An Experiment on Forward-vs Backward Induction: How Fairness and Levels of Reasoning Matter

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or

Return of the Daleks

EXTERMINATE!
ORGANIZATION

• Forwards- vs Backwards Induction
• The Experiment
• Fairness Theories
• Discussion
van Damme’s Game

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A

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I

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Forward Induction

• Player 1 does not play $RR$ because this yields at most 6 while $L$ yields safe 7.
• Player 2, knowing this, must choose $L$ in subgame.
• Knowing this, player 1 chooses $RL \Rightarrow$ Payoffs (9,3).
• Notice: 3 levels of reasoning
• Experiment by Forsyth et al.
Our Game

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Backward Induction

• Forward induction argument still applies and yields (9,3).
• Subgame has a clear focal point, a clear solution (fair, risk dominant).
• Backward induction based on focal point: Player 1 chooses outside option, Payoff (7,4).
• This is the equilibrium selected by the Harsanyi / Selten theory (1988)
Literature

• Brandts, Holt
• Cooper, Forsythe, DeJong, Ross
• Schotter, Weigelt, Wilson
• Van Huyck, Battalio, Beil
• Camerer, Johnson
• Huck, Müller
• Brandts, Cabrales, Charness
• Caminati, Innocenti, Ricciuti
Experimental Design
Experimental Design

• Bonn Laboratory for Experimental Economics, 1989

• Graphic Representation of Extensive Form, Computerized Instructions, payoffs not announced during instruction.

• 13 sessions typically with
  - Six subjects throughout in the role of player 1, six in the role of player 2
  - Main part: 50 rounds, random matching, all matchings equally likely.
Experimental Design

• Every player learns how often each terminal node is reached in all six parallel plays.
• Final part: Five rounds, subjects submit complete strategies, simultaneous evaluations against all six opponents (strategy method).

Variations:
• No info on parallel plays
• Order of moves in 2x2-subgame changed
Result

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<td>5%</td>
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<td>II</td>
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<tr>
<td>I</td>
<td>1%</td>
<td>3%</td>
<td>10%</td>
<td>86%</td>
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<tr>
<td>II</td>
<td>9%</td>
<td>0%</td>
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88% 7 4

L R L R

1/2 1/2
Behaviour in the 2x2-subgames

On the right:
• Player 1: 11% L; 41% L (period 1); 39% L (2-5)
• Player 2: 4% l; 24% l; 11% l;

On the left:
• Player 1: 57% L, 50% L (period 1); 67% L (2-5)
• Player 2: 18% l, 14% l; 12.5 l%

Player 1 in left large subgame:
• 4.6% choice of RR, 22% RR, 6% RR
(PLAYER 1: OUT: 88%, 53%, 80%)
Fairness

- Fehr + Schmidt, Bolton + Ockenfels, Charness + Rabin
- Players maximize a utility which depends on the monetary gains of both players
- Envy versus guilt
- Heterogenous Populations, incomplete information game
- Calibration based on ultimatum + dictator games
- Sequential Equilibrium
Our Game

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Iterated Dominance

Ultimatum game: 40-70% of all responders would reject (9,3) in favour of (0,0).

⇒ Level 1: Dominant for these players 2 to play R in the 2x2-subgame

⇒ Level 2: $7 > 9 \times 0.6$; Dominated for all players 1 to play RL.

⇒ Level 3: All players 2 must play R.

Reduction to a Dictator game!
Iterated Dominance (cont.)

⇒ Level 4: Strict, separating equilibrium:
All selfish types of player 1 choose L,
all sufficiently fair types of player 1
(20% - 40%?) choose RR.
All Types of player 2 play r.
⇒ Unique strategically stable equilibrium,
also H/S solution.

Problem: not observed, maybe
because of 4 Levels of reasoning
Further Fairness Equilibria

There cannot be a Nash equilibrium where RL is played, but possible:

- All Types of player 1 choose outside option (L) (pooling equilibrium).
- Some selfish players 2 would play l in 2x2-subgame.
- After RR: expected monetary payments at most (4,4), given 2/3 of player 2 play R which is less than (7,4) for fair player 1.

Trembling hand error: Player 1 plays RL.
Learning etc

- Pooling fairness equilibrium fits data, but conflicting with equilibrium refinement.
- Learning orthogonal to the question of player’s utility functions or stimuli.
- However, Binmore, Samuelson et al. may be needed (in addition to levels of reasoning) to explain why no convergence to more complex equilibrium occurs.
- Learning based on naive priors gives same prediction (Brandts / Holt).
- Fairness + quantal response
Literature

- Brandts, Holt
- Cooper, Forsythe, DeJong, Ross
- Schotter, Weigelt, Wilson
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Exp. on FI in battle of the sexes with outside option

• Normal vs extensive form
• Results in Cooper et al. contradict Fehr + Schmidt
• Need to introduce intentions
• Similar to Falk + Fehr + Fischbacher
• (3,9) instead of (6,6)?
Cooper DeJong Forsyth Ross (scaled)
Version 0

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L

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Falk Fehr Fischbacher I
Falk Fehr Fischbacher II

26.7% 73.3%
Conclusion

• Pooling fairness equilibrium consistent with data (not “evolutionarily” stable).
• Super fair equilibrium is more complex than forward induction solution.
• Forward induction eliminated by fairness, intuitive fairness equilibrium eliminated because of learning and too many levels of reasoning => backward induction outcome remains.
• Other experiments => fairness + intentions matter.
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Diagram:
- Nodes labeled I, II, A, B, C, D, E
- Connections labeled L, R
- Paths from A to I, II, B, C, D, E
Version 1

\begin{array}{cccccc}
  & B & C & D & E \\
I & 9 & 0 & 0 & 0 & 6 \\
II & 3 & 0 & 0 & 0 & 6 \\
  & A & L & R & L & R \\
A & 7 & 0 \\
I & L & R & II \\
II & L & R \\
\end{array}
### Version 2

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![Diagram](attachment:image.png)

- A: 7 10
  - L: I
  - R: II

- L: I
  - R: I

- L: II
  - R: II
Version 3

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A

7 4

I

II

L R

L R

L

R

L

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I

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