BEE1024 – Mathematics for Economists	Juliette Stephenson Amr Algarhi
Homework -Solutions **Week 3**	Department of Economics University of Exeter

You must submit your solutions by 5pm at the reception. Please do not forget to write *name* and *tutorial group* on your answer sheet.

Exercise 1 Calculate the partial derivatives of

$$z = (2 - x - y)\, x + \left(5 + 2x - 3y^2\right) y - 3x + 2y^2$$

Solution 1

$$\begin{aligned}\frac{\partial z}{\partial x} &= (2 - x - y) - x + 2y - 3 = -1 - 2x + y \\ \frac{\partial z}{\partial y} &= -x + 5 + 2x - 3y^2 - 6y^2 + 4y = x + 5 - 9y^2 + 4y\end{aligned}$$

Exercise 2 Find the critical point of the function

$$z = f(x, y) = 50x^2 + 18y^2 + 24y + 8$$

Solution 2 The critical point is the solution to the simultaneous system of equations

$$\frac{\partial z}{\partial x} = 100x = 0 \text{ and } \frac{\partial z}{\partial y} = 36y + 24 = 0$$

It is hence given by $x^* = 0$, $y^* = -2/3$. (Work out the details!)

Exercise 3 A dairy produces whole milk and skim milk in quantities x and y gallons, respectively. Suppose that the price of whole milk is $p(x) = 20 - 5x$ and that the price of skim milk is $q(y) = 4 - 2y$ and assume that $C(x, y) = 2xy + 4$ is the total (!) joint-cost function of the commodities. What should x and y be to maximize profit, assuming that the first order conditions yield a maximum?

Solution 3 The revenue from selling whole milk is

$$xp(x) = x(20 - 5x),$$

the revenue from selling skim milk is

$$yq(y) = y(4 - 2y)$$

and the total costs are $2xy + 4$. The profit is therefore

$$\Pi(x, y) = x(20 - 5x) + y(4 - 2y) - 2xy - 4$$

The conditions for a critical point are

$$\begin{aligned} \frac{\partial \Pi}{\partial x} &= 20 - 5x - 5x - 2y = 20 - 10x - 2y = 0 \\ \frac{\partial \Pi}{\partial y} &= 4 - 2y - 2y - 2x = 4 - 2x - 4y = 0 \end{aligned}$$

Dividing the second equations by 2 we obtain the simultaneous system of equations

$$\begin{aligned} 10x + 2y &= 20 \\ x + 2y &= 2 \end{aligned}$$

Subtraction yields $9x = 18$ or $x = 2$ and so we obtain from the second equation $y = 1 - \frac{1}{2}x = 1 - 1 = 0$. Thus the profit function has a critical point at $x = 2$ and at $y = 0$.