BEE1024 – Mathematics for Economists	Juliette Stephenson Amr Algarhi
Homework - Solutions **Week 2**	Department of Economics University of Exeter

Exercise 1 Let $F(K,L) = 10K^{\frac{1}{2}}L^{\frac{1}{3}}$ where $K \geq 0$ and $L \geq 0$
Find $F(1,1)$, $F(4,27)$, $F(9,\frac{1}{27})$, $F\left(3,\sqrt{2}\right)$, $F(100,1000)$ and $F(2K,2L)$

Solution 1

$$\begin{aligned}
\text{a) } F(1,1) &= 10 \\
\text{b) } F(4,27) &= 60 \\
\text{c) } F(9,\frac{1}{27}) &= 10 \\
\text{d) } F(3,\sqrt{2}) &= 10\sqrt{3}\sqrt[6]{2} \\
\text{e) } F(100,1000) &= 1000 \\
\text{f) } F(2K,2L) &= 10.2^{\frac{5}{6}}K^{\frac{1}{2}}L^{\frac{1}{3}} = 2^{\frac{5}{6}}F(K,L)
\end{aligned}$$

Exercise 2 Calculate the partial derivatives of

$$\begin{aligned}
\text{a) } z &= x^3 + 3x^2y^2 + y^3 \\
\text{b) } z &= \frac{x+y}{x^2+y^3} \\
\text{c) } z &= (x^2+y)\left(x-y^2\right) \\
\text{d) } z &= \left(x^3+y^2\right)^{0.5}
\end{aligned}$$

Solution 2

$$\begin{aligned}
\text{a) } \frac{\partial z}{\partial x} &= 3x^2 + 6xy^2 \qquad \text{a) } \frac{\partial z}{\partial y} = 6x^2y + 3y^2 \\
\text{b) } \frac{\partial z}{\partial x} &= \frac{(x^2+y^3) - 2x(x+y)}{(x^2+y^3)^2} \qquad \text{b) } \frac{\partial z}{\partial y} = \frac{(x^2+y^3) - 3y^2(x+y)}{(x^2+y^3)^2} \\
\text{c) } \frac{\partial z}{\partial x} &= \left(x^2+y\right) + 2x\left(x-y^2\right) \qquad \text{c) } \frac{\partial z}{\partial y} = -2y\left(x^2+y\right) + \left(x-y^2\right) \\
\text{d) } \frac{\partial z}{\partial x} &= \frac{3}{2}\left(x^3+y^2\right)^{-0.5}x^2 \qquad \text{d) } \frac{\partial z}{\partial y} = \left(x^3+y^2\right)^{-0.5}y
\end{aligned}$$

Exercise 3 Find all second derivatives $\frac{\partial^2 z}{\partial x^2}$, $\frac{\partial^2 z}{\partial y \partial x}$, $\frac{\partial^2 z}{\partial x \partial y}$ and $\frac{\partial^2 z}{\partial y^2}$ of

$$\begin{aligned}
\text{a) } z &= x^3y^3 \\
\text{b) } z &= x^2 + 2x - 3y + y^2
\end{aligned}$$

Solution 3 a)

$$\frac{\partial z}{\partial x} = 3x^2y^3 \qquad \frac{\partial z}{\partial y} = x^3 3y^2$$

$$\begin{bmatrix} \frac{\partial^2 z}{\partial x^2} & \frac{\partial^2 z}{\partial y \partial x} \\ \frac{\partial^2 z}{\partial x \partial y} & \frac{\partial^2 z}{\partial y^2} \end{bmatrix} = \begin{bmatrix} 6xy^3 & 9x^2y^2 \\ 9x^2y^2 & 6x^3y \end{bmatrix}$$

b)

$$\frac{\partial z}{\partial x} = 2x + 2 \qquad \frac{\partial z}{\partial y} = -3 + 2y$$

$$\begin{bmatrix} \frac{\partial^2 z}{\partial x^2} & \frac{\partial^2 z}{\partial y \partial x} \\ \frac{\partial^2 z}{\partial x \partial y} & \frac{\partial^2 z}{\partial y^2} \end{bmatrix} = \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}$$

Exercise 4 For the production function

$$Q = 10K^{0.5}L^{0.5}$$

a) Find the isoquant (level curve) for $Q = 100$ and express in explicit form (i.e. with K the subject)
b) Find the first and second derivatives; what does this tell you about the marginal rate of substitution?
c) Determine the marginal product of labour and capital.

Solution 4

$$\begin{aligned}
\text{a) } 100 &= 10K^{0.5}L^{0.5} \Longrightarrow 10 = K^{0.5}L^{0.5} \qquad K = \frac{100}{L} \\
\text{b)} \frac{dK}{dL} &= -100L^{-2} \qquad \frac{d^2K}{dL^2} = 200L^{-3} \\
MRS &= \text{convex from below, therefore decreasing MRS} \\
\frac{\partial Q}{\partial K} &= 5K^{-0.5}L^{0.5} \\
\frac{\partial Q}{\partial L} &= 5K^{0.5}L^{-0.5}
\end{aligned}$$