BEE1020 – Basic Mathematical Economics	Juliette Stephenson Amr Algarhi
Class Exercises -Solution **Week 6**	Department of Economics University of Exeter

Exercise 1 Suppose £5000 is invested at an annual interest rate of 10%. Compute the balance after 10 years if interest is compounded (a) annually, (b) quarterly, (c) monthly, (d) continuously.

Solution 1

$$\begin{aligned} \text{a) } P &= 5000\,(1+0.1)^{10} \approx 12969 \\ \text{b) } P &= 5000\,(1+0.025)^{40} \approx 13425 \\ \text{c) } P &= 5000\left(1+\frac{1}{120}\right)^{120} \approx 13535 \\ \text{d) } P &= 5000e^{10\times 0.1} \approx 13591 \end{aligned}$$

Exercise 2 How much should be invested today at 7% compounded quaterly so that it will be worth £5000 in five years?

Solution 2

$$\begin{aligned} 5000 &= P_0\left(1+\frac{0.07}{4}\right)^{20} = 1.4148P_0 \\ P_0 &= \frac{5000}{1.4148} = 3534.1 \end{aligned}$$

£3535 must be invested.

Exercise 3 It is estimated that the population of a certain country grows exponentially ($P(t) = P_{1986}e^{r(t-1986)}$). If the population was 60 million in 1986 and 90 million in 1991, what will be the population in 2002?

Solution 3

$$\begin{aligned} 90 &= P\,(1991) = P\,(1986)\,e^{5r} = 60e^{5r} \qquad \frac{3}{2} = e^{5r} \\ 5r &= \ln\frac{3}{2} \approx 0.405,\ r \approx \frac{0.405}{5} = 0.081 \\ P\,(2002) &= 60e^{16\times 0.081} = 219.28 \end{aligned}$$

The population will have grown to 219 million.

Exercise 4 Differentiate the following functions twice:

$$\text{a) } y = e^{\frac{1}{x}} \qquad \text{b) } y = x\ln x^2 \qquad \text{c) } y = e^x \ln x$$

Solution 4 a)

$$\begin{aligned} y' &= e^{\frac{1}{x}}\left(x^{-1}\right)' = -x^{-2}e^{\frac{1}{x}} = -\frac{e^{\frac{1}{x}}}{x^2} \\ y'' &= 2x^{-3}e^{\frac{1}{x}} - x^{-2}\left(e^{\frac{1}{x}}\right)' = \left(2\frac{1}{x^3} + \frac{1}{x^4}\right)e^{\frac{1}{x}} \end{aligned}$$

b)

$$\begin{aligned} y &= 2x\ln x \\ y' &= 2\ln x + 2x \times \frac{1}{x} = 2\ln x + 2 \\ y'' &= \frac{2}{x} \end{aligned}$$

c)

$$\begin{aligned} y' &= e^x \ln x + \frac{1}{x}e^x \\ y'' &= \left(e^x \ln x + \frac{1}{x}e^x\right) - \frac{1}{x^2}e^x + \frac{1}{x}e^x = \left(\ln x + \frac{2}{x} - \frac{1}{x^2}\right)e^x \end{aligned}$$

Exercise 5 Use logarithmic differentiation to find the derivatives of the following functions:

$$\begin{aligned} &\text{a)} \quad y = \frac{(x+2)^5}{\sqrt[6]{3x-5}} \\ &\text{b)} \quad y = (x+1)^3(6-x)^2\sqrt[3]{3x+1} \\ &\text{c)} \quad y = 2^{\left(x^2\right)} \\ &\text{d)} \quad y = (2^x)^2 \end{aligned}$$

Solution 5 a)

$$\begin{aligned} \ln y(x) &= 5\ln(x+2) - \frac{1}{6}\ln(3x-5) \\ (\ln y(x))' &= 5\frac{(x+2)'}{(x+2)} - \frac{1}{6}\frac{(3x-5)'}{3x-5} \\ &= 5\frac{1}{x+2} - \frac{1}{6}\frac{3}{3x-5} = \frac{5(3x-5) - \frac{1}{2}(x+2)}{(x+2)(3x-5)} \\ &= \frac{\left(14\frac{1}{2}\right)x - 26}{(x+2)(3x-5)} \\ y'(x) &= y(x)(\ln y(x))' = \frac{\left(\left(14\frac{5}{6}\right)x - 25\frac{1}{3}\right)(x+2)^4}{\sqrt[6]{3x-5}\,(3x-5)} \end{aligned}$$

b)

$$\begin{aligned}
\ln y(x) &= 3\ln(x+1) + 2\ln(6-x) + \frac{1}{3}\ln(3x+1) \\
(\ln y(x))' &= 3\frac{(x+1)'}{x+1} + 2\frac{(6-x)'}{6-x} + \frac{1}{3}\frac{(3x+1)'}{3x+1} \\
&= \frac{3}{x+1} - \frac{2}{6-x} + \frac{1}{3x+1} \\
y'(x) &= \left[\frac{3}{x+1} - \frac{2}{6-x} + \frac{1}{3x+1}\right](x+1)^3(6-x)^2\sqrt[3]{3x+1}
\end{aligned}$$

c)

$$\begin{aligned}
\ln y(x) &= x^2 \ln 2 \\
(\ln y(x))' &= 2x\ln 2 \\
y'(x) &= y(x)(\ln y(x))' = 2x(\ln 2)\,2^{(x^2)}
\end{aligned}$$

d)

$$\begin{aligned}
\ln y(x) &= 2\ln 2^x = 2\ln e^{x\ln 2} = 2(\ln 2)x \\
(\ln y(x))' &= 2\ln 2 \\
y'(x) &= y(x)(\ln y(x))' = 2(\ln 2)(2^x)^2
\end{aligned}$$

Exercise 6 The demand function for good X is given by:

$$Q_x = 520 - 20P_x + 0.6Y + 2.9P_y$$

Find the price, income and cross-price elasticities of demand at $P_x = 10$, $Y = 700$, $P_y = 21$

Which of the following terms can be used to describe the situation: complement/substitute; necessity/luxury; elastic/inelastic demand; inferior/normal?

Solution 6

$$Q_x = 520 - 20 \times 10 + 0.6 \times 700 + 2.9 \times 21 = 800.90$$

$$\begin{aligned}
PED &= \frac{\partial Q_x}{\partial P_x} \times \frac{P_x}{Q_x} = -20 \times \frac{10}{800.90} \approx -0.25 \\
YED &= \frac{\partial Q_x}{\partial Y} \times \frac{Y}{Q_x} = +0.6 \times \frac{700}{800.90} \approx 0.524 \\
XED &= \frac{\partial Q_x}{\partial P_y} \times \frac{P_y}{Q_x} = +2.9 \times \frac{21}{800.90} \approx 0.076
\end{aligned}$$

(mild) substitute / inelastic / necessity / normal