE1020 – Basic Mathematical Economics	Juliette Stephenson Amr Algarhi
Homework -Solution **Week 4**	Department of Economics University of Exeter

Exercise 1 Use the Lagrangian method to maximize the function

$$f(x, y) = xy$$

subject to the constraint

$$x + 2y \leq 200$$

Solution 1

$$\mathcal{L}(x, y) = xy + \lambda (200 - x - 2y)$$

Provided the constraint is binding the solution must satisfy the three conditions

$$\begin{aligned} \frac{\partial \mathcal{L}}{\partial x} &= y - \lambda = 0 \Leftrightarrow y = \lambda \\ \frac{\partial \mathcal{L}}{\partial y} &= x - 2\lambda = 0 \Leftrightarrow x = 2\lambda \\ 200 &= x + 2y \end{aligned}$$

The first two equations yield $x = 2y$ and hence the last gives $4y = 200$. Overall $\lambda = y = 50 \geq 0$, $x = 100$.

If the constraint were not binding, the complementarity condition would imply $\lambda = 0$ and hence $x = y = 0$. We will later see that this is a saddlepont of the function xy, not a (local) maximum.

Exercise 2 A consumer has the utility function

$$u(x, y) = 5x^2 + 6xy + y^2 + 38x + 18y$$

a) Determine the marginal utility for the two commodities. Is more always better for the consumer?

b) The consumer has a budget of £40. A unit of the first commodity costs £10 and a unit of the second £5. Write down the budget equation.

c) The consumer wants to maximize his utility subject to his budget constraint. Write down the Lagrangian for this problem.

d) Calculate the first order conditions for a critical point of the Lagrangian.

e) Assume only the budget constraint binds. Derive a linear equation to be satisfied by a critical point that does not involve the Lagrange multiplier λ for the budget constraint.

f) Use the equation from e) and the budget equation to find the constrained optimum.

Solution 2 a) marginal utilities are

$$\begin{aligned} \frac{\partial u}{\partial x} &= 10x + 6y + 38 \\ \frac{\partial u}{\partial y} &= 6x + 2y + 18 \end{aligned}$$

These are positive when x and y are positive.

b) budget constraint:

$$10x + 5y \leq 40$$

budget equation:

$$10x + 5y = 40$$

c) Lagrangian:

$$\begin{aligned}
\mathcal{L}(x, y) &= u(x, y) + \lambda[40 - 10x - 5y] + \lambda_2 x + \lambda_3 x \\
&= 5x^2 + 6xy + y^2 + 38x + 18y + \lambda[40 - 10x - 5y] + \lambda_2 x + \lambda_3 x
\end{aligned}$$

d) conditions for critical point (assuming $\lambda_2 = \lambda_3 = 0$)

$$\begin{aligned}
\frac{\partial \mathcal{L}}{\partial x} &= 10x + 6y + 38 - 10\lambda = 0 \\
\frac{\partial \mathcal{L}}{\partial y} &= 6x + 2y + 18 - 5\lambda = 0
\end{aligned}$$

e) Eliminate λ: Divide first equation by 2 and then subtract second one.

$$\begin{array}{cccccc}
5x & +3y & +19 & -5\lambda & = & 0 \\
6x & +2y & +18 & -5\lambda & = & 0 \\
-x & +y & +1 & & = & 0
\end{array}$$

f) solve simultaneous system

$$\begin{aligned}
-x + y + 1 &= 0 \text{ from e)} \\
10x + 5y &= 40 \text{ budget equation from b)}
\end{aligned}$$

division of the budget equation by 5 yields

$$\begin{array}{cccccc}
-x & +y & +1 & = & 0 & \\
2x & +y & & = & 8 & |- \\
\hline
-3x & & +1 & = & -8 & \\
-3x & & & = & -9 &
\end{array}$$

The candidate for the maximum is $x = 3$, $y = x - 1 = 2$.