BEE1024 – Mathematics for Economists	Juliette Stephenson Amr Algarhi
Class Exercises - Solutions **Week 2**	Department of Economics University of Exeter

Exercise 1

$$z = \left(y^2 + 2x\right)\left(y^3 - y^2\right)$$

Solution 1

$$\frac{\partial z}{\partial x} = 2\left(y^3 - y^2\right) \qquad \frac{\partial z}{\partial y} = 2y\left(y^3 - y^2\right) + \left(y^2 + 2x\right)\left(3y^2 - 2y\right)$$

Exercise 2 Let $f(x, y) = x + 2y$. Find $f(0, 1)$, $f(2, -1)$, $f(a, a)$, and $f(a + h, b) - f(a, b)$.

Solution 2

$$\begin{aligned} \text{a) } f(0,1) &= 2 \\ \text{b) } f(2,-1) &= 0 \\ \text{c) } f(a,a) &= 3a \\ \text{d) } f(a+h,b) - f(a,b) &= h \end{aligned}$$

Exercise 3 Calculate the partial derivatives of

$$\begin{aligned} \text{a) } z &= 5y^5 + 4x^4y + 3x^2y^3 + 2xy^4 + 2x + 3y + 5 \\ \text{b) } z &= \frac{xy^2}{x^2y^3 + 1} \\ \text{c) } z &= \left(x^9y + 1\right)\left(xy^8 + 1\right) \end{aligned}$$

Solution 3

$$\begin{aligned} \text{a) } \frac{\partial z}{\partial x} &= 16x^3y + 6xy^3 + 2y^4 + 2 \\ \text{a) } \frac{\partial z}{\partial y} &= 25y^4 + 4x^4 + 9x^2y^2 + 8xy^3 + 3 \\ \text{b) } \frac{\partial z}{\partial x} &= \frac{y^2\left(x^2y^3 + 1\right) - xy^2\left(2xy^3\right)}{\left(x^2y^3 + 1\right)^2} = \frac{y^2 - x^2y^5}{\left(x^2y^3 + 1\right)^2} \\ \text{b) } \frac{\partial z}{\partial y} &= \frac{2xy\left(x^2y^3 + 1\right) - xy^2\left(3x^2y^2\right)}{\left(x^2y^3 + 1\right)^2} = \frac{-x^3y^4 + 2xy}{\left(x^2y^3 + 1\right)^2} \\ \text{c) } \frac{\partial z}{\partial x} &= \left(9x^8y\right)\left(xy^8 + 1\right) + \left(x^9y + 1\right)\left(y^8\right) = 10x^9y^9 + 9x^8y + y^8 \\ \text{c) } \frac{\partial z}{\partial y} &= \left(x^9\right)\left(xy^8 + 1\right) + \left(x^9y + 1\right)\left(8xy^7\right) = 9x^{10}y^8 + x^9 + 8xy^7 \end{aligned}$$

Exercise 4 Find all second derivatives $\frac{\partial^2 z}{\partial x^2}$, $\frac{\partial^2 z}{\partial y \partial x}$, $\frac{\partial^2 z}{\partial x \partial y}$ and $\frac{\partial^2 z}{\partial y^2}$ of

$$\begin{aligned} \text{a) } z &= 5x^2y + 3x^2y^2 + 5y^3 \\ \text{b) } z &= \left(x^2 + y^3\right)^5 \end{aligned}$$

Solution 4 a)

$$\frac{\partial z}{\partial x} = 10xy + 6xy^2 \qquad \frac{\partial z}{\partial y} = 5x^2 + 6x^2y + 15y^2$$

$$\begin{bmatrix} \frac{\partial^2 z}{\partial x^2} & \frac{\partial^2 z}{\partial y \partial x} \\ \frac{\partial^2 z}{\partial x \partial y} & \frac{\partial^2 z}{\partial y^2} \end{bmatrix} = \begin{bmatrix} 10y + 6y^2 & 10x + 12xy \\ 10x + 12xy & 6x^2 + 30y \end{bmatrix}$$

b)

$$\frac{\partial z}{\partial x} = 10\left(x^2 + y^3\right)^4 x \qquad \frac{\partial z}{\partial y} = 15\left(x^2 + y^3\right)^4 y^2$$

$$\begin{bmatrix} \frac{\partial^2 z}{\partial x^2} & \frac{\partial^2 z}{\partial y \partial x} \\ \frac{\partial^2 z}{\partial x \partial y} & \frac{\partial^2 z}{\partial y^2} \end{bmatrix} = \begin{bmatrix} 80\left(x^2 + y^3\right)^3 x^2 + 10\left(x^2 + y^3\right)^4 & 120\left(x^2 + y^3\right)^3 y^2 x \\ 120\left(x^2 + y^3\right)^3 y^2 x & 180\left(x^2 + y^3\right)^3 y^4 + 30\left(x^2 + y^3\right)^4 y \end{bmatrix}$$

Exercise 5 For the production function

$$Q = K^{\frac{1}{3}} L^{\frac{2}{5}} + 3K + 2L^2$$

determine the marginal product of labour and capital.

Solution 5

$$\begin{aligned} \frac{\partial Q}{\partial K} &= \frac{1}{3} K^{-\frac{2}{3}} L^{\frac{2}{5}} + 3 \\ \frac{\partial Q}{\partial L} &= \frac{2}{5} K^{\frac{1}{3}} L^{-\frac{3}{5}} + 4L \end{aligned}$$