BEE1020 – Basic Mathematical Economics	Dieter Balkenborg, Giovanni Caggiano Axel Dreher, Ioannis Krassas
Class Exercises – Solutions **Week 16** 17-18/02/2004	Department of Economics University of Exeter

Solution 1

$$\begin{aligned}
\text{a) } P &= 5000\,(1+0.1)^{10} \approx 12969 \\
\text{b) } P &= 5000\,(1+0.025)^{40} \approx 13425 \\
\text{c) } P &= 5000\left(1+\frac{1}{120}\right)^{120} \approx 13535 \\
\text{d) } P &= 5000e^{10\times 0.1} \approx 13591
\end{aligned}$$

Solution 2

$$\begin{aligned}
5000 &= P_0\left(1+\frac{0.07}{4}\right)^{20} = 1.4148P_0 \\
P_0 &= \frac{5000}{1.4148} = 3534.1
\end{aligned}$$

£3535 must be invested.

Solution 3

$$\begin{aligned}
90 &= P\,(1991) = P\,(1986)\,e^{5r} = 60e^{5r} \qquad \frac{3}{2} = e^{5r} \\
5r &= \ln\frac{3}{2} \approx 0.405,\ r \approx \frac{0.405}{5} = 0.081 \\
P\,(2002) &= 60e^{16\times 0.081} = 219.28
\end{aligned}$$

The population will have grown to 219 million.

Solution 4 a)

$$\begin{aligned}
y' &= e^{\frac{1}{x}}\left(x^{-1}\right)' = -x^{-2}e^{\frac{1}{x}} = -\frac{e^{\frac{1}{x}}}{x^2} \\
y'' &= -2x^{-3}e^{\frac{1}{x}} - x^{-2}\left(e^{\frac{1}{x}}\right)' = \left(-2\frac{1}{x^3}+\frac{1}{x^4}\right)e^{\frac{1}{x}}
\end{aligned}$$

b)

$$\begin{aligned}
y &= 2x\ln x \\
y' &= 2\ln x + 2x\times\frac{1}{x} = 2\ln x + 2 \\
y'' &= \frac{2}{x}
\end{aligned}$$

c)

$$\begin{aligned} y' &= e^x \ln x + \frac{1}{x} e^x \\ y'' &= \left(e^x \ln x + \frac{1}{x} e^x \right) - \frac{1}{x^2} e^x + \frac{1}{x} e^x = \left(\ln x + \frac{2}{x} - \frac{1}{x^2} \right) e^x \end{aligned}$$

Solution 5 a)

$$\begin{aligned} \ln y(x) &= 5 \ln(x+2) - \frac{1}{6} \ln(3x-5) \\ (\ln y(x))' &= 5 \frac{(x+2)'}{(x+2)} - \frac{1}{6} \frac{(3x-5)'}{3x-5} \\ &= 5 \frac{1}{x+2} - \frac{1}{6} \frac{3}{3x-5} = \frac{5(3x-5) - \frac{1}{2}(x+2)}{(x+2)(3x-5)} \\ &= \frac{\left(14\frac{1}{2}\right) x - 26}{(x+2)(3x-5)} \\ y'(x) &= y(x)(\ln y(x))' = \frac{\left(\left(14\frac{5}{6}\right) x - 25\frac{1}{3}\right)(x+2)^4}{\sqrt[6]{3x-5}\,(3x-5)} \end{aligned}$$

b)

$$\begin{aligned} \ln y(x) &= 3 \ln(x+1) + 2 \ln(6-x) + \frac{1}{3} \ln(3x+1) \\ (\ln y(x))' &= 3 \frac{(x+1)'}{x+1} + 2 \frac{(6-x)'}{6-x} + \frac{1}{3} \frac{(3x+1)'}{3x+1} \\ &= \frac{3}{x+1} - \frac{2}{6-x} + \frac{1}{3x+1} \\ y'(x) &= \left[\frac{3}{x+1} - \frac{2}{6-x} + \frac{1}{3x+1} \right] (x+1)^3 (6-x)^2 \sqrt[3]{3x+1} \end{aligned}$$

c)

$$\begin{aligned} \ln y(x) &= x^2 \ln 2 \\ (\ln y(x))' &= 2x \ln 2 \\ y'(x) &= y(x)(\ln y(x))' = 2x (\ln 2)\, 2^{\left(x^2\right)} \end{aligned}$$

d)

$$\begin{aligned} \ln y(x) &= 2 \ln 2^x = 2 \ln e^{x \ln 2} = 2 (\ln 2)\, x \\ (\ln y(x))' &= 2 \ln 2 \\ y'(x) &= y(x)(\ln y(x))' = 2 (\ln 2) (2^x)^2 \end{aligned}$$