

BEE1020 – Basic Mathematical Economics	Juliette Stephenson Amr Algarhi
Class Exercises	Department of Economics
Week 5	University of Exeter

Exercise 1 a) Find all second derivatives $\frac{\partial^2 z}{\partial x^2}$, $\frac{\partial^2 z}{\partial y \partial x}$, $\frac{\partial^2 z}{\partial y \partial x}$ and $\frac{\partial^2 z}{\partial y^2}$ of the function

$$z = f(x, y) = 2x^2 - 2xy - 16x + 5y^2 + 2y + 34$$

b) Calculate the determinant of the Hessian matrix

$$\det H = \det \begin{bmatrix} \frac{\partial^2 z}{\partial x^2} & \frac{\partial^2 z}{\partial y \partial x} \\ \frac{\partial^2 z}{\partial y \partial x} & \frac{\partial^2 z}{\partial y^2} \end{bmatrix} = \begin{vmatrix} \frac{\partial^2 z}{\partial x^2} & \frac{\partial^2 z}{\partial y \partial x} \\ \frac{\partial^2 z}{\partial y \partial x} & \frac{\partial^2 z}{\partial y^2} \end{vmatrix} = \frac{\partial^2 z}{\partial x^2} \frac{\partial^2 z}{\partial y^2} - \left(\frac{\partial^2 z}{\partial y \partial x} \right)^2$$

Is it positive or negative?

c) Is the function convex, concave or saddle shaped? Does the function have a peak, a trough or a maximum?

Exercise 2 A T-shirt shop carries two competing shirts, one endorsed by Michael Jordan and the other by Shaq O'Neal. The owner of the store can obtain both at a cost of \$2 per shirt and estimates that if Jordan shirts are sold for x dollars apiece and O'Neal shirts for y dollars apiece, consumers will buy approximately $40 - 50x + 40y$ Jordan shirts and $20 + 60x - 70y$ O'Neil shirts each day.

a) Express as functions of x and y : i) the revenue from selling Jordan shirts, ii) the revenue from selling O'Neal shirts iii) the costs for shirts and iv) the overall profit.

b) Find the critical point of the profit function.

c) How should the owner price the shirts in order to generate the largest possible profit?

d) Calculate the Hessian matrix for this problem and its determinant. Is the solution found in b) indeed an absolute maximum?

Exercise 3 Calculate the partial derivatives of

$$z = \frac{x}{y} = xy^{-1}$$

Suppose now that x and y are functions of t , i.e., $x = u(t)$, $y = v(t)$. Using the chain rule for two variables, what is $\frac{dz}{dt}$?

Exercise 4 A price-taking firm in a competitive market has the total costs $TC(Q)$. Let $\Pi(Q, P) = PQ - TC(Q)$, let $Q^*(P)$ be the optimal quantity and let $\Pi^*(P)$ be the optimal profit the firm can get when the price or output is P . Use the chain rule to calculate $\frac{d\Pi^*}{dP}$. Assuming an interior solution, simplify the expression using the first order condition for a maximum. Give an intuition for the result.