BEE1024 – Mathematics for Economists	Juliette Stephenson Amr Algarhi
Class Exercises -Solution **Week 8**	Department of Economics University of Exeter

Exercise 1 Evaluate

$$\int_{-2}^{2} \left(x^2 - 4\right) dx$$

Solution 1

$$\begin{aligned}\int_{-2}^{2} \left(x^2 - 4\right) dx &= \left(\frac{1}{3}x^3 - 4x\right)\Big|_{-2}^{2} \\ &= \left(\frac{1}{3}2^3 - 4 \times 2\right) - \left(\frac{1}{3}\left(-2\right)^3 - 4 \times \left(-2\right)\right) = -\frac{32}{3} = -10\frac{2}{3}\end{aligned}$$

Exercise 2 Use integration by part (and, in b) also integration by substitution) to find

$$\text{a)} \int xe^x dx$$

$$\text{b)} \int x\left(x+1\right)^8 dx$$

Solution 2 a) Recall $\int uv'dx = uv - \int u'vdx$

$$\begin{array}{ll} u = x & v = e^x \\ u' = 1 & v' = e^x \end{array}$$

$$\int xe^x dx = xe^x - \int 1 \times e^x dx = xe^x - e^x + C$$

b)

$$\begin{array}{ll} u = x & v = \frac{1}{9}\left(x+1\right)^9 \\ u' = 1 & v' = \left(x+1\right)^8 \end{array}$$

$$\int x\left(x+1\right)^8 dx = \frac{1}{9}x\left(x+1\right)^9 - \frac{1}{9}\int \left(x+1\right)^9 dx$$

To evaluate the last integral by substituting $u = x+1$ and hence $du = dx$, so $\int \left(x+1\right)^9 dx = \int u^9 du = \frac{1}{10}u^{10} + c$. Overall

$$\int x\left(x+1\right)^8 dx = \frac{1}{9}x\left(x+1\right)^9 - \frac{1}{90}\left(x+1\right)^{10} + C$$

Exercise 3 Use the substitution $u = 1 - x$ to find the indefinite integral

$$\text{a)} \int \frac{1}{1-x} dx$$

$$\text{b)} \int \frac{x}{1-x} dx$$

Check your answer by differentiating.

Solution 3 a) Since $u = 1 - x$, then $\frac{du}{dx} = -1$, so $du = -dx$ or $dx = -du$. Hence

$$\begin{aligned} \text{a)} \int \frac{1}{1-x} dx &= \int \frac{1}{u} (-du) = -\int \frac{1}{u} du = -\ln|u| + C = -\ln|1-x| + C \\ \frac{d}{dx}(-\ln|1-x| + C) &= -\frac{1}{1-x}(-1) = \frac{1}{1-x} \end{aligned}$$

To solve b) we use that $x = 1 - u$:

$$\begin{aligned} \text{b)} \int \frac{x}{1-x} dx &= \int \frac{x}{u}(-1)\, du = -\int \frac{1-u}{u} du = -\int \left(\frac{1}{u} - 1\right) du \\ &= -\int \frac{1}{u} du + \int du = -\ln|u| + u + C = -\ln|1-x| + 1 - x + C \end{aligned}$$

$$\frac{d}{dx}(-\ln|1-x| + 1 - x + C) = -\frac{1}{1-x}(-1) - 1 = \frac{1}{1-x} - 1 = \frac{1-(1-x)}{1-x} = \frac{x}{1-x}$$

Exercise 4 (a)"Multiply"

$$\begin{bmatrix} 1 & 0 & 4 & 3 \end{bmatrix} \begin{bmatrix} 3 \\ 2 \\ 1 \\ 0 \end{bmatrix} = ?$$

(b)Find a non-zero solution to

$$\begin{bmatrix} 3 & 5 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = 0$$

Solution 4 (a) & (b)

$$\begin{aligned} \begin{bmatrix} 1 & 0 & 4 & 3 \end{bmatrix} \begin{bmatrix} 3 \\ 2 \\ 1 \\ 0 \end{bmatrix} &= (1 \times 3) + (0 \times 2) + (4 \times 1) + (3 \times 0) = 7 \\ \begin{bmatrix} 3 & 5 \end{bmatrix} \begin{bmatrix} 5 \\ -3 \end{bmatrix} &= 0 \end{aligned}$$

Exercise 5 Consider the matrices

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix}, \; B = \begin{bmatrix} 5 & 4 & 3 \\ 2 & 1 & 0 \end{bmatrix}, \; C = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}$$

a) Which of the following products is defined: AA, AB, AC', AA', AB'?
b) Calculate AB' and AC.

Solution 5 a)

AA	AB	AC'	AA'	AB'
2×3, 2×3	2×3, 2×3	2×3, 3×3	2×3, 3×2	2×3, 3×2
no	no	yes (2×3)	yes (2×2)	yes (2×2)

In the last three cases the product is defined and yields a matrix of the types indicated.

b)

$$
\begin{aligned}
AB' &= \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix} \begin{bmatrix} 5 & 2 \\ 4 & 1 \\ 3 & 0 \end{bmatrix} \\
&= \begin{bmatrix} (1 \times 5) + (2 \times 4) + (3 \times 3) & (1 \times 2) + (2 \times 1) + (3 \times 0) \\ (4 \times 5) + (5 \times 4) + (6 \times 3) & (4 \times 2) + (5 \times 1) + (6 \times 0) \end{bmatrix} = \begin{bmatrix} 22 & 4 \\ 58 & 13 \end{bmatrix} \\
AC &= \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix} \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 4 & 2 & 4 \\ 10 & 5 & 10 \end{bmatrix}
\end{aligned}
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