

BEE1024 – Mathematics for Economists	Juliette Stephenson Amr Algarhi
Class Exercises	Department of Economics
Week 9	University of Exeter

Exercise 1 Use the determinant and the adjoint of the matrix of coefficients to find the solution to the linear system of equations

$$\begin{aligned} 3x + 5y &= 2 \\ x + 2y &= 4. \end{aligned}$$

Exercise 2 For the matrix

$$A = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ -1 & 0 & 1 \end{bmatrix}$$

- find the matrix of 2×2 -minors M ,
- find the matrix of cofactors C ,
- find the adjoint matrix $\text{ad } A$,
- calculate the product $(\text{ad } A)A$
- find the inverse A^{-1} ,
- find the determinant using the Sarrus rule.

Exercise 3 a) and b) verifies a result due to Hamilton verified for a particular square matrix. Let A be the matrix

$$A = \begin{bmatrix} 1 & 2 \\ 4 & 3 \end{bmatrix}$$

- Calculate the “characteristic polynomial”

$$\det(tId_2 - A)$$

Notice that this is indeed a polynomial of the form $\alpha t^2 + \beta t + \gamma$ because t is a variable.

- With α , β and γ as just found, what matrix is

$$\alpha A^2 + \beta A + \gamma Id_2?$$

c) What are the roots of the characteristic polynomial (called the “Eigenvalues” of the matrix).

d) For each Eigenvalue t find a corresponding “Eigenvector”, i.e. a vector $\vec{v} \neq 0$ such that

$$A\vec{v} = t\vec{v}.$$